FORCED RESPONSE OF MISTUNED BLADED DISKS

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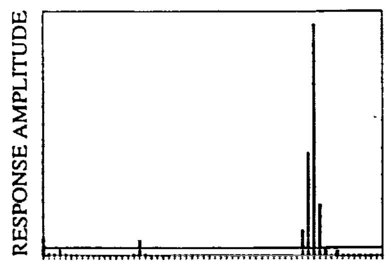
Mistuning

- Manufacturing tolerances, material non-uniformities, non-identical root fixtures, and in-service degradation result in blade-to-blade differences that destroy cyclic symmetry

- *Small* mistuning can cause large, *catastrophic* changes in blade vibrational response
  - amplitudes of vibration of some blades may increase by several hundred percent, producing "rogue" blades and HCF failure
  - free and forced responses may be highly *sensitive* to mistuning
  - tuned system predictions may be *qualitatively* in error and grossly underestimate blade forced response and overestimate fatigue life

- A credible forced response prediction system for turbomachinery vibration must take mistuning into account

An Example of Mistuning Effects on the Free Aeroelastic Response



(Pierre and Murthy, 1991)

- ☐ In tuned analysis, all of the blades vibrate with equal amplitudes and share the excitation energy.
- ☐ Real system, with 1% random blade mistuning, has most of the vibration energy confined to 4 blades out of 56.

Mistuning causes *vibration localization*

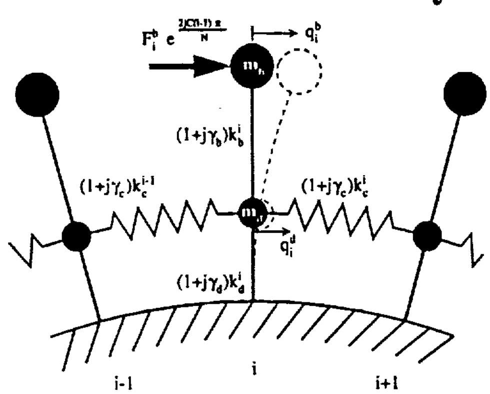
⟶ much larger amplitudes for some blades

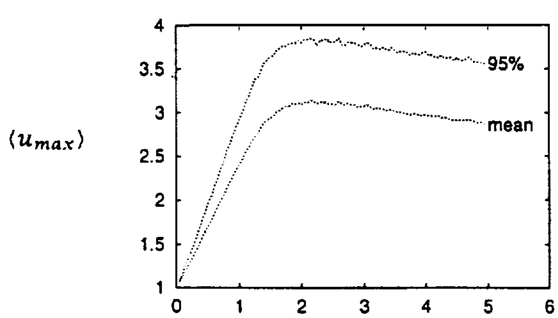
⟶ high stresses

⟶ blade fatigue

If unaccounted for, mistuning could cause cracks and catastrophic blade failures.

Effect of mistuning on forced response for a common blade assembly model





Obstacles

- Mistuned assembly analyses are very expensive. Parametric studies cannot be performed

  $\longrightarrow$ need for accurate reduced-order models

- Mistuning is random by nature

  - ☐ mistuning pattern (and sometimes mistuning strength) is typically not available
  - ☐ mistuning differs from rotor to rotor
  - ☐ mistuning that results from in-service degradation cannot be modeled deterministically

  $\longrightarrow$ calls for *statistical* and parametric tools

- Studies of mistuning by Afolabi, Bendiksen, Ewins, Griffin, Kaza, Kielb, Pierre, Sinha, Srinivasan, Mignolet, *etc.*, have led to general conclusions:
  - ☐ helps flutter
  - ☐ increases forced response amplitudes
- However — quantitatively and even qualitatively different findings regarding other issues
  - ☐ blade with largest amplitude
  - ☐ forced response amplitude increase over tuned system

- A new perspective of the mistuning problem (Bendiksen, Pierre):
  - ☐ Mistuning belongs to the broader topic of repetitive structures with periodicity-breaking irregularities
  - ☐ identification of the basic physical mechanism governing mistuning effects: sensitivity of aeroelastic eigensolution to mistuning is inverse proportional to the distance between the eigenvalues

$$\delta^2 \lambda_j \propto \frac{1}{\lambda_{oj} - \lambda_{ok}}$$

- ☐ closeness of eigenvalues is governed by the *interblade coupling* and number of blades
  - ⟶ weakly coupled assemblies are highly sensitive to mistuning (interblade coupling depends on frequency)
  - ⟶ assemblies with many blades are more sensitive
  - ⟶ mistuning effects increase with frequency (tip modes)
- ☐ highly sensitive mistuned assemblies feature localized responses

- ■ Formulation of a preliminary unifying theory of mistuning

- ■ Demonstration of the importance of considering mistuning effects at the design stage

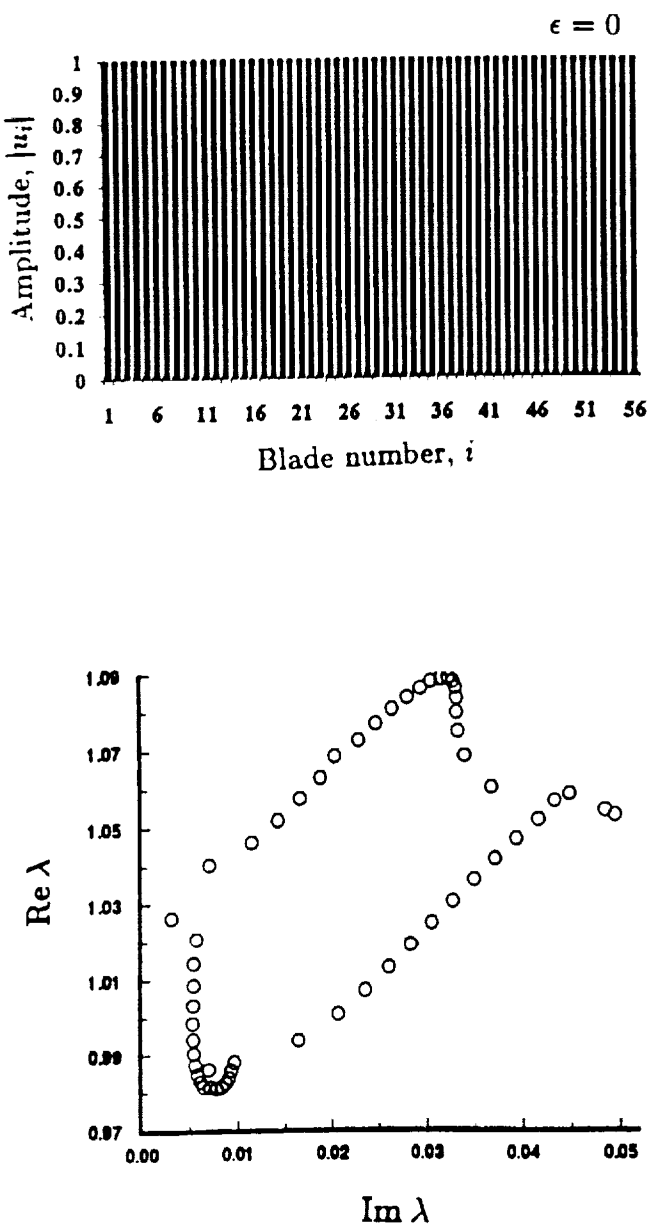
Objectives

Provide the designer with tools for predicting the forced response amplitudes of real (*i.e.*, mistuned) bladed disks. Incorporate a mistuning analysis capability into forced response prediction system (FREPS)

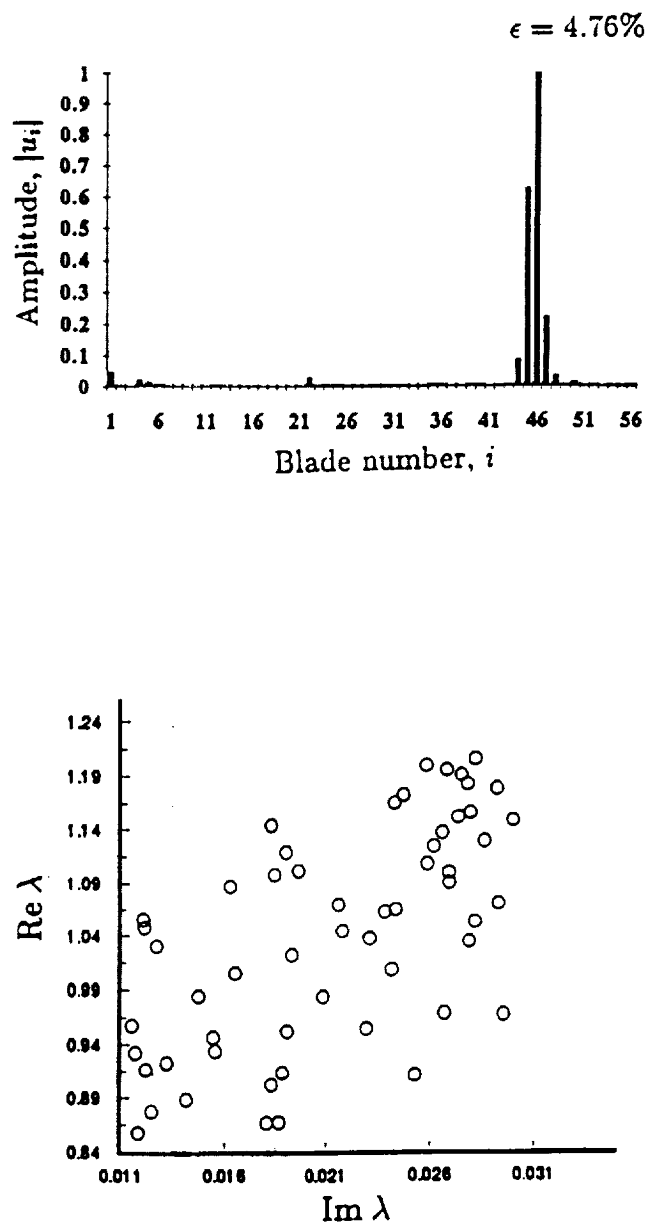
- ■ develop low-dimensional reduced-order models
- ■ evaluate the significance of mistuning effects in terms of system parameters. Identify key parameters governing sensitivity to mistuning.
- ■ predict the sensitivity of the system dynamics to blade mistuning .
- ■ determine true response amplitudes for typical mistuned bladed disks
- ■ obtain confidence intervals for amplitudes and stresses and estimates of fatigue life

NASA research program thrusts

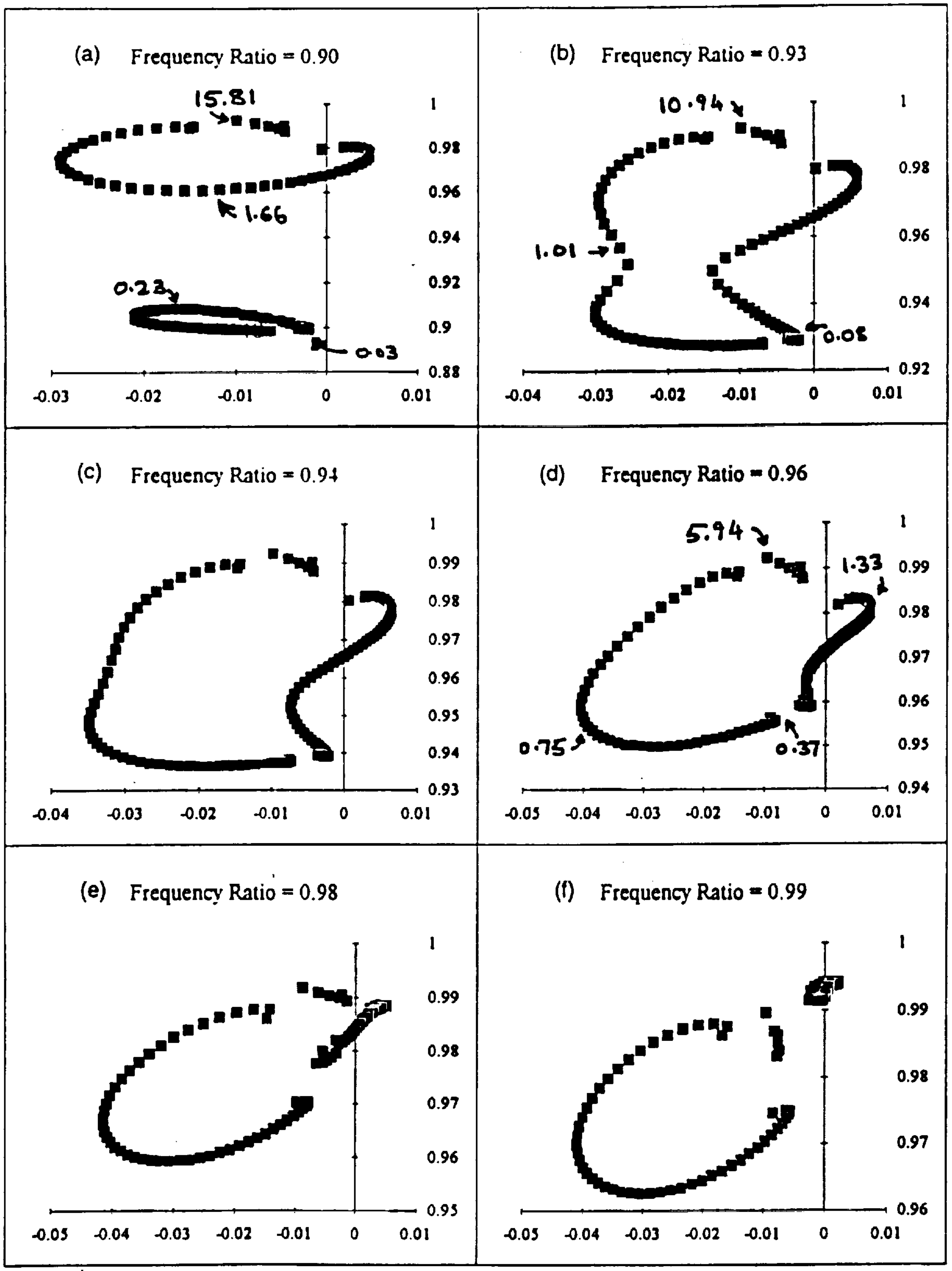
- Aeroelastic characteristics of mistuned assemblies: mode localization and root locus scattering
- Stochastic measures of sensitivity to mistuning
  - transfer matrix based
  - eigenvalue perturbation based
  - localization factors
  - composite sensitivity measure for structurally and aerodynamically coupled rotors
- Dynamics of mistuned assemblies with several component modes per blade.
  Effect of close blade modes on tuned and mistuned system dynamics.
- Design for low sensitivity to mistuning: formulation of an optimization constraint.
- Forced response of mistuned assemblies:
  - → physical mechanisms governing mistuning effects
  - → efficient statistical computational methods
- Mistuned bladed disk formulation via component mode analysis and validation of simple models

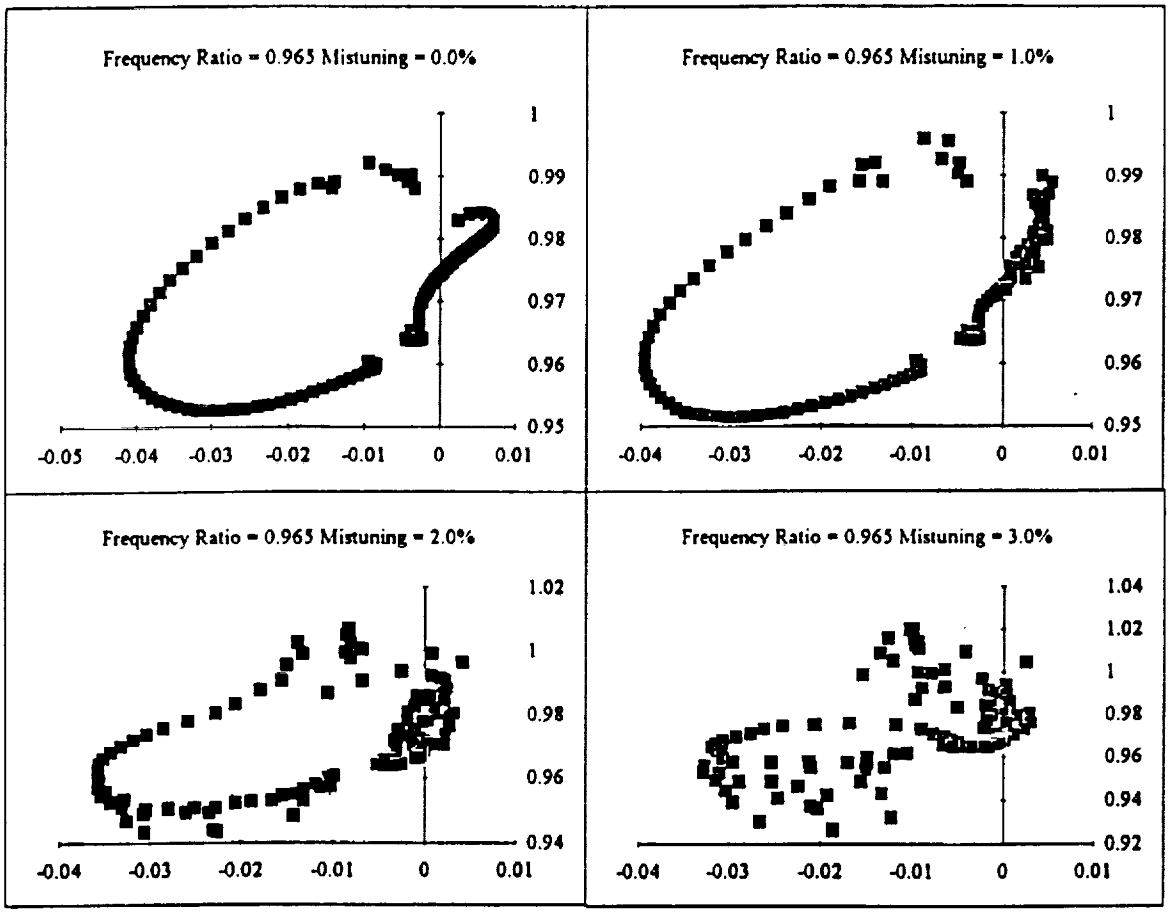


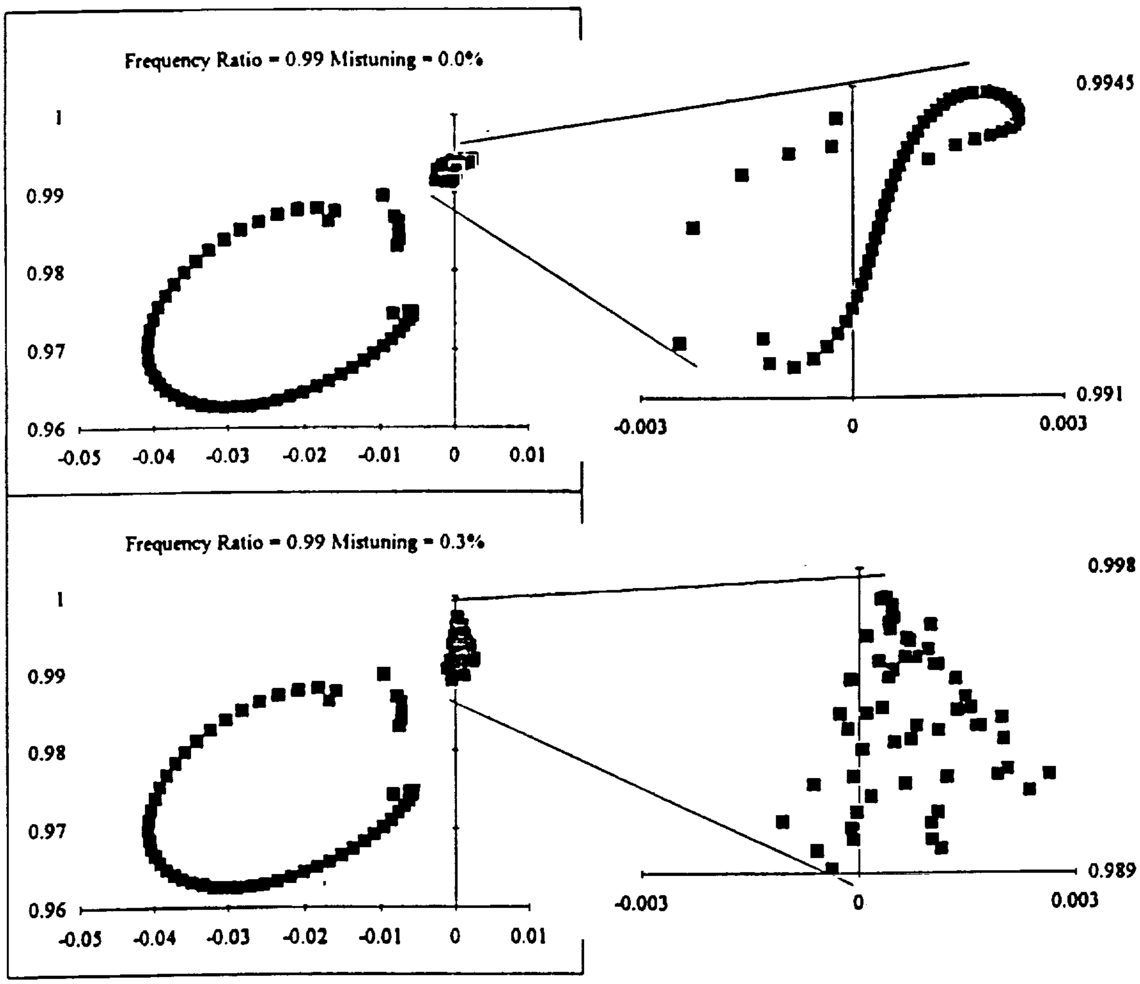
Tuned, $\epsilon = 0$



Mistuned, $\epsilon = 4.76\%$


(a) Frequency Ratio = 0.90
15.81
1.66
0.23
0.03
1
0.98
0.96
0.94
0.92
0.9
0.88
-0.03
-0.02
-0.01
0
0.01
(b) Frequency Ratio = 0.93
10.94
1.01
0.05
1
0.98
0.96
0.94
0.92
-0.04
-0.03
-0.02
-0.01
0
0.01
(c) Frequency Ratio = 0.94
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
-0.04
-0.03
-0.02
-0.01
0
0.01
(d) Frequency Ratio = 0.96
5.94
1.33
0.75
0.37
1
0.99
0.98
0.97
0.96
0.95
0.94
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
(e) Frequency Ratio = 0.98
1
0.99
0.98
0.97
0.96
0.95
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
(f) Frequency Ratio = 0.99
1
0.99
0.98
0.97
0.96
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01

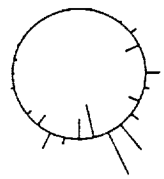

Frequency Ratio = 0.965 Mistuning = 0.0%
Frequency Ratio = 0.965 Mistuning = 1.0%
Frequency Ratio = 0.965 Mistuning = 2.0%
Frequency Ratio = 0.965 Mistuning = 3.0%



Frequency Ratio = 0.99 Mistuning = 0.0%
1
0.99
0.98
0.97
0.96
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.9945
0.991
-0.003
0
0.003
Frequency Ratio = 0.99 Mistuning = 0.3%
1
0.99
0.98
0.97
0.96
-0.05
-0.04
-0.03
-0.02
-0.01
0
0.01
0.998
0.989
-0.003
0
0.003

Practical Significance of the Localization Factor

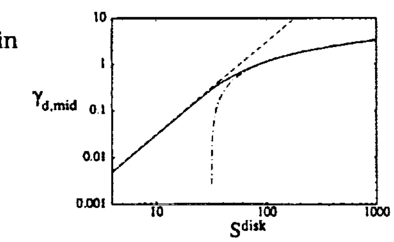
decay according to $e^{-\gamma}$



$S_{mid} \simeq 25$
$\gamma = 0.2$
90% amplitude decay
by the 11th blade

- For $\gamma = 0.1$, amplitude decays by a factor $e^{-0.1} \simeq 0.9$ from one bay to the next (on *average*)

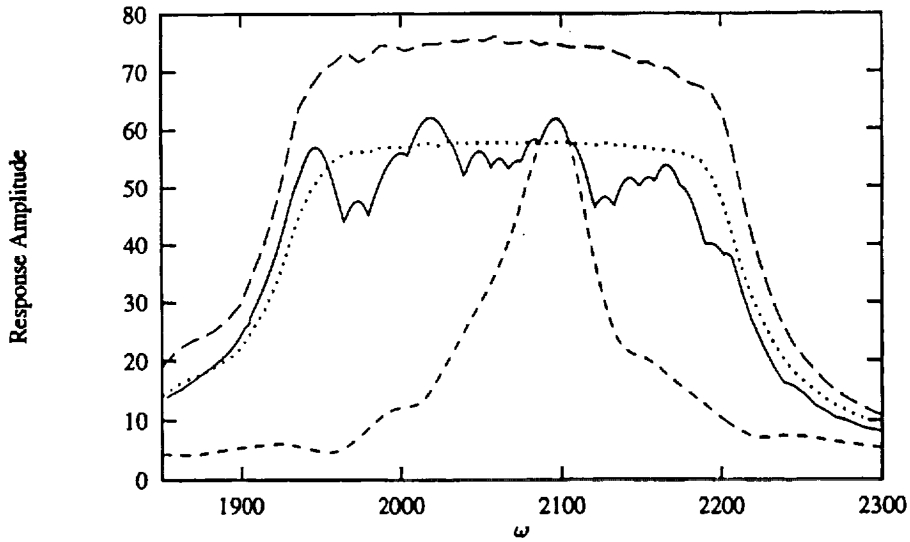
  56% of the energy is transmitted to the 3rd bay
- For $\gamma = 1.0$, average energy transmitted to next bay is 13.5% and less than 0.25% of the energy reaches the 3rd bay!
- γ is an average quantity and specific realizations of mistuned systems may exhibit different behavior.
- γ can be calculated in terms of a universal sensitivity measure for simple models.

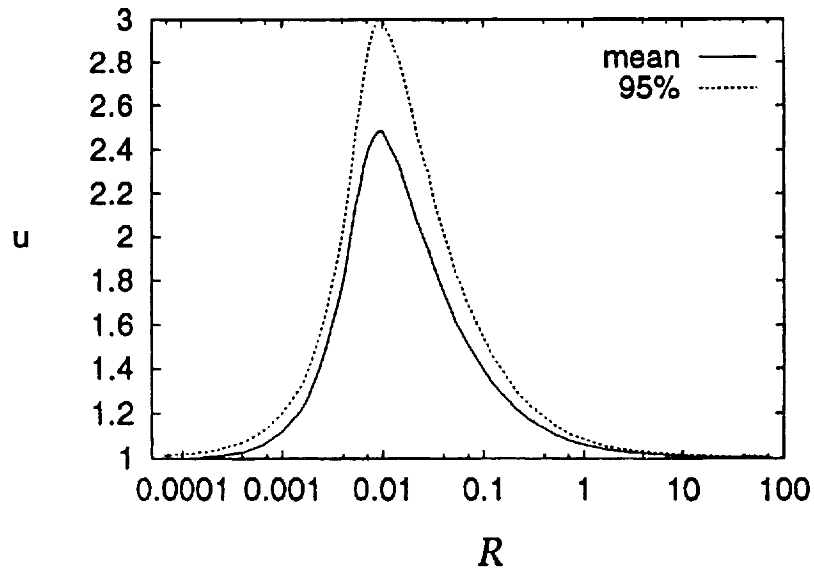


- Use in design:

  Maximum allowable localization strength $\Rightarrow \gamma \Rightarrow S \Rightarrow$ corresponding permitted regions in parameter space.

Forced Response of Mistuned Assemblies





$$u = \frac{\text{Maximum blade amplitude in mistuned system}}{\text{Blade amplitude in tuned system}}$$

Closing

- Because of its potentially catastrophic effects such as single blade failure, mistuning must be accounted for in the design and analysis of blade assemblies
- Simple and effective mistuning capability must be incorporated into FREPS
- Underlying physical mechanisms must be understood to generate proper reduced-order models

Future work:

- Forced response: develop physical understanding and associated efficient computational techniques
- Mistuning experiment: corroborate occurrence of localization and high sensitivity in nonrotating/rotating conditions
- Beneficial mistuning patterns: practical only if mistuning can be controlled