

**RADIATION/CONVECTION COUPLING IN
ROCKET MOTORS AND PLUMES**52-34
~~43777~~p. 29
1995 116994R. C. Farmer
A. J. SaladinoSECA, Inc.
Huntsville, AL**Abstract**

The three commonly used propellant systems; H₂/O₂, RP-1/O₂, and solid propellants; primarily radiate as: molecular emitters, non-scattering small particles, and scattering larger particles, respectively. Present technology has accepted the uncoupling of the radiation analysis from that of the flowfield. This approximation becomes increasingly inaccurate as one considers plumes, interior rocket chambers, and nuclear rocket propulsion devices. This study will develop a hierarchy of methods which will address radiation/convection coupling in all of the aforementioned propulsion systems.

The nature of the radiation/convection coupled problem is that the divergence of the radiative heat flux must be included in the energy equation and that the local, volume-averaged intensity of the radiation must be determined by a solution of the radiative transfer equation (RTE). The intensity is approximated by solving the RTE along several lines of sight (LOS) for each point in the flowfield. Such a procedure is extremely costly; therefore, further approximations are needed. Modified differential approximations are being developed for this purpose. It is not obvious which order of approximations are required for a given rocket motor analysis. Therefore, LOS calculations have been made for typical rocket motor operating conditions in order to select the type approximations required. The results of these radiation calculations, and the interpretation of these intensity predictions are presented herein.

The study is still in progress. The inclusion of the selected radiation model into a coupled CFD solution will be reported at a later date.

**RADIATION/CONVECTION COUPLING
IN ROCKET MOTORS & PLUMES**

R. C. Farmer

A. J. Saladino

SECA, INC.

Presentation for:

Workshop for CFD Applications in Rocket Propulsion

April 20-22, 1993

NASA/MSFC, MSFC, AL

OBJECTIVE:

Develop a radiation/convection coupled analysis of rocket motors & plumes

993

Tasks:

- 1 - Characterize radiation for various type motors**
- 2 - Develop radiation/convection coupling methodology**

PARAMETERS AFFECTING RADIATION

H₂/O₂ Propellant System

- Radiator H₂O vapor
- Requires spectral integration

HC/O₂ Propellant System

- Radiators soot, H₂O, CO₂
- Requires soot concentration prediction

SRM

- Radiators Al₂O₃ particles, H₂O, CO₂, HCl
- Requires
 - Predicting particle size distributions
 - Predicting particle temperatures
 - Specifying optical property data

Internal Surface Characteristics

- Not well known
- Assume reflectivity of 0.2

Radiation Analyses

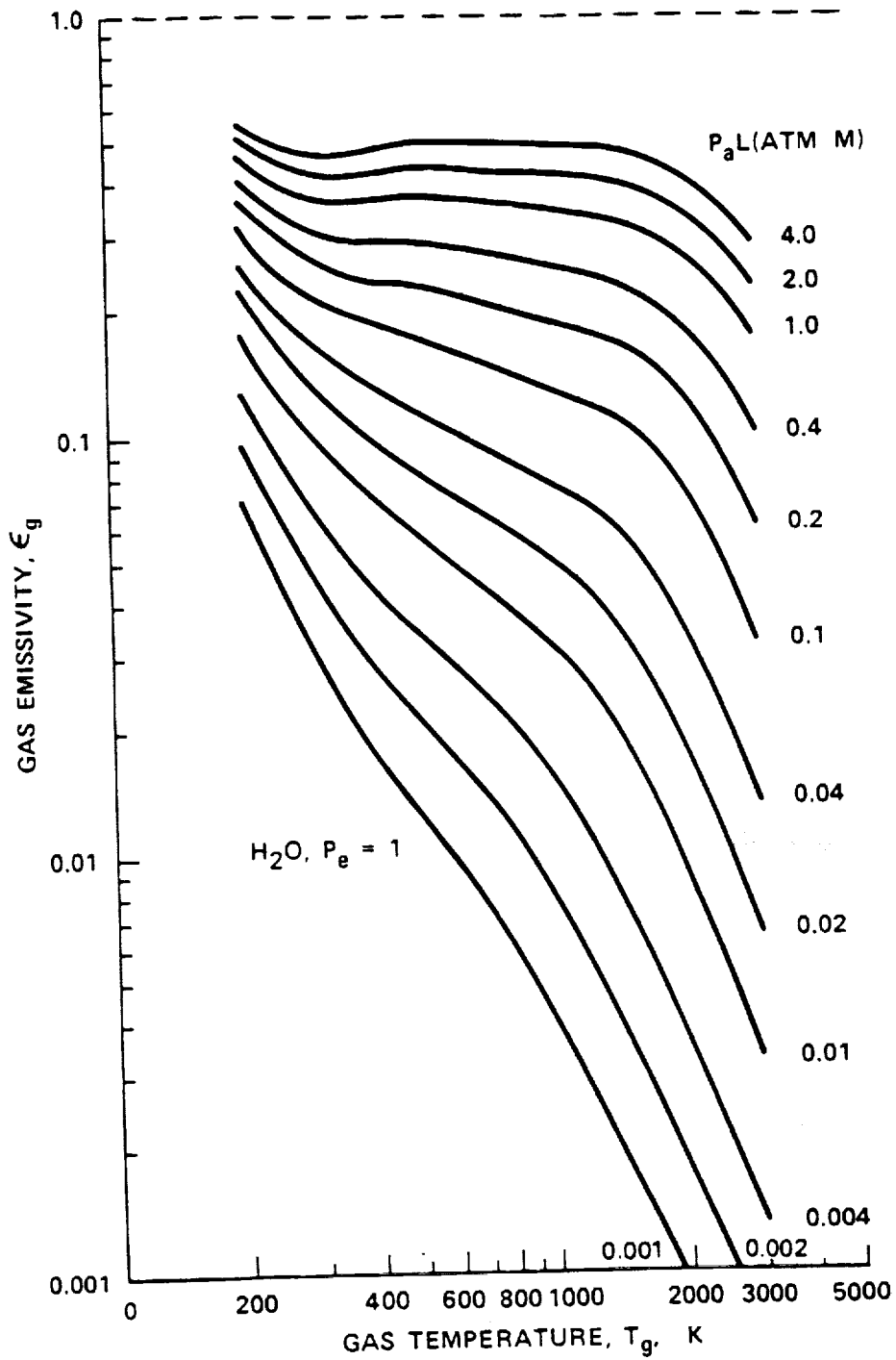
Molecular Radiators

- Narrow Band Models
- Wide Band Models
- Mean of all Bands

Particle Radiators

- Small particles - like soot emit & absorb but do not scatter
- large particles - like Al_2O_3 emit, absorb, & scatter non-isotropically.

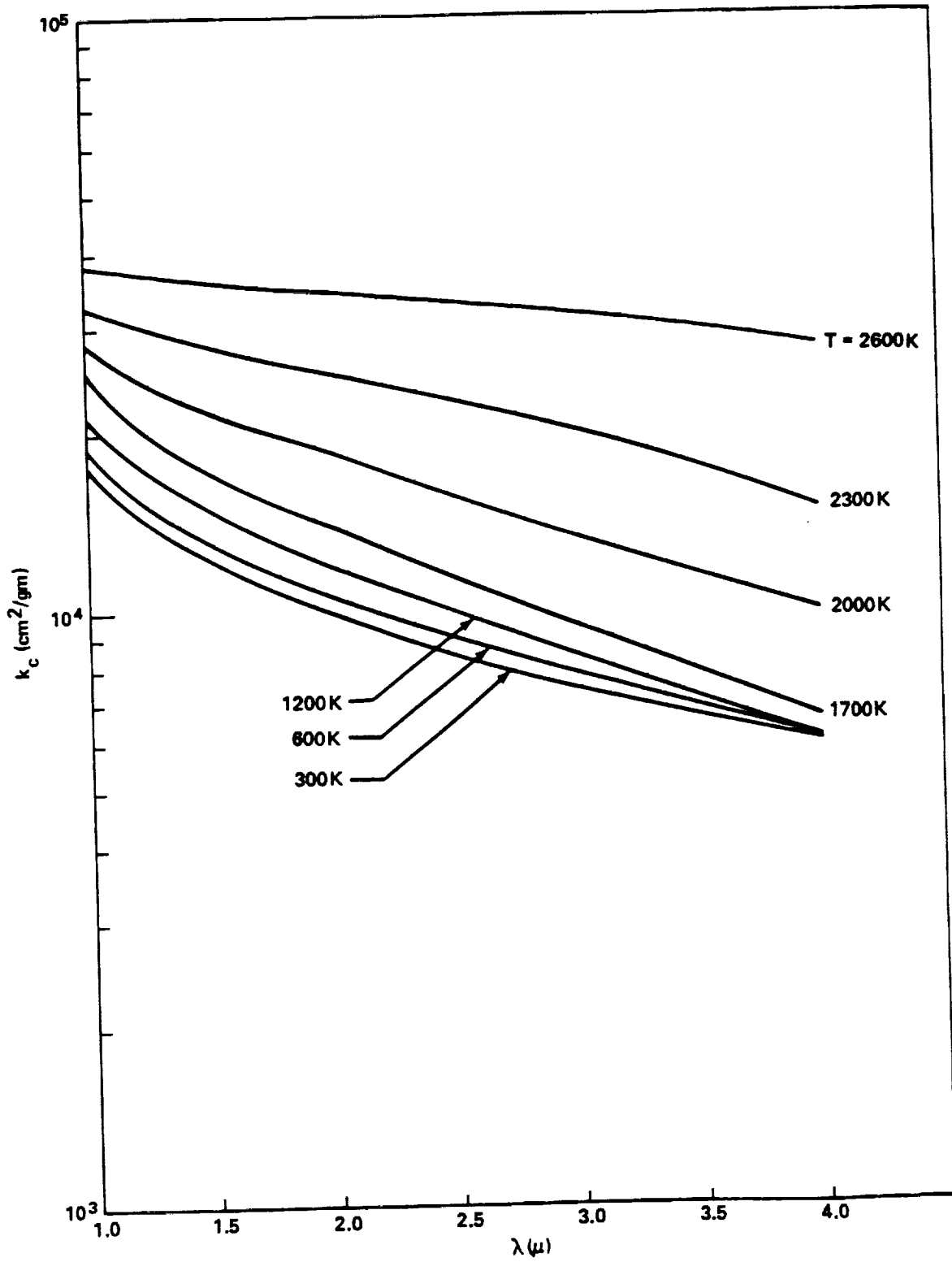
Evaluations made with line-of-sight (LOS) calculation for typical motors.



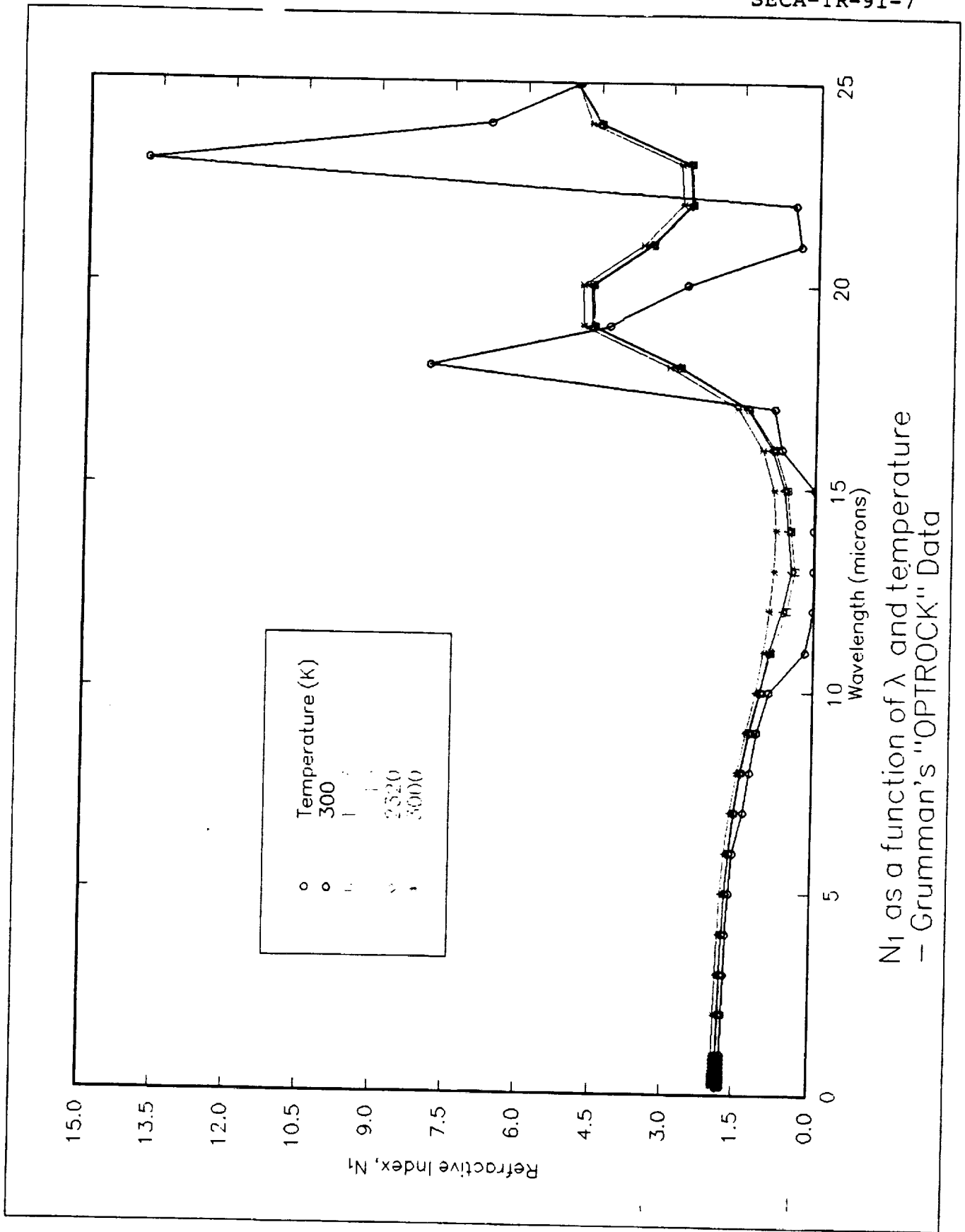
Total emissivity of H_2O .

Wide band model correlation parameters for various gases

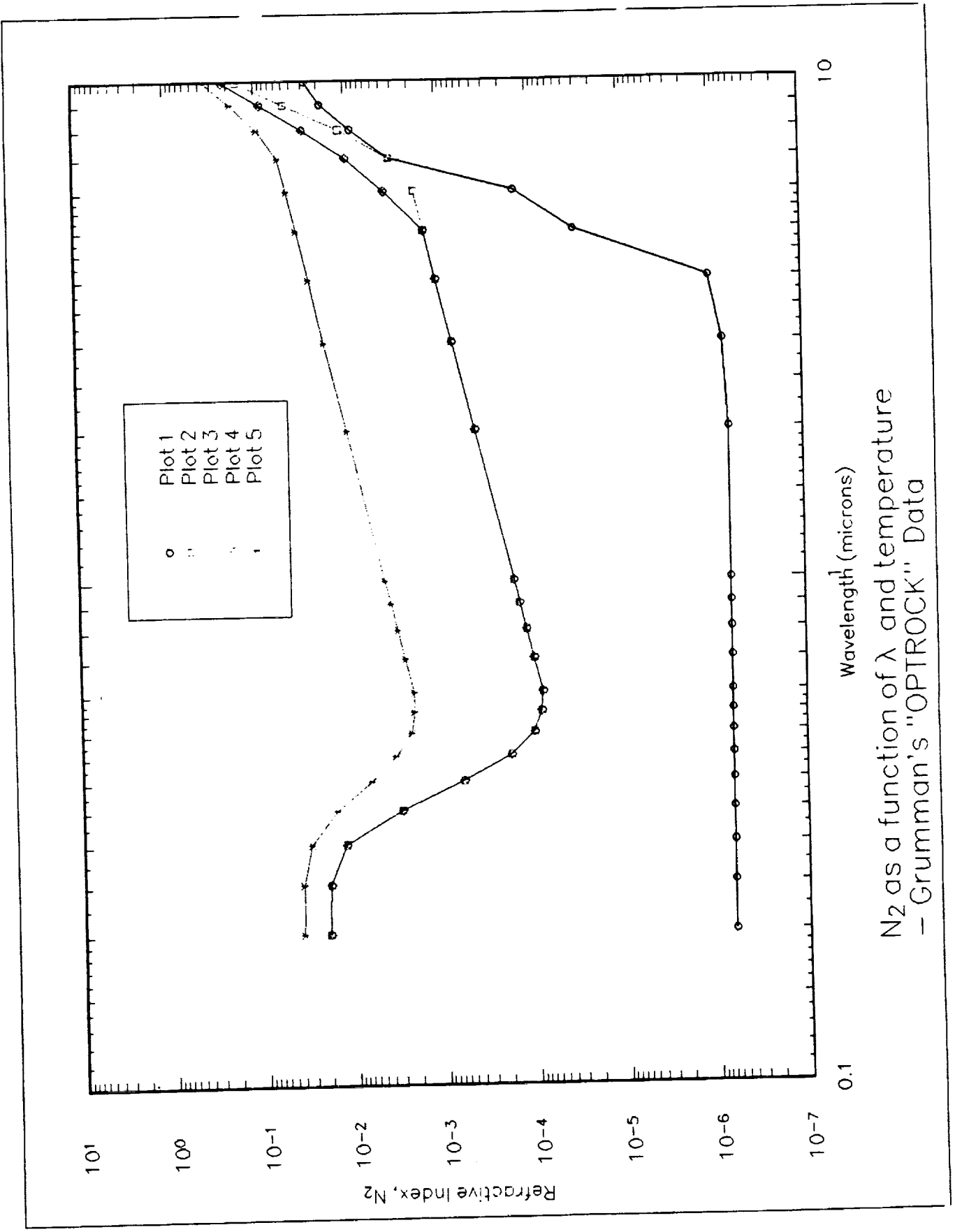
Band Location		Pressure Parameters		Correlation Parameters		
λ [μm]	(δ_k)	n	b	α_0 [$\text{cm}^{-1}/(\text{g}/\text{m}^2)$]	γ_0	ω_0 [cm^{-1}]
H₂O $m = 3, \eta_1 = 3652 \text{ cm}^{-1}, \eta_2 = 1595 \text{ cm}^{-1}, \eta_3 = 3756 \text{ cm}^{-1}, g_k = (1, 1, 1)$						
71 μm^a	$\eta_c = 140 \text{ cm}^{-1}$ (Rotational) (0,0,0)	1	$8.6 \sqrt{\frac{T_0}{T}} + 0.5$	44,205	0.14311	69.3
6.3 μm	$\eta_c = 1600 \text{ cm}^{-1}$ (0,1,0)	1	$8.6 \sqrt{\frac{T_0}{T}} + 0.5$	41.2	0.09427	56.4
2.7 μm	$\eta_c = 3760 \text{ cm}^{-1}$ (0,2,0) (1,0,0) (0,0,1)	1	$8.6 \sqrt{\frac{T_0}{T}} + 0.5$	0.2 2.3 23.4	0.13219 ^{b,c}	60.0 ^b
1.87 μm	$\eta_c = 5350 \text{ cm}^{-1}$ (0,1,1)	1	$8.6 \sqrt{\frac{T_0}{T}} + 1.5$	3.0	0.08169	43.1
1.38 μm	$\eta_c = 7250 \text{ cm}^{-1}$ (1,0,1)	1	$8.6 \sqrt{\frac{T_0}{T}} + 1.5$	2.5	0.11628	32.0
CO₂ $m = 3, \eta_1 = 1351 \text{ cm}^{-1}, \eta_2 = 666 \text{ cm}^{-1}, \eta_3 = 2396 \text{ cm}^{-1}, g_k = (1, 2, 1)$						
15 μm	$\eta_c = 667 \text{ cm}^{-1}$ (0,1,0)	0.7	1.3	19.0	0.06157	12.7
10.4 μm	$\eta_c = 960 \text{ cm}^{-1}$ (-1,0,1)	0.8	1.3	2.47×10^{-9}	0.04017	13.4
9.4 μm	$\eta_c = 1060 \text{ cm}^{-1}$ (0,-2,1)	0.8	1.3	2.48×10^{-9}	0.11888	10.1
4.3 μm	$\eta_u = 2410 \text{ cm}^{-1}$ (0,0,1)	0.8	1.3	110.0	0.24723	11.2
2.7 μm	$\eta_c = 3660 \text{ cm}^{-1}$ (1,0,1)	0.65	1.3	4.0	0.13341	23.5
2.0 μm	$\eta_c = 5200 \text{ cm}^{-1}$ (2,0,1)	0.65	1.3	0.060	0.39305	34.5



Experimentally Determined Carbon Absorption Coefficients
(from Radiation Handbook)

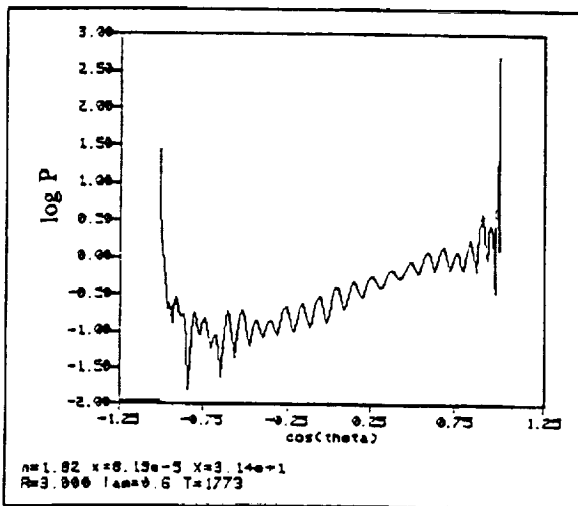
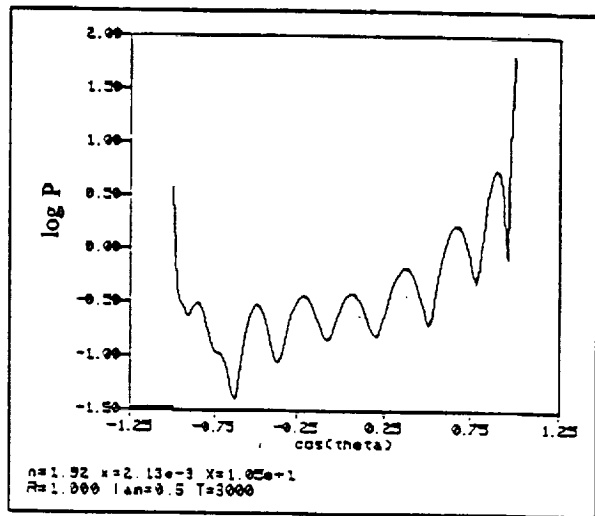
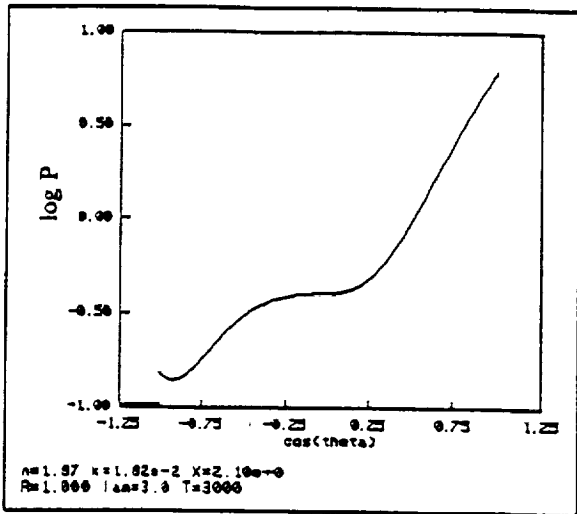


N_1 as a function of λ and temperature
 - Grumman's "OPTROCK" Data



N₂ as a function of λ and temperature
 - Grumman's "OPTROCK" Data

PHASE FUNCTION AS
DETERMINED BY
MIE THEORY
CALCULATION



Optical properties: n, κ
 Particle Radius: R
 Wavelength: λ
 Temperature: T
 Particle Size
 Parameter: $X=2\pi R/\lambda$

ORIGINAL PAGE IS
OF POOR QUALITY

Al₂O₃ RADIATION PROPERTIES

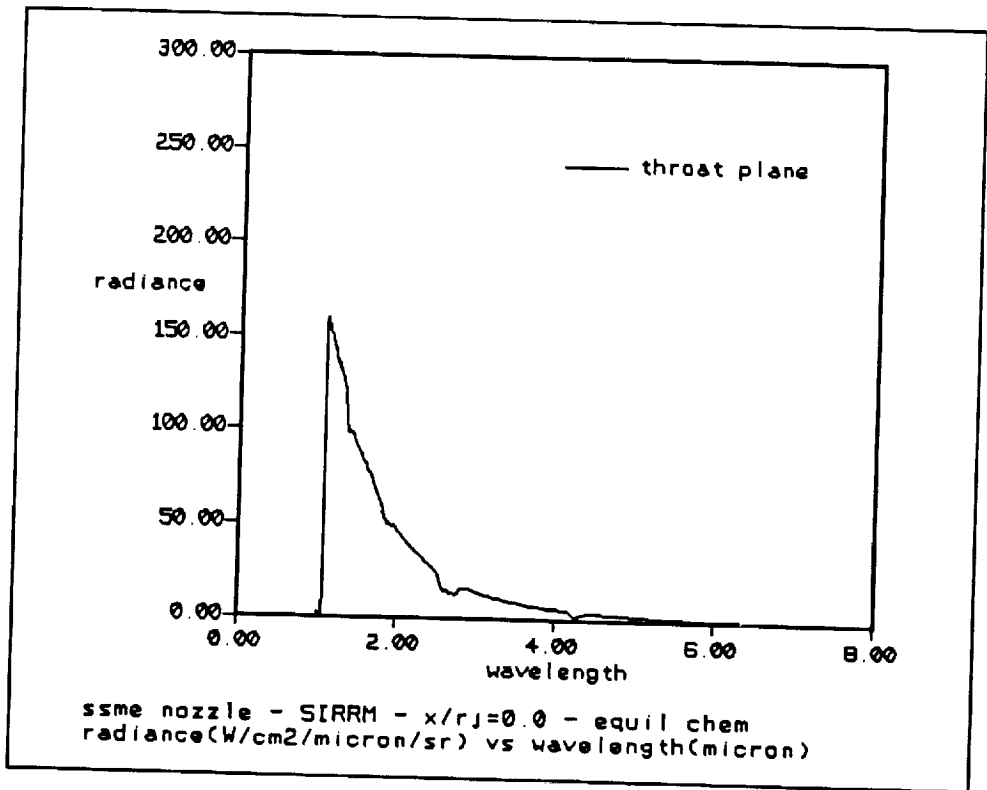
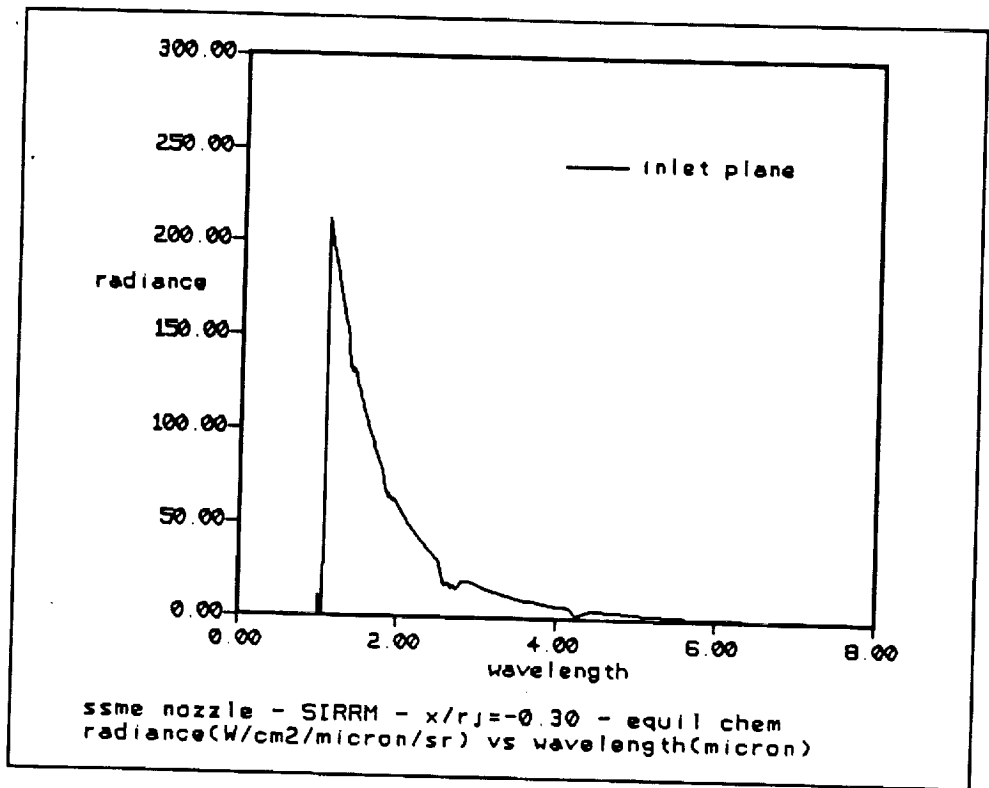
r , (μm)	Wave Number	σ_e , (cm^2)	Albedo	s_b	b_b	b_2	T(K)	
1.0E-3	400	6.181E-19 6.181E-19	5.565E-10 5.565E-10	.14352 .1439	.21295 .2120	.49999 .5065	3000	OS OM
3	5000	6.7430E-7 5.956E-7	.98982 .9905	.073316 .07461	.061334 .06874	.087562 .14723	2320	RS RM
3	10000	6.3054E-7 6.618E-7	.99193 .9924	.059637 .05366	.053251 .05025	.070317 .10287	2320	RS RM
3	3333	6.841E-7 8.103E-7	.98754 .9831	.08270 .06509	.07050 .08108	.11115 .17645	2320	RS RM
3	10000	6.723E-7 6.752E-7	.98954 1.000	.07248 .05096	.060389 .05813	.085913 .11004	1773	RS RM
3	10000	6.843E-7 6.445E-7	.81057 .8666	.06522 .05318	.04385 .03896	.060442 .09529	3000	RS RM

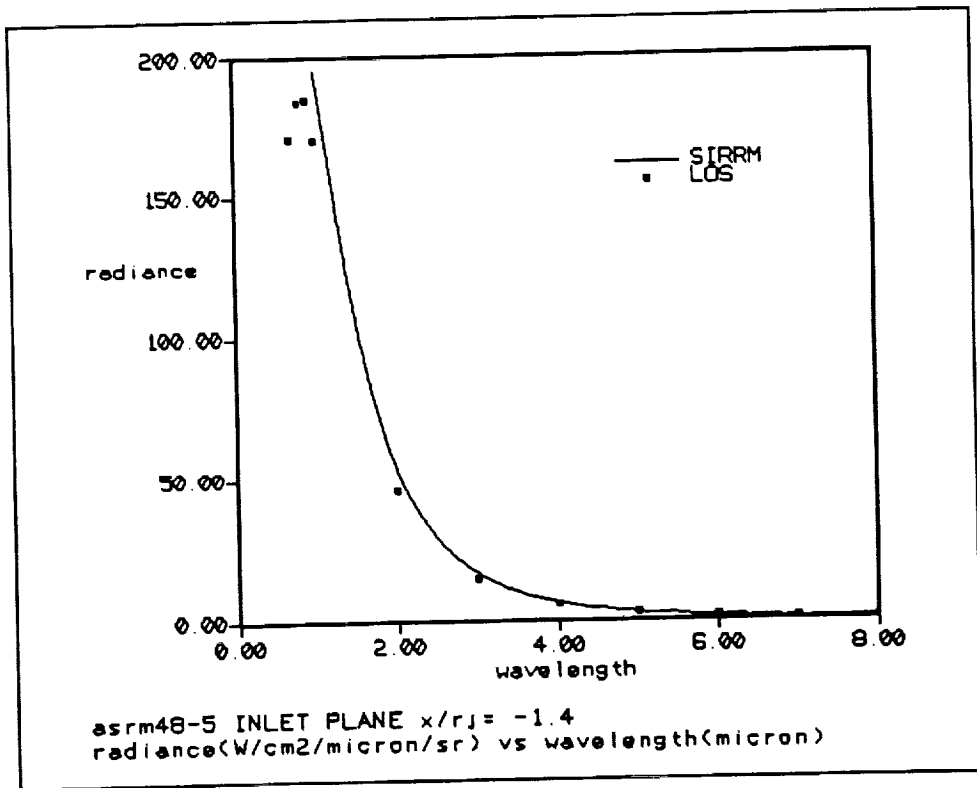
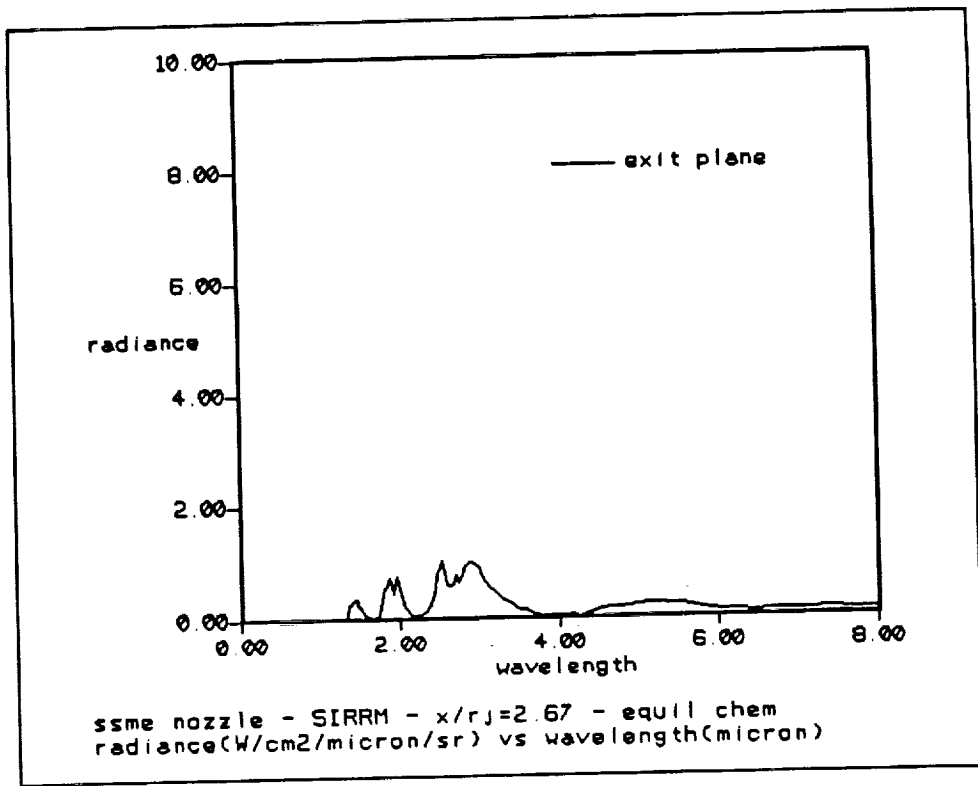
OS Original Al₂O₃ Optical Properties in SIRR

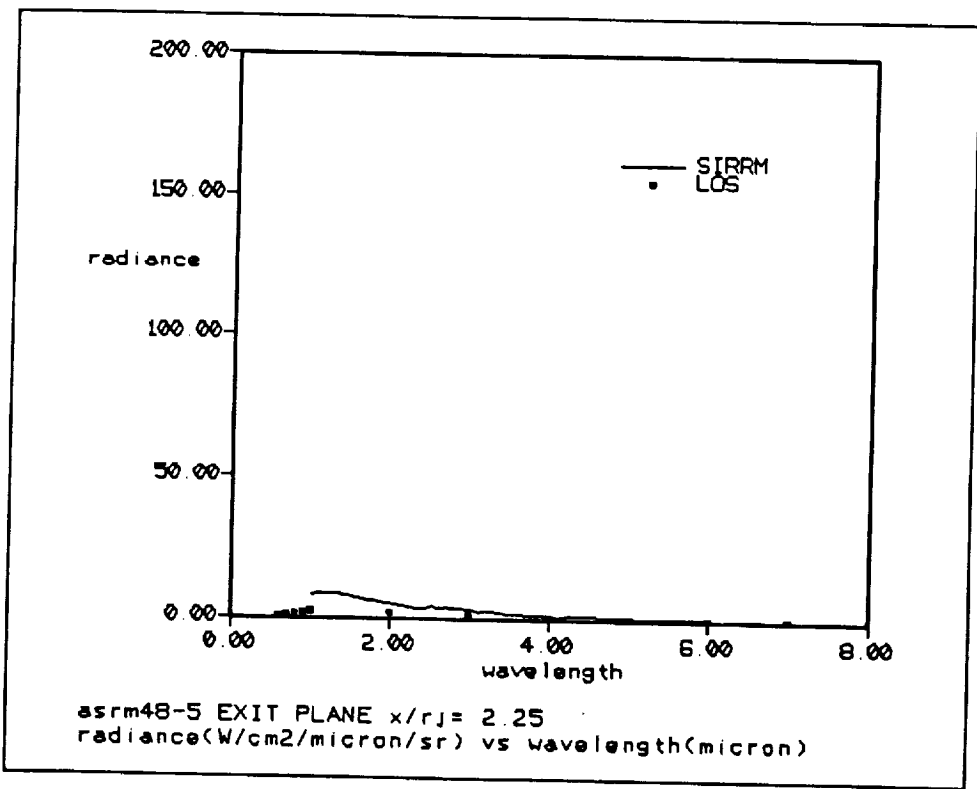
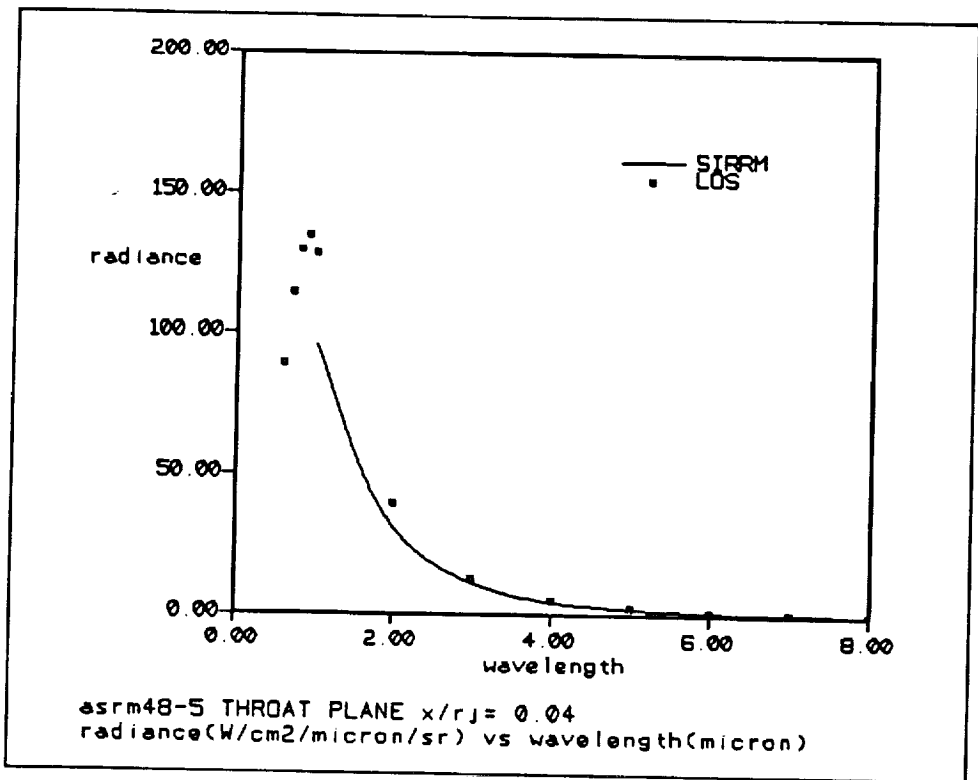
OM Original Al₂O₃ Optical Property Data With MIE Code

RS OPTROCK Properties in SIRR

RM OPTROCK Properties Calculated With A MIE Code







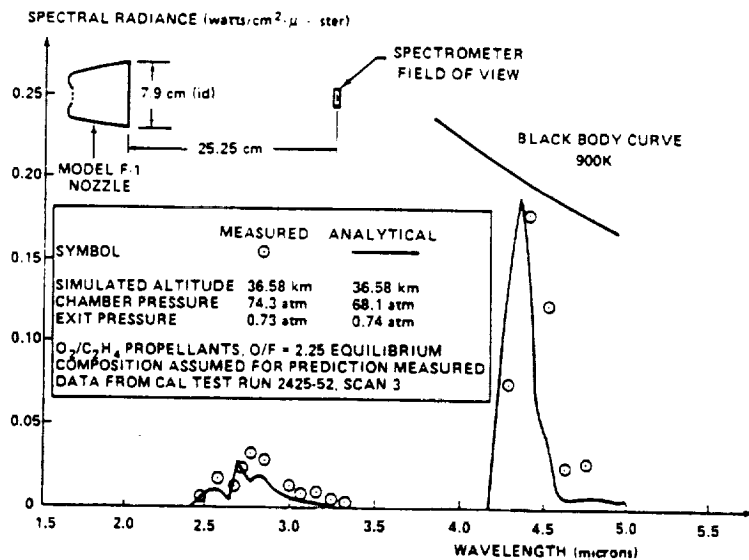
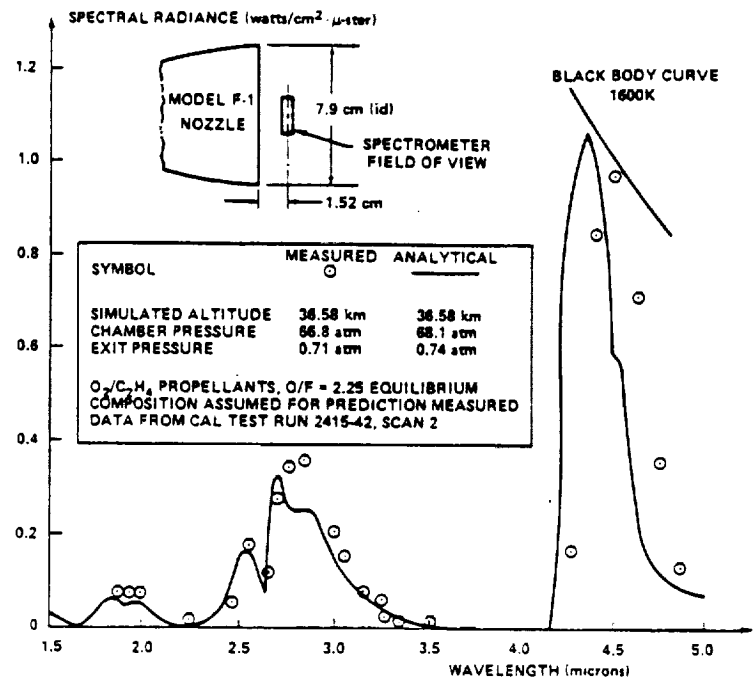
TOTAL GAS RADIATION - cal/cm²-s

SSME				
	Mean	Wide	Narrow	σT^4
CHAMBER P = 194.4 atm T = 3626.02K L = 43.98 cm P _w = 134.5 atm	234	90.7	124	234
THROAT P = 115 atm T = 3450K L = 25.42 cm P _w = 81.1 atm	192	65.8	98.4	192
EXIT P = 0.1943 atm T = 1279K L = 230.436 cm P _w = 0.15 atm	0.692	0.621	0.63	3.62

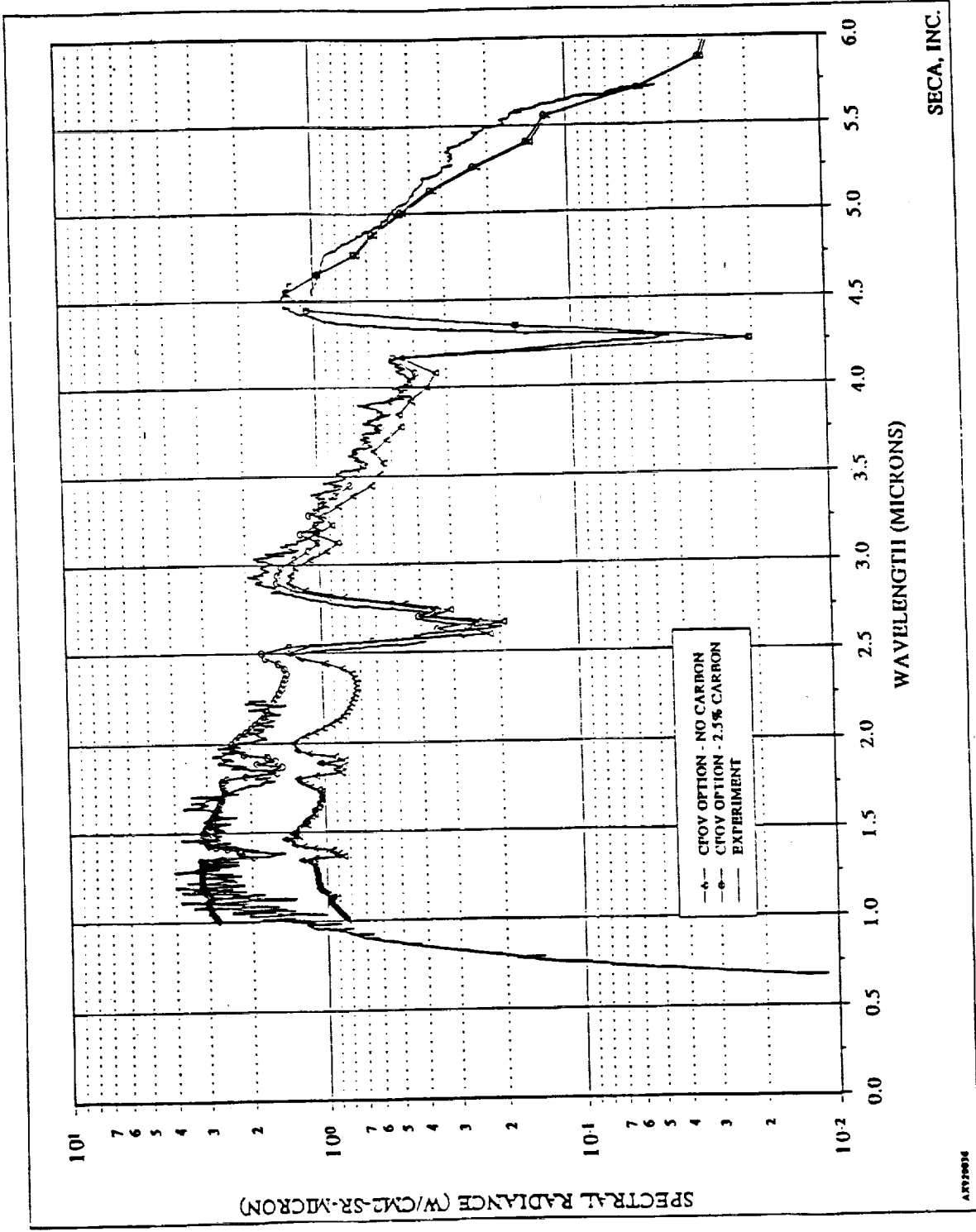
TOTAL GAS RADIATION - cal/cm²-s

ASRM 48-5			
	Mean	Wide	σT^4
CHAMBER P=43 atm T=3513K L=105.184 cm P _w = 5.76 atm P _c = 0.57 atm	91.1	51.4	206
THROAT P=25 atm T=3230K L=24.918 cm P _w = 3.40 atm P _c = 0.33 atm	21.6	22.1	147
EXIT P=1.04 atm T=2200 K L=67.95 cm P _w = 0.14 atm P _c = 0.014 atm	2.48	2.53	31.7

Al ₂ O ₃ PARTICLE RADIATION - cal/cm ² -s		
	LOS	σT^4
CHAMBER	182	206.3
THROAT	95.0	156.8
EXIT	12.5	39.1



Plume Radiation from a Small Scale F-1 Engine
(Reardon, Radiation Handbook)



RSRM48-1 SIRR-11 Predicted Plume Radiance at $X/R_j = 2.24$

TRENDS IN Al_2O_3 PARTICLE SIZE FOR SRM

(from Netzer's experiments)

- Chamber near propellant surface - 130 μm
- Chamber at nozzle entrance - 4-17 μm
- Converging Nozzle Section - particles breakup
- Diverging Nozzle Section - particles agglomerate
- Plume Nearfield - multi-modal PSD w/large particles - near centerline
- Plume Farfield - small particles

PRESENT PLUME RADIATION METHODOLOGY

- Flowfields - Standard Axisymmetric Models (RAMP2, SPF/2, SPF/3)
- Radiation - SIRRM, Reverse Monte Carlo
- Particle Size Based on MNASA 48" Measurements of Sambamurthi
- Improve Particle-Gas Heat Transfer Method of Moylan

RADIATION/CONVECTION COUPLING

Transport Equation Solver: the FDNS-EL Code

Energy Equation contains:

$$\nabla \cdot \vec{q}_r = \int_0^\infty \kappa_\lambda [4\pi I_{\lambda b}\{T\} - \int_{4\pi} I_\lambda\{\vec{r}, \vec{\Omega}\} d\vec{\Omega}] d\lambda$$

1013

The last term in this equation requires an evaluation of the intensity in all directions at each point in the flowfield.

The intensity is obtained from the Radiative Transfer Equation (RTE):

$$\vec{\Omega} \cdot \nabla I_\lambda\{\vec{r}, \vec{\Omega}\} = \kappa_\lambda I_{\lambda b}\{T\} - (\kappa_\lambda + \sigma_\lambda) I_\lambda\{\vec{r}, \vec{\Omega}\} + \frac{\sigma_\lambda}{4\pi} \int_{4\pi} P\{\vec{r}, \vec{\Omega}' - \vec{\Omega}\} I_\lambda\{\vec{r}, \vec{\Omega}'\} d\Omega'$$

FORMAL SOLUTION TO RTE

$$I\{\vec{r}, \vec{\Omega}\} = I\{\vec{r}', \vec{\Omega}\} e^{-\tau} + \int_0^{\tau} e^{-\kappa t} \left[\kappa I_b\{\vec{r}\} + \frac{\sigma}{4\pi} \int_{4\pi} I\{\vec{r}, \vec{\Omega}\} P\{\vec{r}, \vec{\Omega}' - \vec{\Omega}\} d\vec{\Omega}' \right] dt$$

(Subscript λ is suppressed for clarity, but intensity is still monochromatic)

Solutions have been obtained by:

- the **PIM method** (Tan)
 - numerical evaluation of the integral terms.
- the **YIX method** (Tan & Howell)
 - uses piecewise-constant interpolation and stores intermediate integral evaluations to improve the PIM method.

SOLUTIONS TO THE RTE USING SPHERICAL HARMONICS

(P_N - Approximation)

The P_1 - Approximation (ODA) requires solutions of PDE's to evaluate the integrated form of the RTE rather than integral equations:

1. Express the intensity as a generalized Fourier Series using Legendre Polynomials.

2. Express the Phase Function with Legendre Polynomials

$$P\{\vec{r}, \vec{\Omega}' - \vec{\Omega}\} = 1 + A_1 \vec{r} \cdot (\vec{\Omega}' - \vec{\Omega})$$

Requires solution to:

$$\nabla_{\tau} \cdot \left(\frac{1}{(1 - A_1 \omega/3)} \nabla_{\tau} G \right) = -3(1 - \omega)(4\pi I_b - G)$$

If $A_1 \omega$ is constant, this is an elliptic PDE. When solved,

$$\vec{q} = - \left(\frac{1}{3 - A_1 \omega} \right) \nabla_{\tau} G$$

G is incident radiation at a point

$$G\{r\} = \int_{4\pi} I\{\vec{r}, \vec{\Omega}'\} d\Omega'$$

$$I\{\vec{r}, \vec{\Omega}\} = \frac{1}{4\pi} [G\{\vec{r}\} + 3\vec{q}\{\vec{r}\} \cdot \vec{\Omega}]$$

Limitation: For $N=1$ or 3 , accurate for only optically thick media.

THE MODIFIED DIFFERENTIAL APPROXIMATION

The MDA generalizes P_1 solution to treat arbitrary optical thickness.

Let surface & media contribute to intensity:

$$I\{\vec{r}, \vec{s}\} = I_w\{\vec{r}, \vec{s}\} + I_m\{\vec{r}, \vec{s}\}$$

Solution requires evolution of:

$$G_w\{\vec{r}\} = \frac{1}{\pi} \int_{4\pi} J_w\{\vec{r}_w\} e^{-\tau_s} d\Omega$$

$$\vec{q}_w\{\vec{r}\} = \frac{1}{\pi} \int_{4\pi} J_w\{\vec{r}_w\} e^{-\tau_s} \vec{s} d\Omega$$

$$J_i = \epsilon_i \pi I_{bi} + (1 - \epsilon_i) \sum_{j=1}^N J_j e^{-\tau_{ij}} F_{i-j}$$

$$\nabla_\tau G_m = A_1 \omega (\vec{q}_w + \vec{q}_m) - 3 \vec{q}_m$$

$$\nabla_\tau \cdot \vec{q}_m = (1 - \omega) 4\pi I_b + \omega (G_w + G_m) - G_m$$

THE IMPROVED DIFFERENTIAL APPROXIMATION

The IDA has the same solution features as the MDA but is more computationally efficient.

Solution requires:

The ODA, i.e. the P_1 solution, and also:

$$J_w\{\vec{r}\} = \epsilon\pi I_{bw}\{\vec{r}\} + (1-\epsilon)\int_A [J_w\{\vec{r}'\}e^{-\tau_s} + \pi S^* \{\vec{r}-s_0\hat{s}, \hat{s}\}(1-e^{-\tau_s})] \frac{\cos\theta \cos\theta'}{\pi S^2} dA$$

$$G\{\vec{r}\} = \int_A [J_w\{\vec{r}'\}e^{-\tau_s} + \pi S^* \{\vec{r}-s_0\hat{s}, \hat{s}\}(1-e^{-\tau_s})] \frac{\cos\theta'dA}{\pi S^2}$$

$$q\{\vec{r}\} = \int_A [J_w\{\vec{r}'\}e^{-\tau_s} + \pi S^* \{\vec{r}-s_0\hat{s}, \hat{s}\}(1-e^{-\tau_s})] \hat{s} \frac{\cos\theta'dA}{\pi S^2}$$

SOLUTIONS TO THE RTE USING THE METHOD OF DISCRETE COORDINATES

Solves a number of LOS's to obtain:

$$G\{\vec{r}\} = \sum_{i=1}^n w_i I_i\{\vec{r}\}$$
$$\vec{q}\{\vec{r}\} = \sum_{i=1}^n w_i I_i\{\vec{r}\} \vec{S}_i$$

w_i 's are weighting functions which must sum to the surface of a unit sphere.

Radiation/convection coupled solution has been reported for a 2-D sooty, ethylene flame in which 12 LOS's were used to evaluate each grid point (Kaplan, NRL)

CONCLUSIONS

1. For H_2/O_2 motors -
 - use narrow band models for H_2O
 - within chamber use $\epsilon = 0.95$ between 1-4 μm
2. For SRM motors -
 - use narrow band models for CO_2 & H_2O
 - use linear anisotropic scattering model for Al_2O_3 particles
 - within chamber use $\epsilon = 0.97$ between 0.6-8 μm
3. For HC/ O_2 motors -
 - reasonable prediction of soot concentration is needed
4. Radiation/convection coupling
 - use FDNS (or FDNS-EL) for flowfield
 - use ODA in chamber & converging nozzle
 - use IDA in diverging nozzle & plume

