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PRECONDITIONING FOR THE NAVIER-STOKES EQUATIONS WITH FINITE-RATE CHEMISTRY

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ABSTRACT

The extension of Van Leer's preconditioning procedure to generalized finite-rate chemistry is discussed. Application to viscous flow is begun with the proper preconditioning matrix for the one-dimensional Navier-Stokes equations. Eigenvalue stiffness is resolved and convergence-rate acceleration is demonstrated over the entire Mach-number range from nearly stagnant flow to hypersonic. Specific benefits are realized at the low and transonic flow speeds typical of complete propulsion-system simulations. The extended preconditioning matrix necessarily accounts for both thermal and chemical non-equilibrium. Numerical analysis reveals the possible theoretical improvements from using a preconditioner for all Mach number regimes. Numerical results confirm the expectations from the numerical analysis. Representative test cases include flows with previously troublesome embedded high-condition-number areas.

Van Leer, Lee, and Roe recently developed an optimal, analytic preconditioning technique to reduce eigenvalue stiffness over the full Mach-number range. By multiplying the flux-balance residual with the preconditioning matrix, the acoustic wave speeds are scaled so that all waves propagate at the same rate, an essential property to eliminate inherent eigenvalue stiffness. This session discusses a synthesis of the thermo-chemical non-equilibrium flux-splitting developed by Grossman and Cinnella and the characteristic wave preconditioning of Van Leer into a powerful tool for implicitly solving two and three-dimensional flows with generalized finite-rate chemistry.

For finite-rate chemistry, the state vector of unknowns is variable in length. Therefore, the preconditioning matrix extended to generalized finite-rate chemistry must accommodate a flexible system of moving waves. Fortunately, no new kind of wave appears in the system. The only existing waves are entropy and vorticity waves, which move with the fluid, and acoustic waves, which propagate in Mach-number dependent directions. The non-equilibrium vibrational energies and species densities in the unknown state vector act strictly as convective waves. The essential concept for extending the preconditioning to generalized chemistry models is determining the differential variables which symmetrize the flux Jacobians. The extension is then straight-forward.

This algorithm research effort will be released in a future version of the production-level computational code coined the General Aerodynamic Simulation Program (GASP), developed by Walters, Slack and McGrory.

Preconditioning for the Navier-Stokes Equations with Finite-Rate Chemistry

Dr. Andrew G. Godfrey

What's Preconditioning?

Preconditioning ...

- reformulates the governing equations so that all wave fronts propagate at the same rate.
- scales the acoustic waves to reduce stiffness .
- accelerates convergence for all Mach numbers.
- maintains hyperbolic influence of the governing equations.

Singular Systems

- Solution to a linear system

$$A\,x = b$$

$$Ax - b = R$$

- Condition Number, $k(A) = \|A\|\,\|A^{-1}\|$

$$\frac{\|\partial x\|}{\|x\|} = k(A)\frac{\|\partial R\|}{\|R\|}$$

- Compressible Euler equations decouple at low speed.

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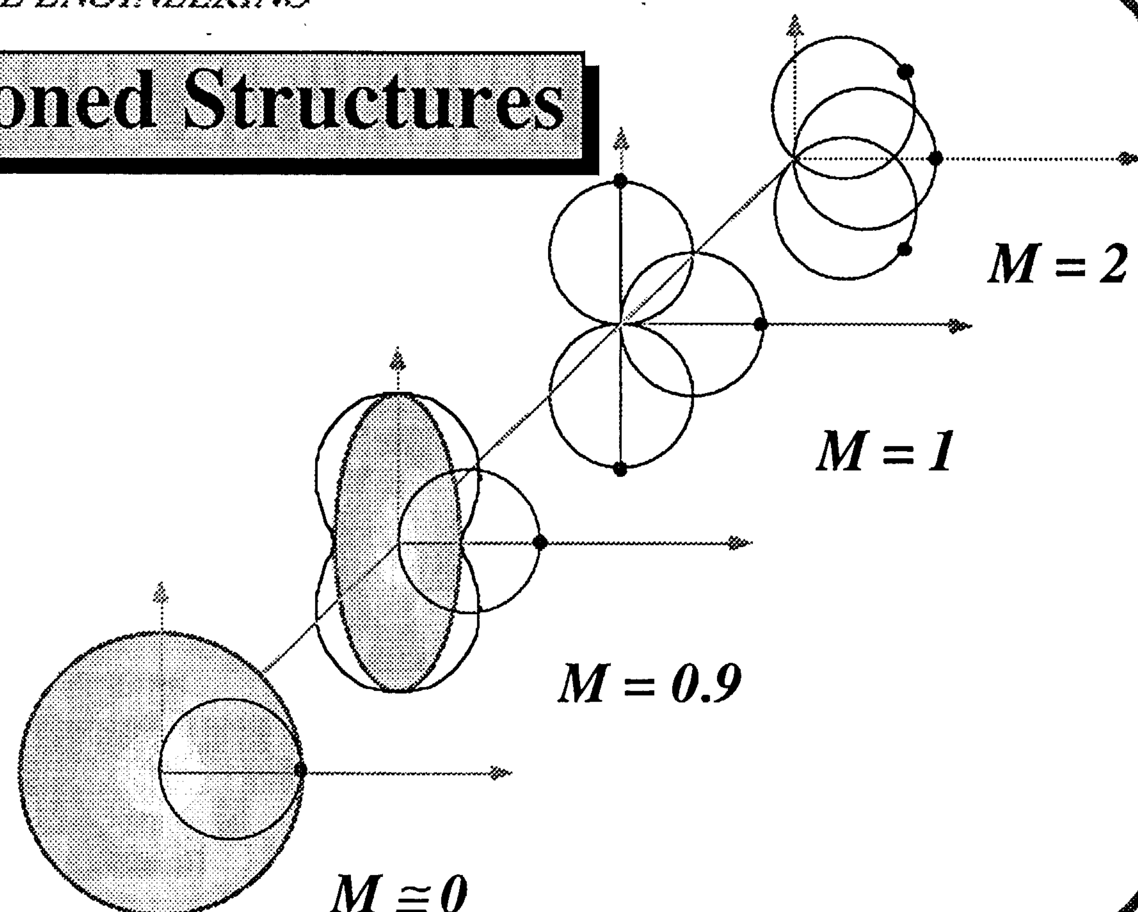
Preconditioned Structures



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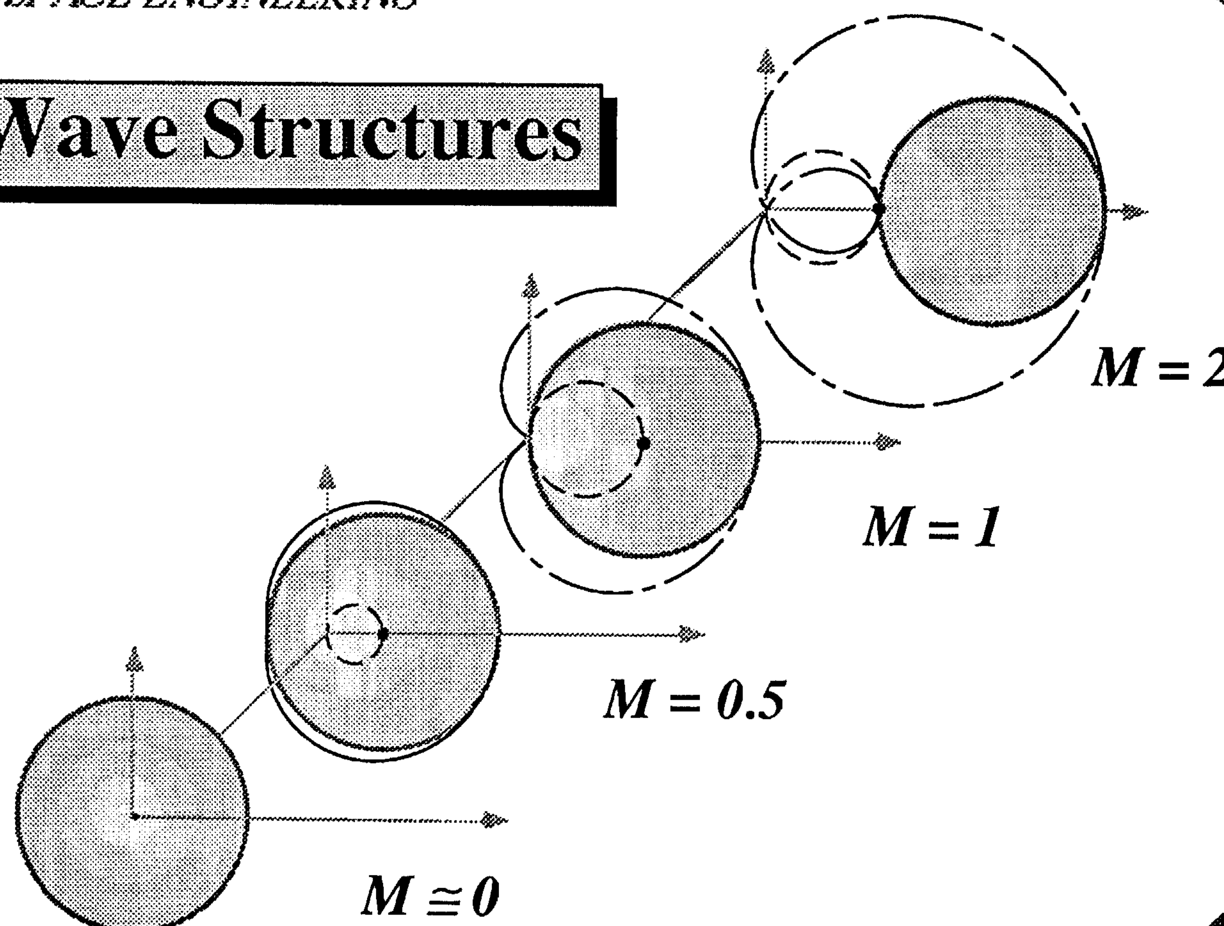
Euler Wave Structures



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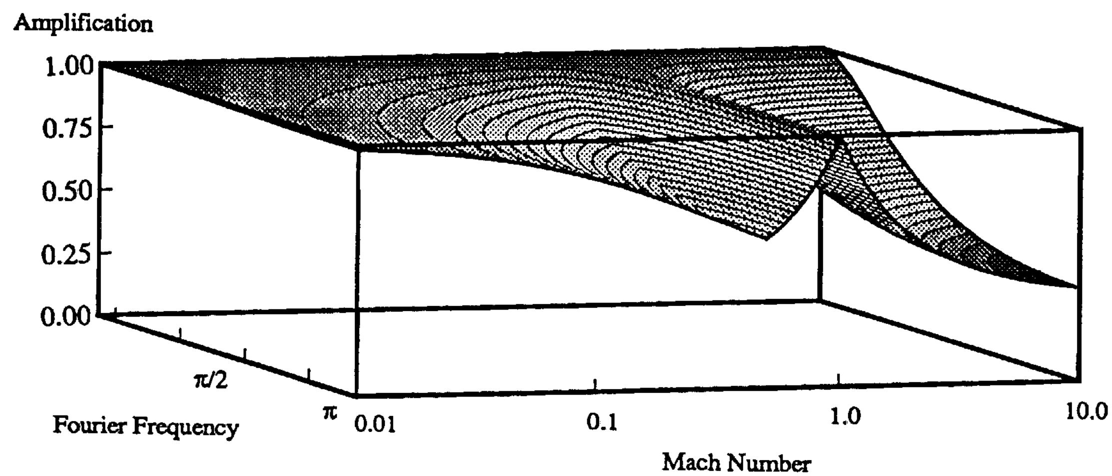
WITHOUT Preconditioning



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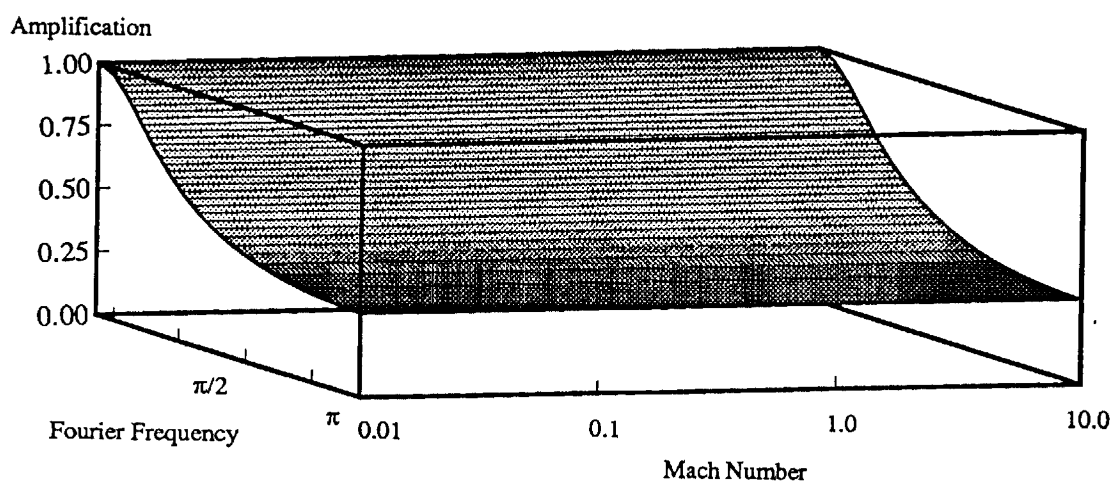
WITH Preconditioning



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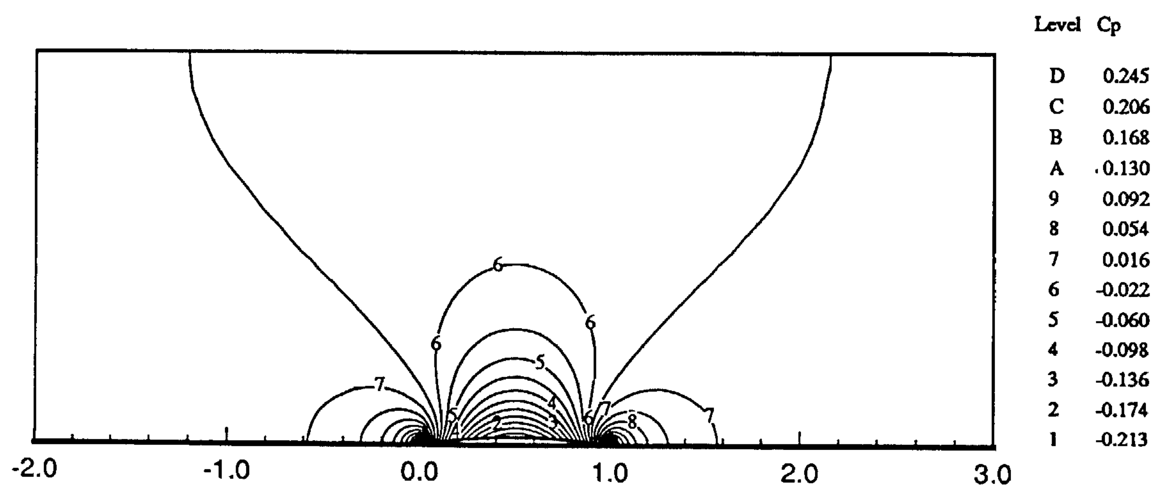
Low-Speed Channel Flow



M=0.001 WITH Preconditioning

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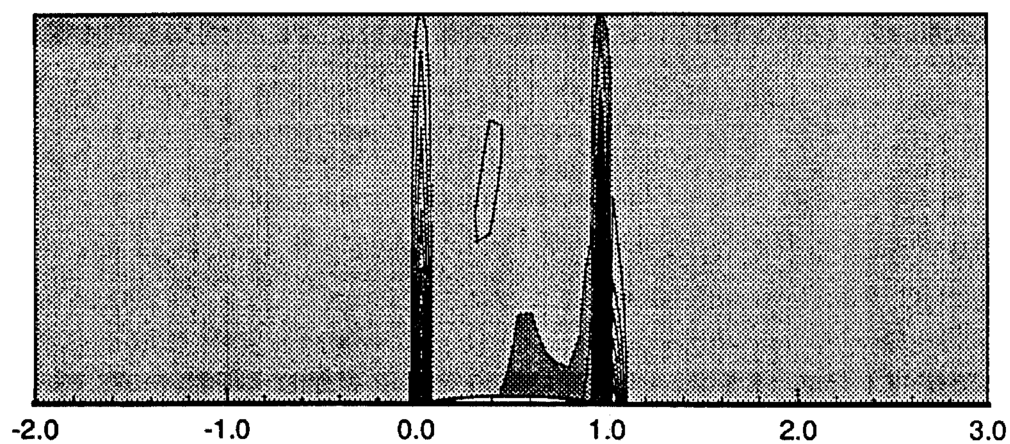
Low-Speed Channel Flow



M=0.001 WITHOUT Preconditioning

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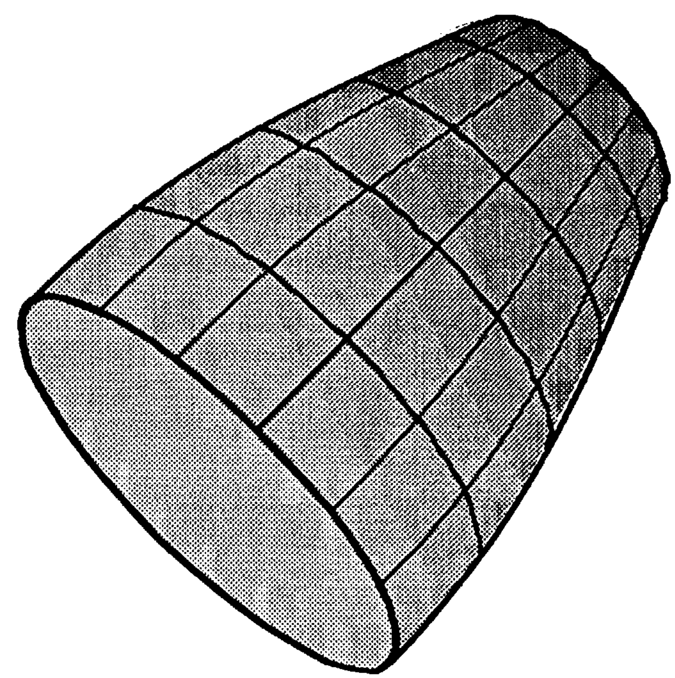
Axi-Symmetric Nozzle

$$r(x) = \frac{L}{4}\left[1+\sin\left(\frac{\pi x}{2L} \right)\right]$$



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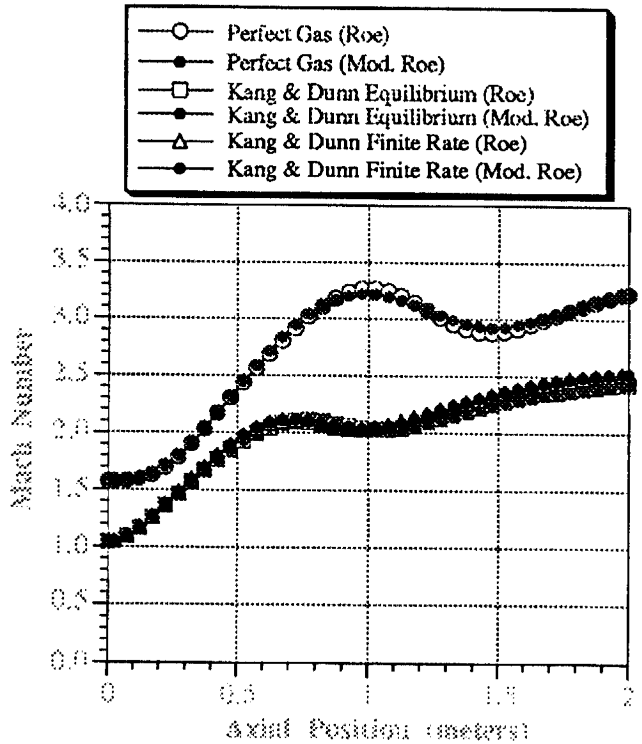
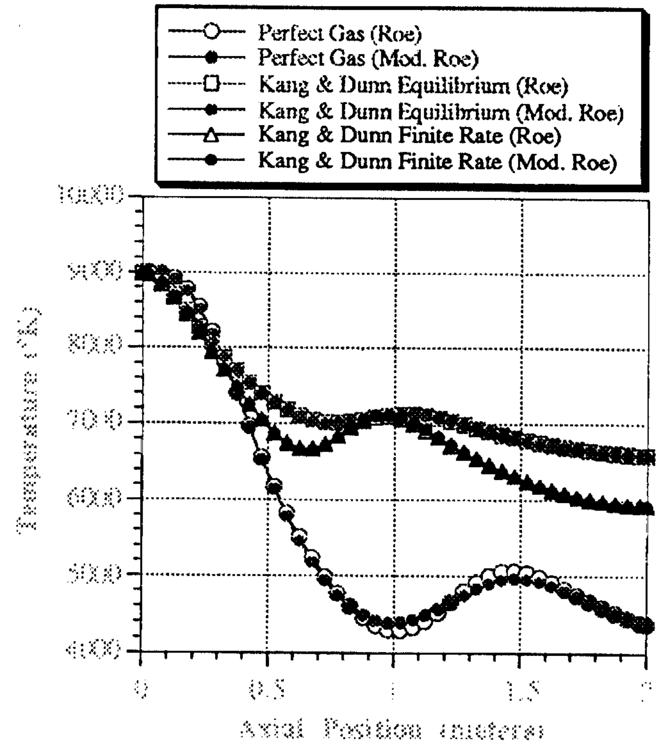
Mach Number & Temperature





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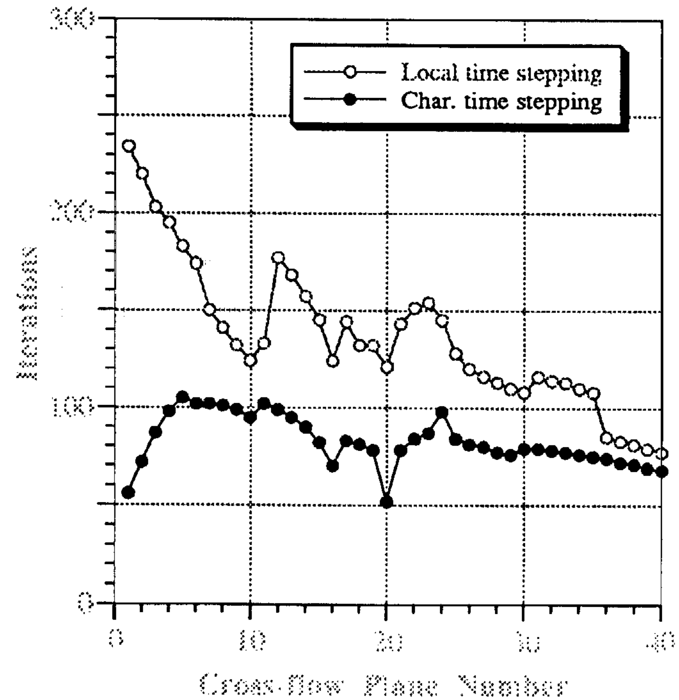
Residual History - Finite Rate



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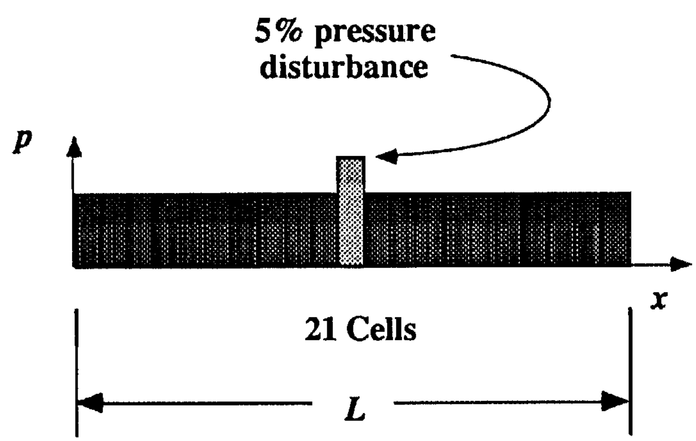
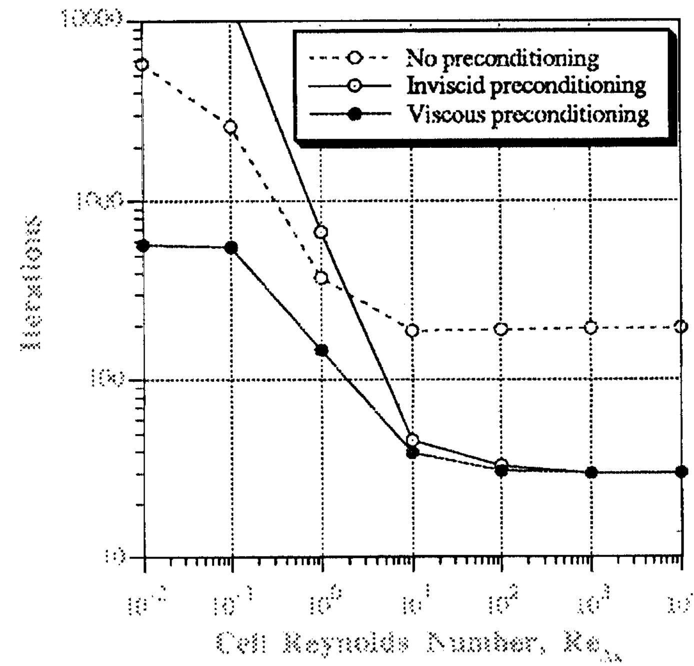
Pressure Damping



$M_\infty = 0.1$
$T_\infty = 400\ °K$
$\rho_\infty = 0.7068\ kg/m^3$



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Future Work

- Boundary condition stiffness.
  Discretize characteristic variables.
- Smoothing of singular preconditioning matrix.
  Stagnation points.
- Multi-dimensional Navier-Stokes equations
  Laminar airfoils and separation bubbles.