UNSTEADY AERODYNAMIC ANALYSES FOR TURBOMACHINERY AEROELASTIC PREDICTIONS

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UNSTEADY AERODYNAMIC ANALYSES

- Applications
  - Aeroelastic: blade flutter and forced vibration
  - Aeroacoustic: noise generation
  - Vibration and noise control
  - Effects of unsteadiness on performance
- Requirements
  - Accuracy/efficiency
    * Realistic operating conditions
    * Arbitrary modes of unsteady excitation
- Approaches
  - Numerical simulation/analytical modeling

ASSUMPTIONS

- Turbulence and transition can be modeled

  $\Longrightarrow$ Reynolds averaged, Navier-Stokes equations
- High Reynolds number, "attached" flow

  $\Longrightarrow$ Thin-layer Navier-Stokes equations, or Inviscid/viscid interaction analyses
- Small-amplitude unsteady excitations

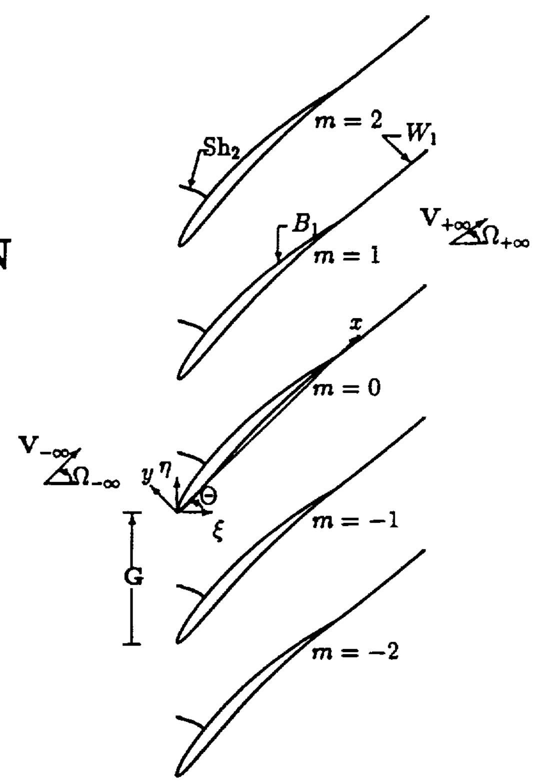
  $\Longrightarrow$ Nonlinear steady + linearized unsteady analyses
- Re $\rightarrow \infty$ $\Longrightarrow$ inviscid flow
  - Potential steady background flow $\Longrightarrow$ LINFLO
  - Uniform steady background flow $\Longrightarrow$ CLT

CONTRACT NAS3-25425

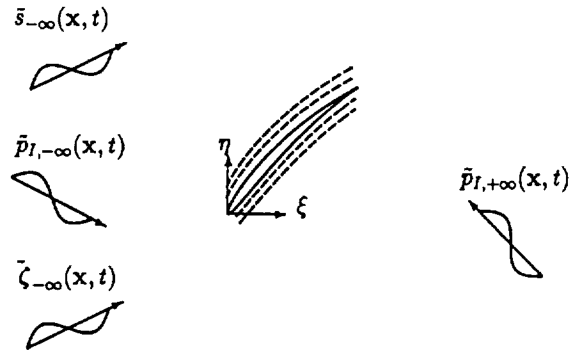
NASA Program Managers: J. Gauntner, G. Stefko

- **Linearized inviscid unsteady aerodynamic analysis: LINFLO**
- **Unsteady viscous layer analysis: UNSVIS**
- **Steady, inviscid/viscid interaction analysis: SFLOW-IVI**
- **Coupled SFLOW-IVI/LINFLO analysis**

EXAMPLE CONFIGURATION



UNSTEADY EXCITATIONS



- Far-field conditions (uniform mean flow)

$$\bar{s}(\mathbf{x}, t) = Re\{s_{-\infty} \exp[i(\boldsymbol{\kappa}_{-\infty} \cdot \mathbf{x} + \omega t)]\}, \quad \xi < \xi_-$$

$$\bar{\zeta}(\mathbf{x}, t) = \mathrm{Re}\{\zeta_{-\infty} \exp[i(\boldsymbol{\kappa}_{-\infty} \cdot \mathbf{x} + \omega t)]\}, \quad \xi < \xi_-$$

$$\bar{p}_{I,\mp\infty}(\mathbf{x}, t) = \mathrm{Re}\{p_{I,\mp\infty} \exp[-\beta_{\mp\infty}\xi + i(\boldsymbol{\kappa}_{\mp\infty} \cdot \mathbf{x} + \omega t)]\}, \quad \xi \lessgtr \xi_{\mp}$$

LINEARIZED INVISCID ANALYSES

- Linearization

$$\tilde{P}(\mathbf{x}, t) = P(\mathbf{x}) + \mathrm{Re}\{p(\mathbf{x}) \exp(i\omega t)\} + \ldots$$

$\Longrightarrow$

- Nonlinear BVP for steady background flow
- Linear variable-coefficient problem for each Fourier component of first-order unsteady flow
  - Time independent
  - Surface conditions imposed at mean surfaces
  - Analytic far-field solutions for s, ζ, and p
  - Single extended blade-passage solution domain

$$\tilde{P}(\mathbf{x} + m\mathbf{G}, t) = P(\mathbf{x}) + \mathrm{Re}\{p(\mathbf{x}) \exp[i(\omega t + m\sigma)]\} + \ldots$$

- Prescribed quantities:

$$\omega,\ \sigma,\ \mathbf{r}_{\mathrm{B}},\ s_{-\infty}\ \zeta_{-\infty},\ \text{and}\ \ p_{I,\mp\infty}$$

LINFLO

- Unsteady perturbation of a potential mean flow
- Steady flow: $\nabla \cdot (\bar{\rho}\nabla\Phi) = 0$
- Unsteady velocity decomposition: $\mathbf{v} = \nabla(\phi + \phi_*) + \mathbf{v}_R$
  - $p = -\bar{\rho}\bar{D}\phi/Dt$
  - $\nabla \cdot \mathbf{v}_R = 0$ far upstream
  - $\bar{D}\phi_*/Dt \equiv 0$; $(\nabla\phi_* + \mathbf{v}_R) \cdot \mathbf{n} \equiv 0$ on B_m & W_m
- Entropy & rotational velocity: $\mathbf{X} = \Delta\mathbf{e}_T + \Psi\mathbf{e}_N \to \mathbf{x}$ as $\xi \to -\infty$

$$s(\mathbf{x}) = s_{-\infty}\exp(i\boldsymbol{\kappa}_{-\infty} \cdot \mathbf{X})$$

$$\mathbf{v}_R(\mathbf{x}) = [\nabla(\mathbf{X} \cdot \mathcal{A}_{-\infty}) + s_{-\infty}\nabla\Phi/2] \times \exp(i\boldsymbol{\kappa}_{-\infty} \cdot \mathbf{X})$$

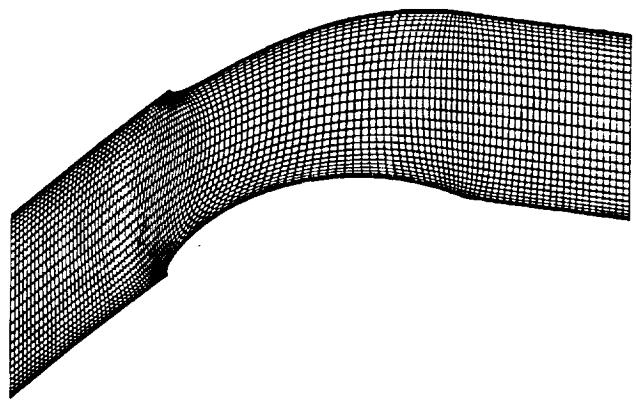
- Unsteady velocity potential

$$\bar{D}(A^{-2}\bar{D}\phi/Dt)/Dt - \bar{\rho}^{\,-1}\nabla \cdot (\bar{\rho}\nabla\phi) = \bar{\rho}^{\,-1}\nabla \cdot [\bar{\rho}\nabla\phi_*]$$

  where $\phi_* = F(\mathcal{A}_{-\infty}, \Psi)\exp(i\boldsymbol{\kappa}_{-\infty} \cdot \mathbf{X})$
- Surface conditions:
  - Blades: $\nabla\phi \cdot \mathbf{n} = f(\mathbf{r}_B)$
  - Wakes: $[\![\bar{D}\phi/Dt]\!] = 0$ and $[\![\nabla\phi]\!] \cdot \mathbf{n} = 0$
  - Shocks: $[\![\bar{\rho}\nabla\phi + \rho\nabla\Phi]\!] \cdot \mathbf{n} = f(\mathbf{r}_{Sh} \cdot \mathbf{n}, \nabla\Phi)$; $\mathbf{r}_{Sh} \cdot \mathbf{n} = -[\![\phi]\!]/[\![\Phi_n]\!]$
- Far field conditions:
  - $\phi_{I,\mp\infty}$ prescribed; $\phi_{R,\mp\infty}$ must be determined
  - Analytic far-field solutions for $\phi = \phi_I + \phi_R$

NUMERICAL SOLUTION DOMAIN

- Extended blade-passage region of finite extent in axial-flow direction



NUMERICAL APPROXIMATION

- Implicit, least-squares, finite-difference model

$$(\mathcal{L}\phi)_o \approx (L\phi)_o = q^o\phi_o + \sum_{m=1}^{m} \beta_m(\phi_m - \phi_o)$$

- Transonic differencing strategies
- Cascade, local and composite mesh solutions
- Direct solution procedure
  - Block tridiagonal system of algebraic equations for subsonic flow
  - Block pentadiagonal system for transonic flow with fitted shocks

AERODYNAMIC RESPONSE AT A BLADE SURFACE

- **Surface pressure (transonic flow):**

$$\tilde{P}(\tau_B, t) = P(\tau_B) + \mathrm{Re}\{p_B(\tau_B)\exp(i\omega t)\} + \sum_n \tilde{P}_{Sh_n}(\tau_B, t) + \ldots$$

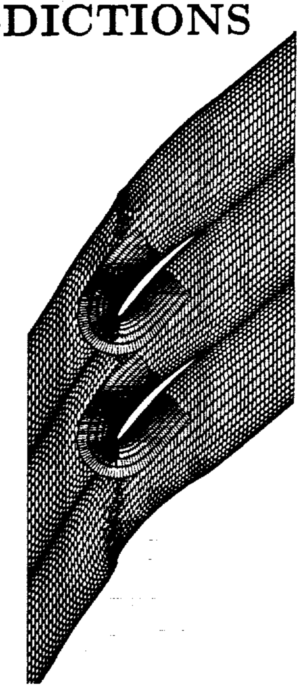
- **Blade motion:** $\mathbf{r}_B(\mathbf{x}) = \sum_{i=1}^{I} \delta_i \mathbf{R}_i(\mathbf{x})$
- **Unsteady airloads:**

$$q_i = \oint q_{i,\tau} d\tau = -\oint_B [P\frac{\partial \mathbf{r}_B}{\partial \tau} \times \mathbf{e}_z + p_B\mathbf{n} - \sum_n r_{Sh_n}[P]\mathbf{n}] \cdot \mathbf{R}_i d\tau$$

- **Work per cycle/pressure-displacement function**

$$W_C = \oint \frac{d\tilde{W}}{dt} dt = \oint_B w(\tau) d\tau = \pi \oint \mathrm{Im}\{\delta_i^* q_{i,\tau}\} d\tau = \pi \mathrm{Im}\{\sum_{i=1}^{I} \delta_i^* q_i\}$$

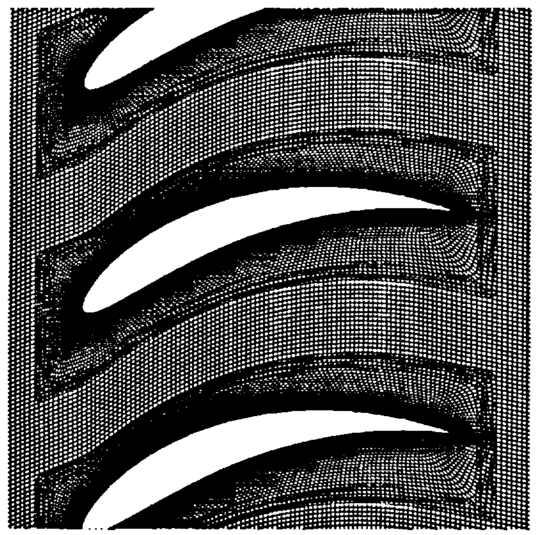
EXAMPLE RESPONSE PREDICTIONS



- **Compressor exit guide vane (EGV): $\Theta = 15\,\mathrm{deg}$, $G = 0.6$**
  - **Thick, highly-cambered NACA 0012 airfoils**
  - **Subsonic flow: $M_{-\infty} = 0.3$, $\Omega_{-\infty} = 40\,\mathrm{deg}$**
  - **Vortical excitation: $\omega = 10$, $\sigma = -2\pi$**
  - **Acoustic excitation from downstream: $\omega = 10$, $\sigma = 0$**
- **High speed compressor cascade: $\Theta = 45\,\mathrm{deg}$, $G = 1$**
  - **Cambered NACA 0006 airfoils**
  - **Subsonic flow: $M_{-\infty} = 0.7$, $\Omega_{-\infty} = 58\,\mathrm{deg}$**
  - **Transonic flow: $M_{-\infty} = 0.8$, $\Omega_{-\infty} = 55\,\mathrm{deg}$**
  - **SDOF blade motions: $\delta_i = (1,0)$, $\omega = 1$**
- **Linear/nonlinear result comparisons**
  - **NGUST analysis (Navier-Stokes)**
  - **NPHASE analysis (Euler)**

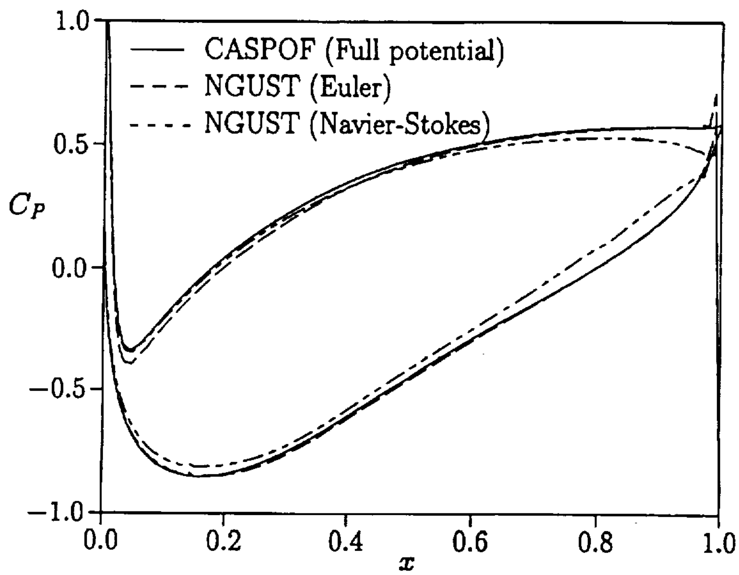
COMPRESSOR EXIT GUIDE VANE

NGUST Computational Grid



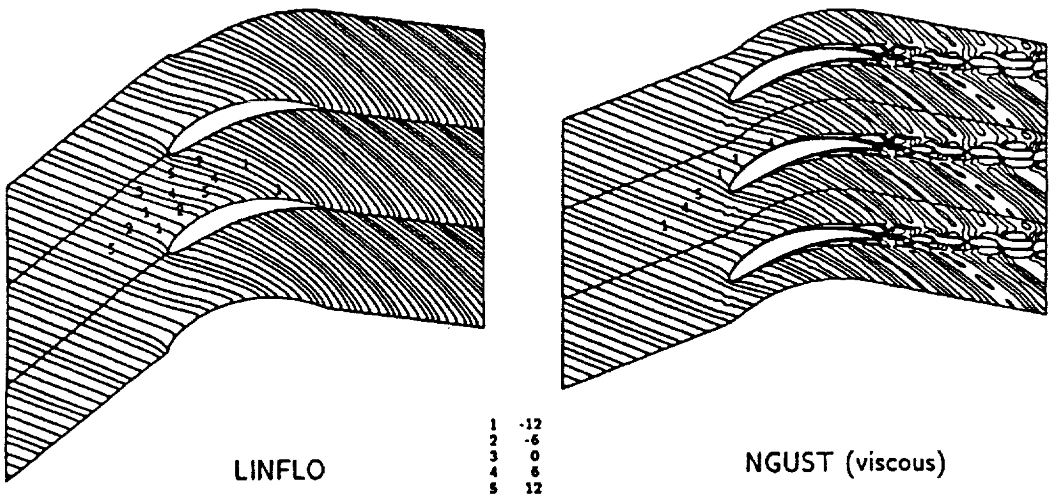
COMPRESSOR EXIT GUIDE VANE

Steady surface pressure coefficient



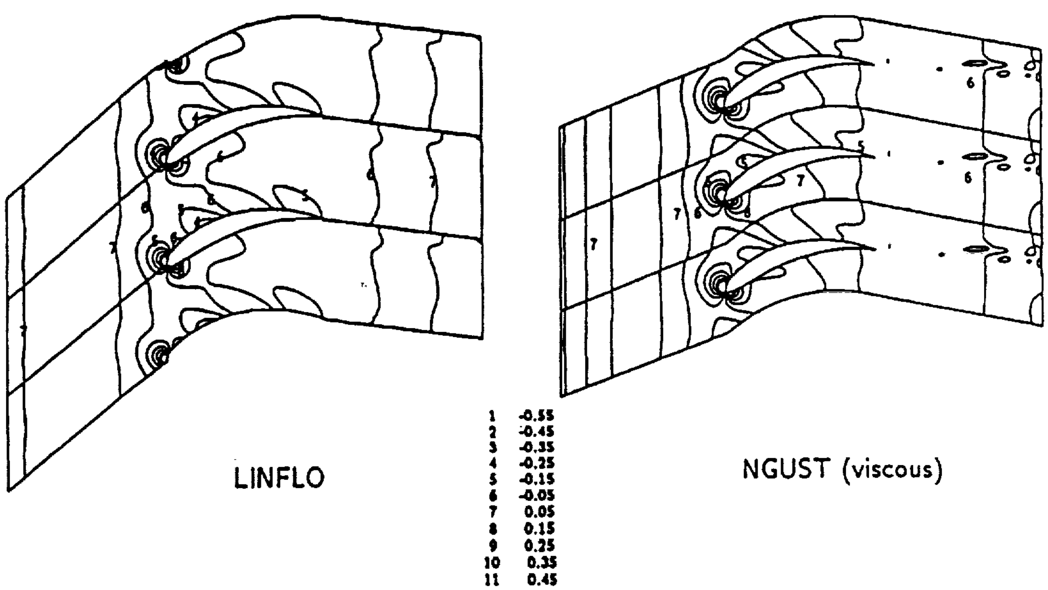
VORTICITY WAVE IN AN EGV CASCADE

Unsteady vorticity, $\vec{v}_{R,-\infty} = (0.05\bar{q}, 0)$, $\sigma = -2\pi$, $\omega = 10.0$



VORTICITY WAVE IN AN EGV CASCADE

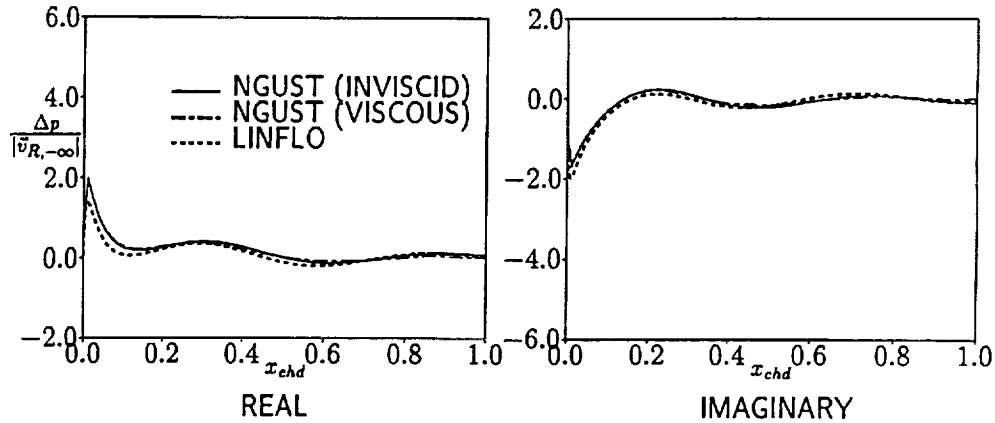
Unsteady pressure, $\vec{v}_{R,-\infty} = (0.05\bar{q}, 0)$, $\sigma = -2\pi$, $\omega = 10.0$



VORTICITY WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE

$\vec{v}_{R,-\infty} = (0.05\bar{q}, 0), \quad \sigma = -2\pi, \quad \omega = 10.0$



VORTICITY WAVE IN AN EGV CASCADE

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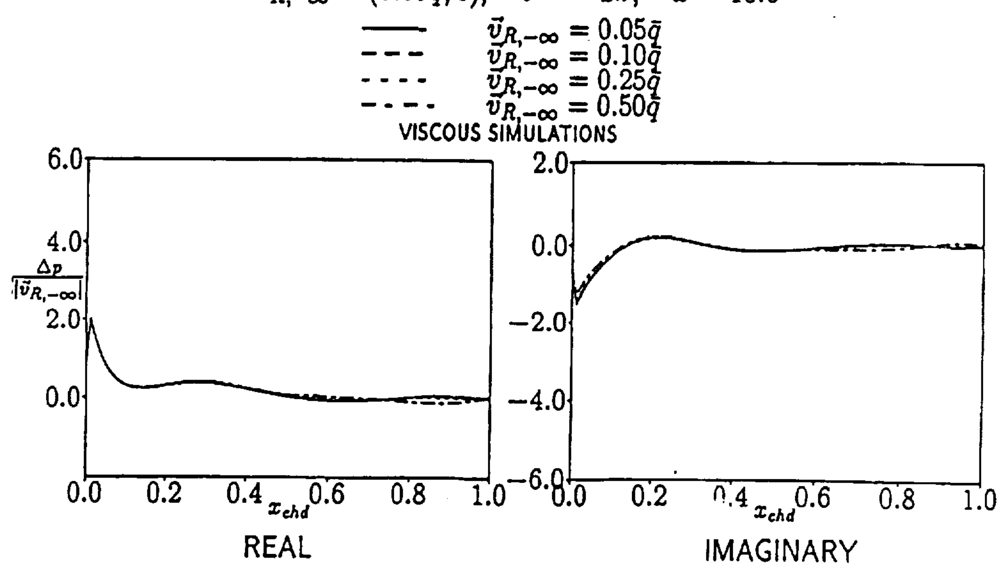
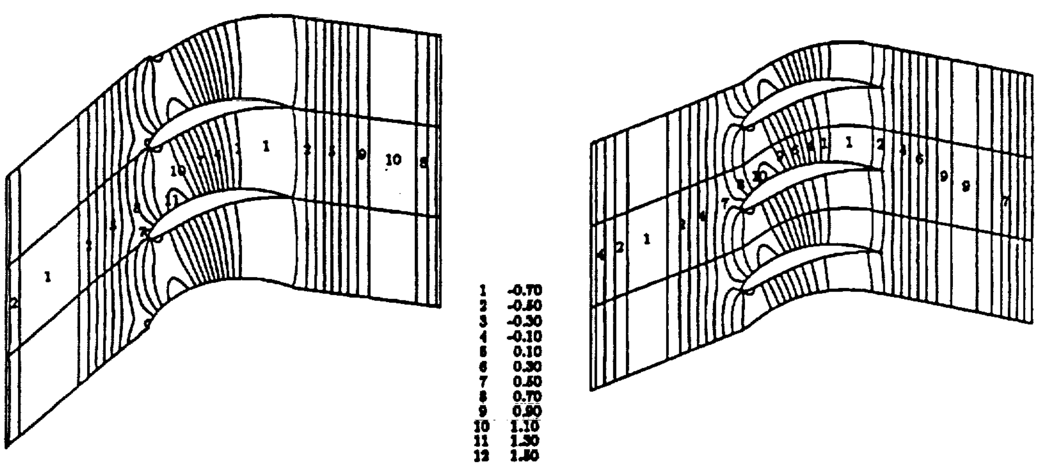


COMPRESSOR EXIT GUIDE VANE

Unsteady Pressure Response

$p_{+\infty} = (0.04, 0),\ \omega = 10.0,\ \sigma = 0.0$

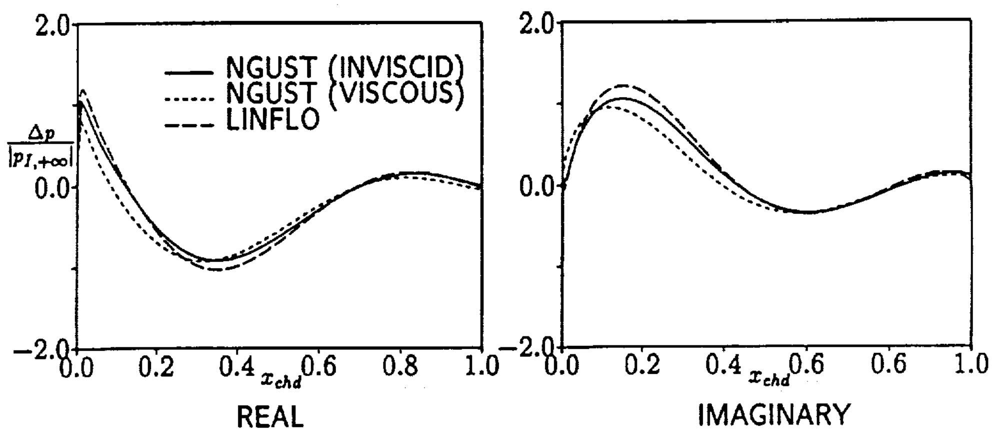
Linearized Inviscid (LINFLO) | Navier-Stokes (NGUST)



EXIT ACOUSTIC WAVE IN AN EGV CASCADE

FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE

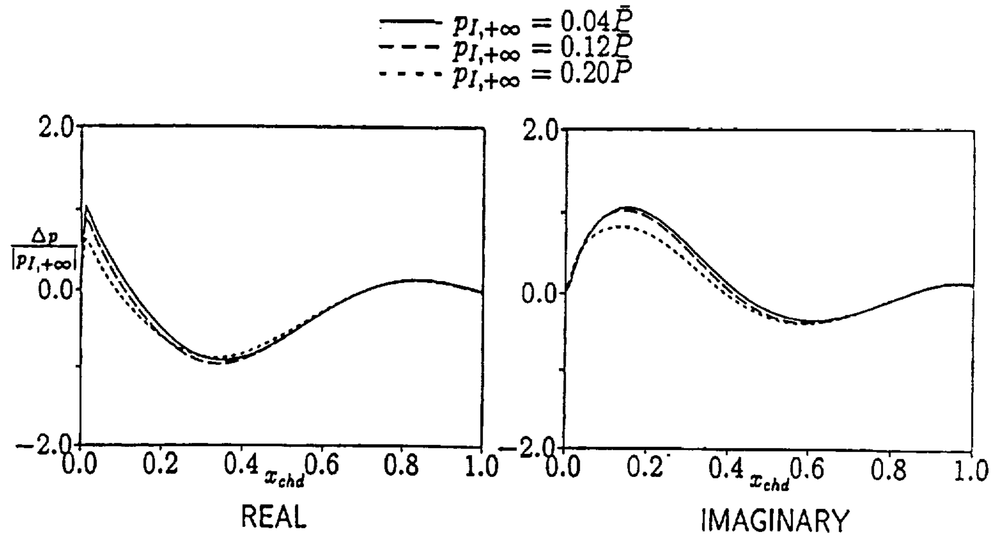
$p_{I,+\infty} = (0.04\bar{P}, 0),\quad \sigma = 0,\quad \omega = 10.0$



EXIT ACOUSTIC WAVE IN AN EGV CASCADE

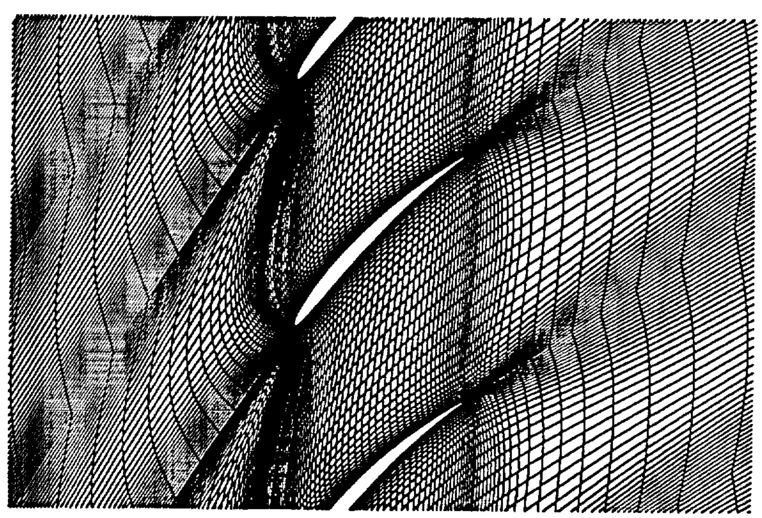
FIRST HARMONIC UNSTEADY PRESSURE DIFFERENCE

$p_{I,+\infty} = (0.04\bar{P}, 0), \quad \sigma = 0, \quad \omega = 10.0$



NACA 0006 CASCADE

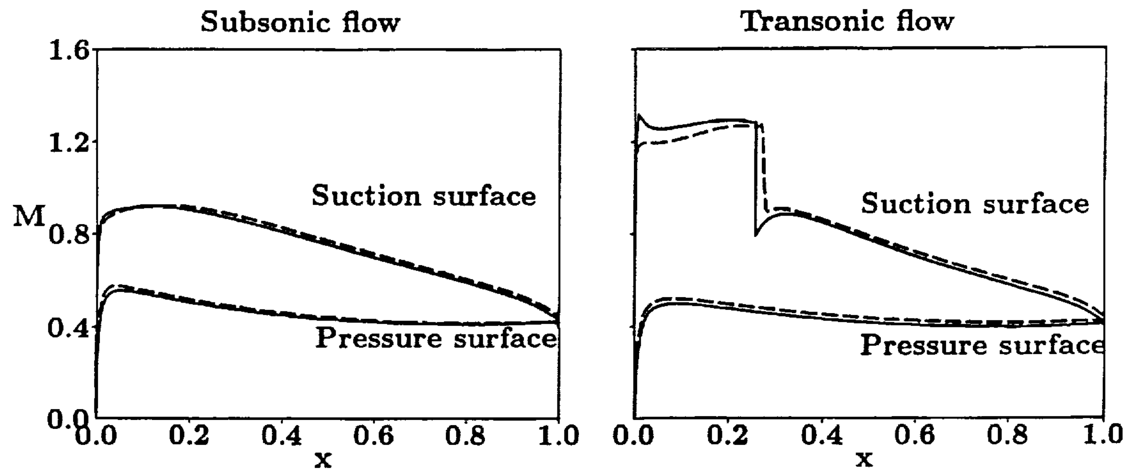
NPHASE Computational Grid



HIGH SPEED COMPRESSOR CASCADE

Surface Mach Number Distributions

—— Potential, – – –– Euler

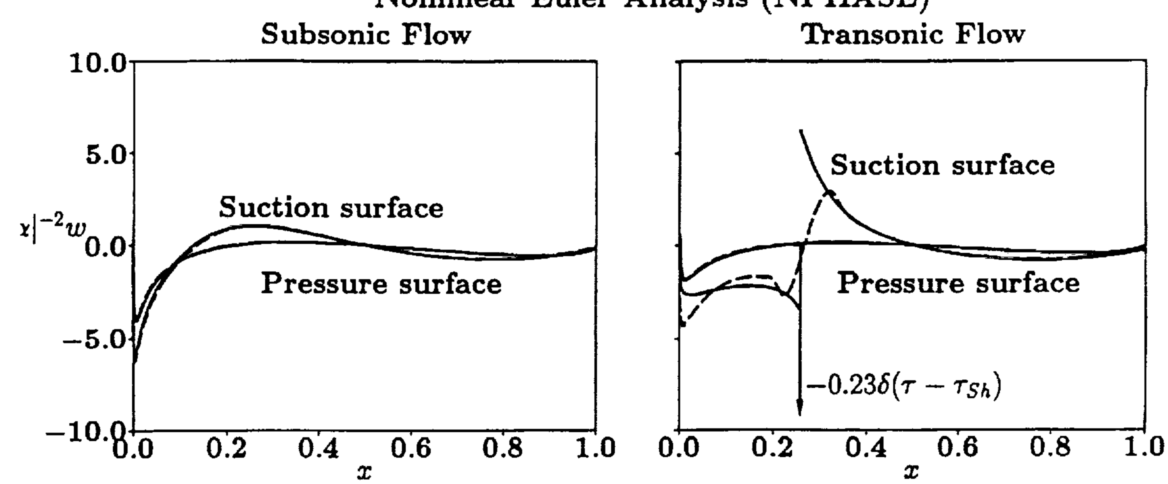


HIGH SPEED COMPRESSOR CASCADE

Pressure Displacement Function Distributions for Torsional Blade Vibrations at $\alpha = 2\,deg$, $\omega = 1$

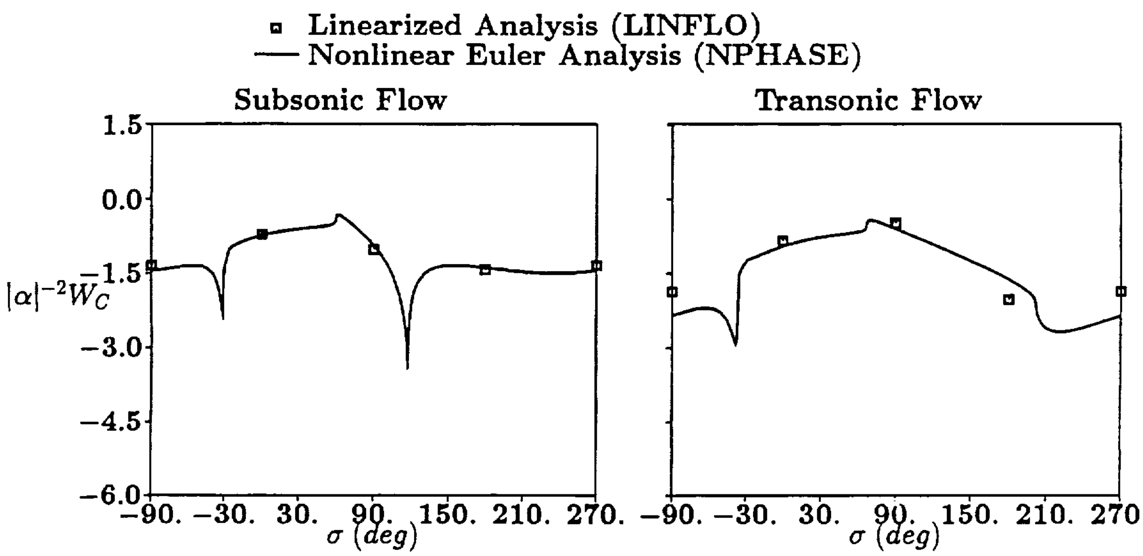
—— Linearized Analysis (LINFLO)

--- Nonlinear Euler Analysis (NPHASE)



HIGH SPEED COMPRESSOR CASCADE

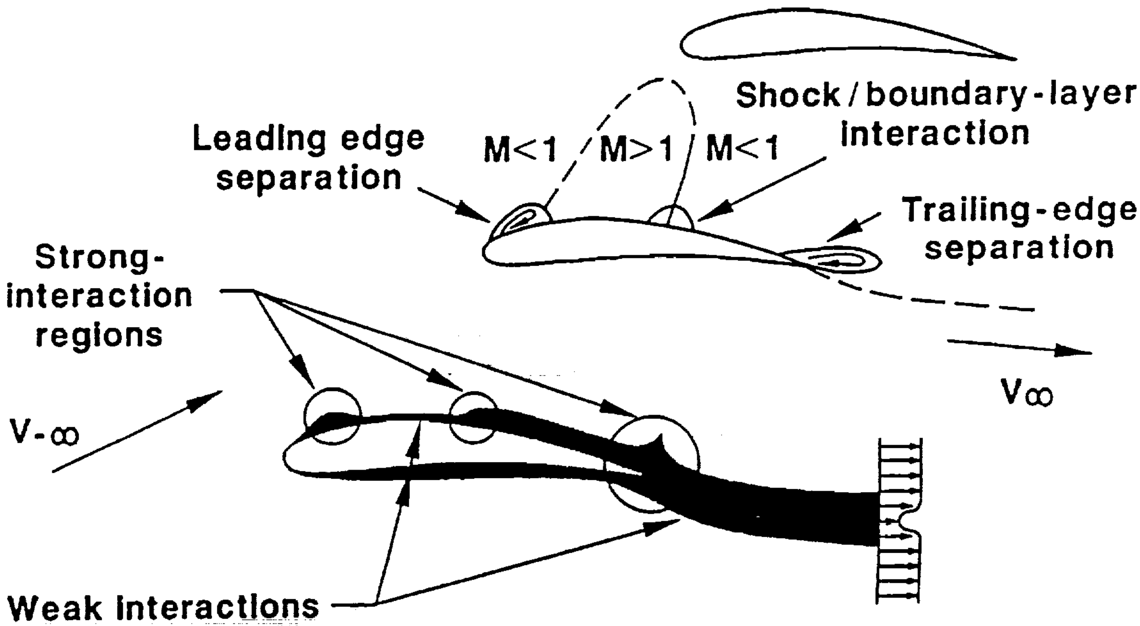
Work per Cycle versus Interblade Phase Angle for Torsional Blade Vibrations at $\alpha = 2\ deg$, $\omega = 1$



INVISCID/VISCID INTERACTION ANALYSES

- **High Reynolds Number Flow**
- **Inviscid region: Euler or potential flow equations**
  - **Surface conditions modified to account for viscous displacement effects**
- **Viscous region: Prandtl's equations**
  - **Direct solution:** $P \rightarrow \bar{\delta}$
  - **Inverse solution:** $\bar{\delta} \rightarrow P$
- **Inviscid viscid interaction law**
  - **Weak interaction $\Rightarrow$ sequential solution, pressure determined by inviscid flow**
  - **Strong interaction $\Rightarrow$ simultaneous solution, pressure determined by inviscid and viscous flows**

CASCADE FLOW WITH LOCAL REGIONS OF STRONG INTERACTION



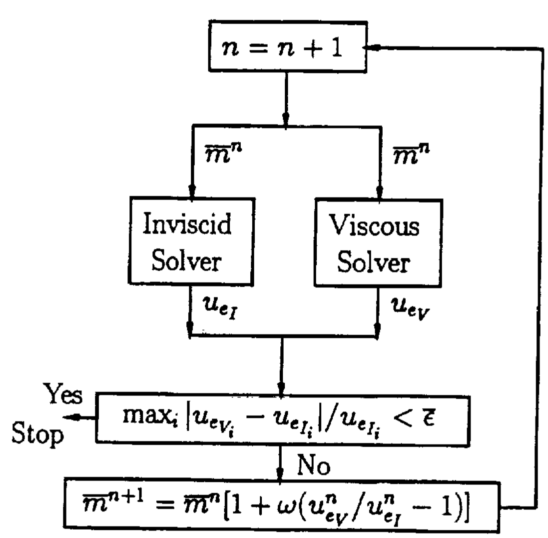
SFLOW-IVI: INVISCID REGION

- Field equation
  - $\bar{\rho}\nabla\Phi = 0$ or $A^2\nabla^2\Phi = \nabla\Phi \cdot \nabla(\nabla\Phi)^2/2$
- Surface b.c.'s account for viscous displacement effects; i.e.,
  - Blades: $\nabla\Phi \cdot \vec{n}|_S = \rho_e^{-1} d(\rho_e u_e \delta)/ds$
  - Wakes: $[\nabla\Phi] \cdot \vec{n}_+|_W = \rho_e^{-1} d(\rho_e u_e \delta_W)/ds$
- Inlet flow conditions prescribed
- Exit flow conditions determined by Kutta cond. & global mass conservation

SFLOW-IVI: VISCOUS REGION

- Classical Viscous-Layer Eqs. (Boundary layers & Wakes)
  - Weak interaction:
    specify $du_e/ds \rightarrow$ calc. δ^* (direct)
  - Strong interaction:
    specify $\overline{m} = \rho_e u_e \delta^* \rightarrow$ calc. u_e (inverse)
- Turbulence and transition
  - Algebraic eddy-viscosity model
    * Blade: Cebeci-Smith w/separation modification
    * Wake: Chang, et al
  - Instantaneous transition
- Solutions in terms of Levy Lees variables

SFLOW-IVI: COUPLING PROCEDURE

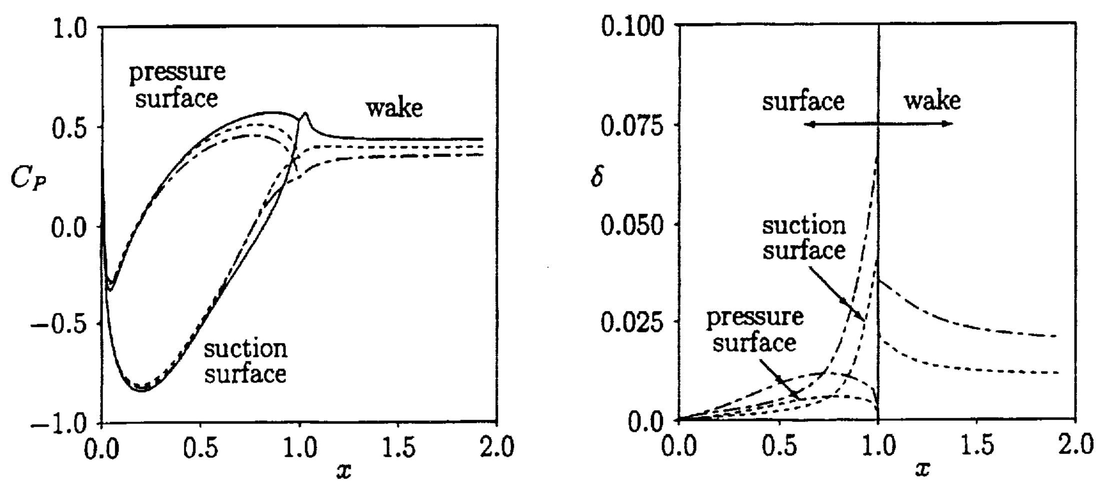


NUMERICAL RESULTS

- Two Cascade Configurations
  - Compressor exit guide vane (EGV)
  - High-speed compressor cascade
- Effect of Varying Re
- Comparison with Navier-Stokes solutions
- Incidence Angle Study (EGV)

COMPRESSOR EXIT GUIDE VANE

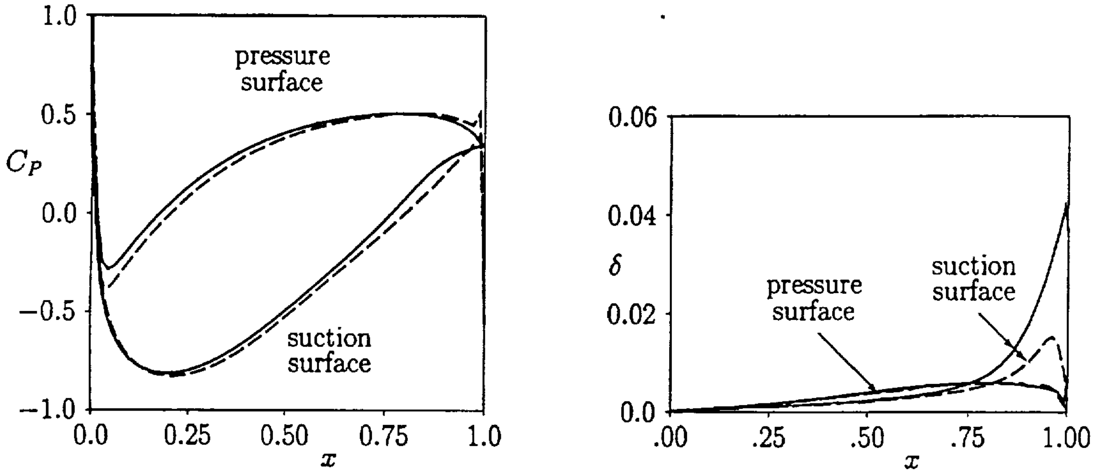
Effect of Varying *Re*: — — — $Re = 10^5$
- - - - $Re = 10^6$
——— Inviscid



COMPRESSOR EXIT GUIDE VANE

Comparison with Navier-Stokes Solution: $Re = 10^6$

—— IVI
- - - N-S



COMPRESSOR EXIT GUIDE VANE

Comparison with Navier-Stokes Solution: $Re = 10^6$

—— IVI
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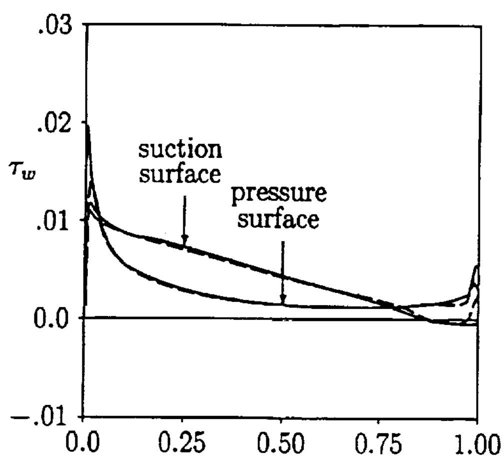
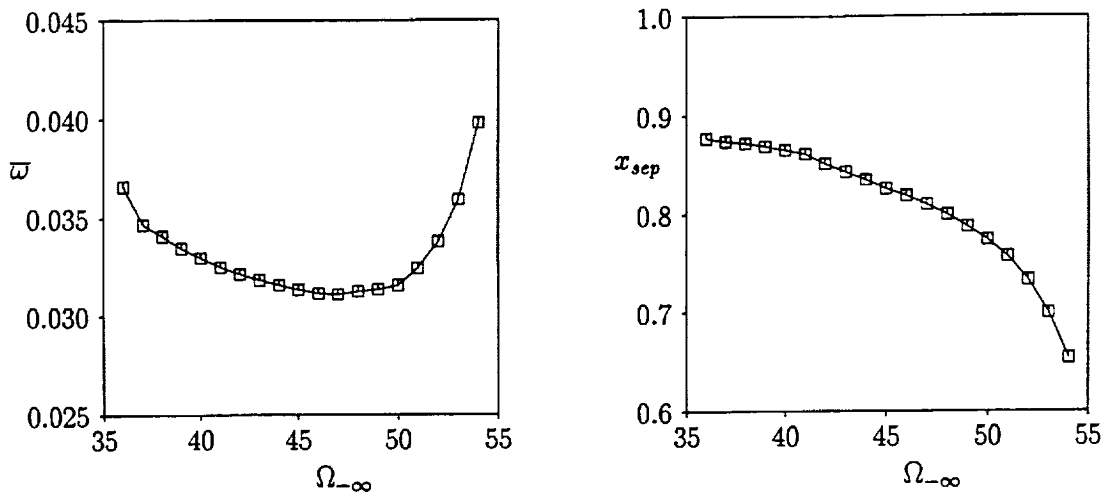


COMPRESSOR EXIT GUIDE VANE

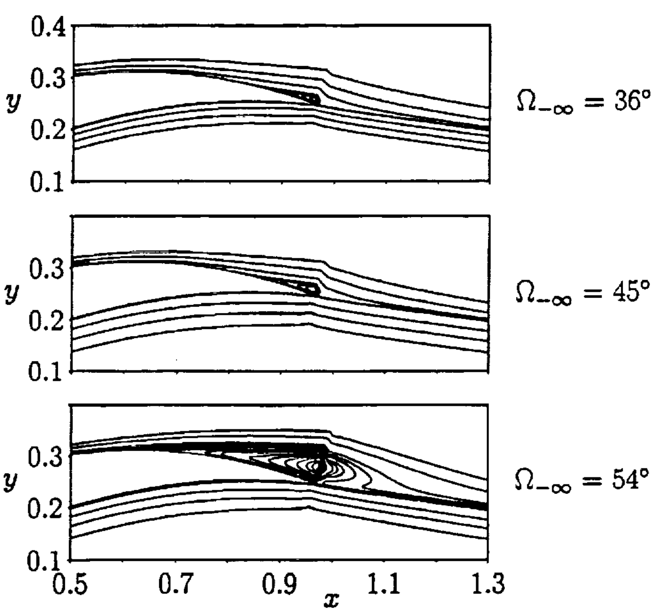
Loss Parameter, $\bar{\omega}$, & separation point location, x_{sep}, versus Inlet Flow Angle:

$Re = 10^6$, $M_{-\infty} = 0.3$



COMPRESSOR EXIT GUIDE VANE

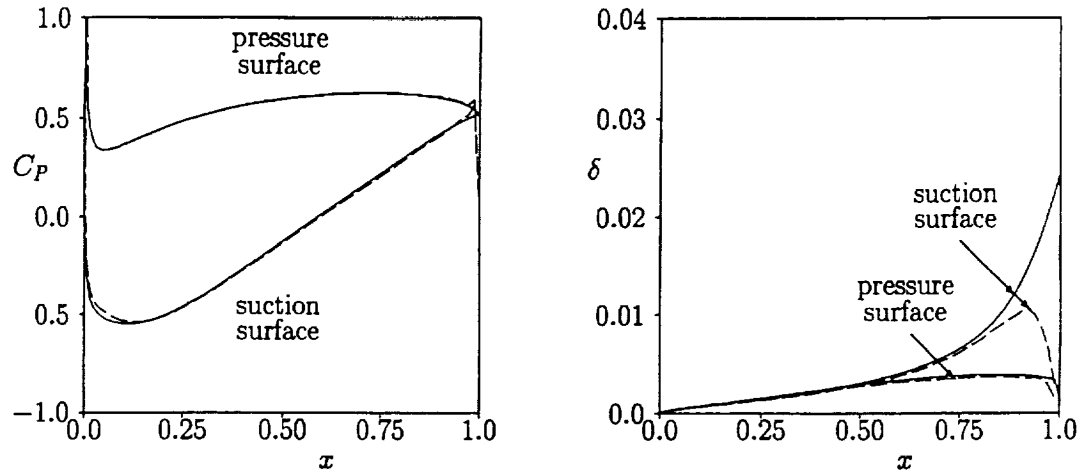
Streamlines in Trailing-Edge Region: $Re = 10^6$



HIGH SPEED COMPRESSOR CASCADE

Comparison with Navier-Stokes Solution: $Re = 10^6$

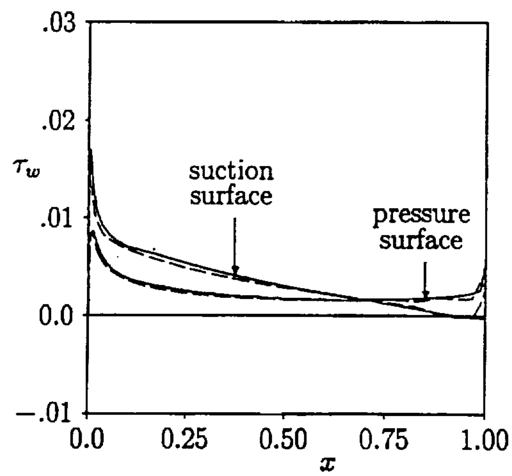
——— IVI
- - - - N-S



HIGH SPEED COMPRESSOR CASCADE

Comparison with Navier-Stokes Solution: $Re = 10^6$

——— IVI
- - - - N-S



GOAL: UNSTEADY IVI ANALYSIS FOR AEROELASTIC APPLICATIONS

- High Re unsteady cascade flows
- Inviscid region
  - Nonlinear steady (SFLOW) $\Rightarrow \Phi$
  - Linearized unsteady (LINFLO) $\Rightarrow s, \mathbf{v}_R, \phi$
    note: $\tilde{\mathbf{V}} = \nabla\Phi + Re\{[\nabla(\phi + \phi^*) + \mathbf{v}_R]\exp(i\omega t)\}$
  - Surface conditions
    - Blades: $(\tilde{\mathbf{V}} - \dot{\mathcal{R}}) \cdot \mathbf{n} = f_B\{\bar{\bar{\delta}}\}$
    - Wakes: $[\![\tilde{\mathbf{V}}]\!] \cdot \mathbf{n} = f_W\{\bar{\bar{\delta}}\}$
- Viscous region
  - Unsteady viscous layer analysis UNSVIS
  - UNSVIS is a direct, time marching solution procedure
- Inviscid/viscid coupling
  - Procedure must be developed for unsteady flows
- Issues
  - Must modify UNSVIS to deal with moving blades
  - Matching of inviscid and viscid solutions for s and $\mathbf{v}_R$ excitations
  - Need inverse unsteady viscous layer calculation
  - Inviscid/viscid coupling $\Rightarrow$ long computer run times; unless
    - $\tilde{\delta} \approx \bar{\delta} + \delta\exp(i\omega t)$, i.e., linearization, or
    - Integral boundary layer calculation

INTERMEDIATE STEP: COUPLED SFLOW-IVI/LINFLO

- Effects of strong steady interactions on unsteady pressure response
- Assumptions
  - $\bar{\tilde{\delta}}(\mathrm{x}, t) = \bar{\delta}(\mathrm{x}) + \tilde{\delta}(\mathrm{x}, t)$
  - Strong steady inviscid/viscid interaction
  - Weak unsteady interaction
- SFLOW-IVI will provide steady background flow information for LINFLO calculation
  - Unsteady surface pressure determined by linearized inviscid calculation
  - Unsteady viscous layer determined by direct solution procedure

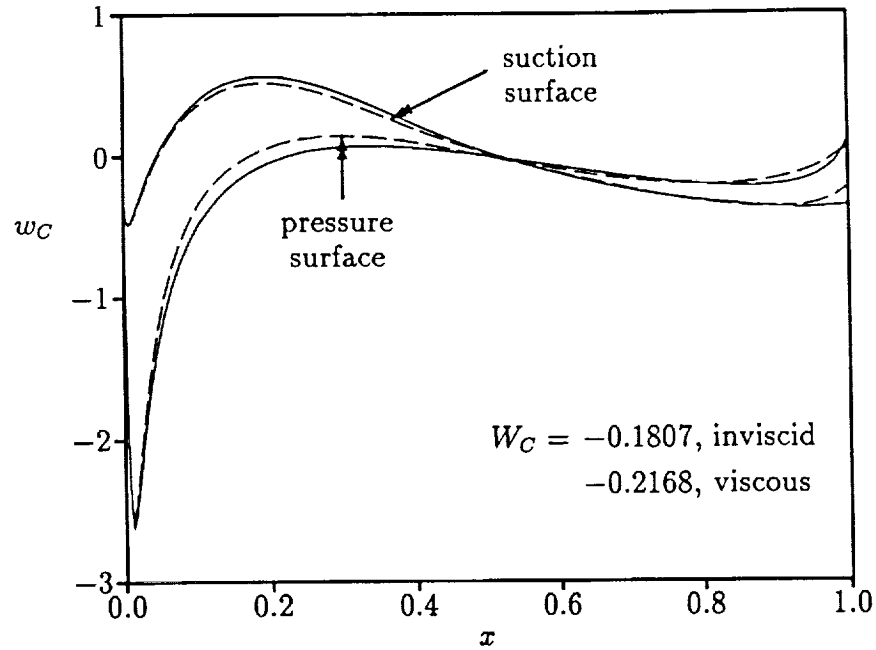


Figure 1: LINFLO results for EGV cascade undergoing torsional vibration ($\alpha = (1,0)$, $\sigma = 0$ deg, $\omega = 1$); $\Omega_{-\infty} = 40$ deg, $M_{-\infty} = 0.30$: (— — — —) inviscid; (———) viscous, $Re = 10^6$.

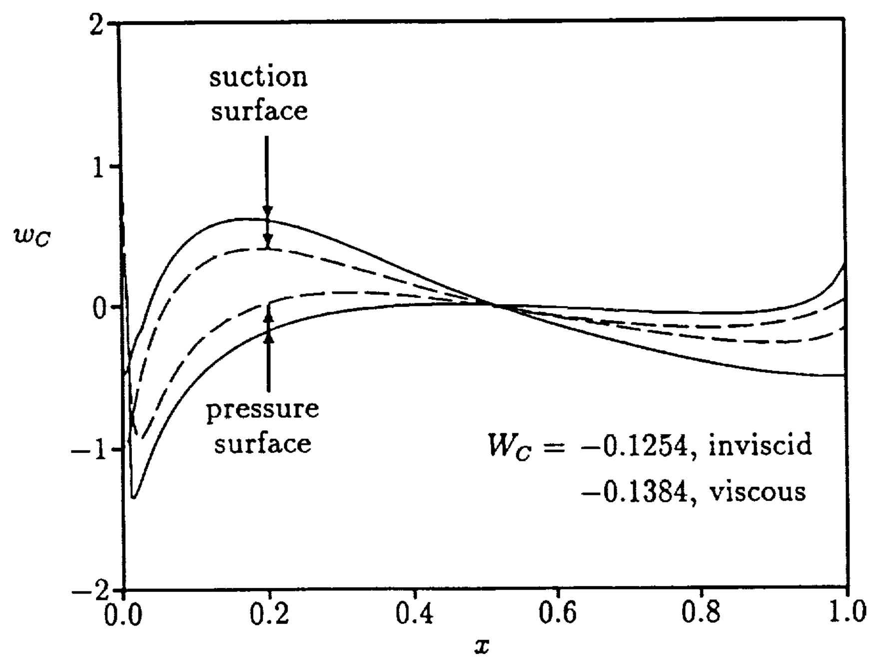


Figure 2: LINFLO results for EGV cascade undergoing torsional vibration ($\alpha = (1,0)$, $\sigma = 0$ deg, $\omega = 1$); $\Omega_{-\infty} = 54$ deg, $M_{-\infty} = 0.30$: (— — — —) inviscid; (———) viscous, $Re = 10^6$.

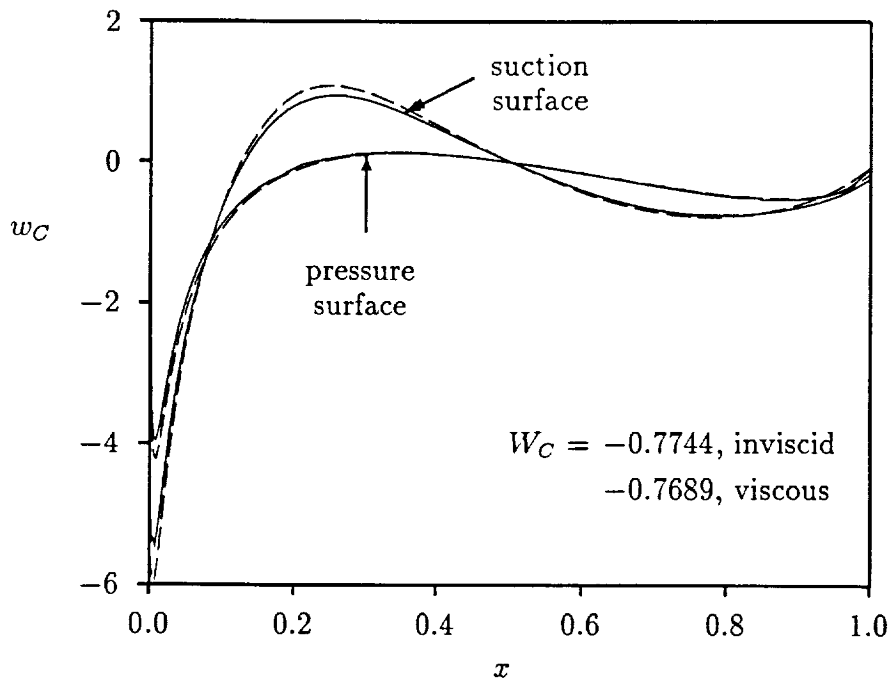


Figure 1: LINFLO results for HSC cascade undergoing torsional vibration [$\alpha = (1, 0)$, $\sigma = 0$ deg, $\omega = 1$]; $\Omega_{-\infty} = 55$ deg, $M_{-\infty} = 0.70$: (— — — —) inviscid; (———) viscous, $Re = 10^6$.

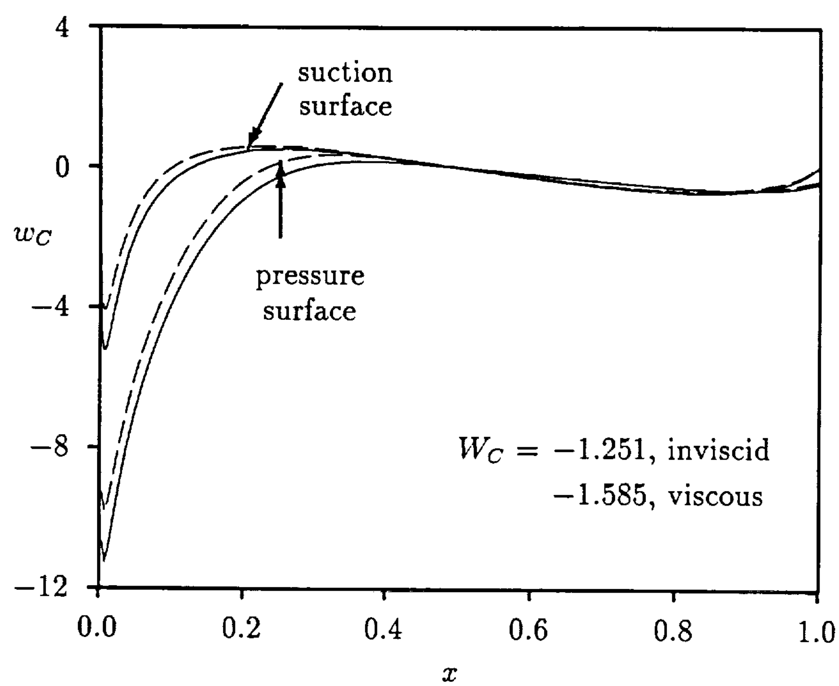


Figure 2: LINFLO results for HSC cascade undergoing torsional vibration [$\alpha = (1, 0)$, $\sigma = 180$ deg, $\omega = 1$]; $\Omega_{-\infty} = 55$ deg, $M_{-\infty} = 0.70$: (— — — —) inviscid; (———) viscous, $Re = 10^6$.

CONCLUDING REMARKS

- Linearized unsteady aerodynamic analysis: LINFLO
  - Realistic 2D flow configurations
  - Arbitrary modes and frequencies of excitation
  - Efficient prediction of unsteady pressure response
- Steady inviscid/viscid interaction analysis: SFLOW-IVI
  - 2D cascade flows
  - Local strong inviscid/viscid interactions
  - Efficient: CPU < 5 min
  - Robust: wide range of operating conditions
- Future work
  - Transonic/supersonic gust response analysis
  - SFLOW-IVI/LINFLO coupling
  - Unsteady IVI analysis