

SUPERSONIC FLOW AND SHOCK FORMATION IN TURBINE TIP GAPS <sup>1995-117009</sup>

John Moore

Mechanical Engineering Department  
Virginia Polytechnic Institute and State University  
Blacksburg, Virginia 24061-0238

57-34  
~~43792~~  
P. 11

Summary

Shock formation due to overexpansion of supersonic flow at the inlet to the tip clearance gap of a turbomachine has been studied.

As the flow enters the tip gap, it accelerates around the blade pressure-side corner creating a region of minimum static pressure. The "free streamline" separates from the wall at the corner; and, for Mach numbers greater than about 1.3, it curves back to intersect the blade tip. At this point, the freestream flow is abruptly turned parallel to the surface, giving rise to an oblique shock.

The results are consistent with compressible sharp-edged orifice flow calculations found in the literature and with the theory of oblique shock wave formation in supersonic flow over a wedge. For freestream Mach numbers of 1.4 to 1.8, wave angles are 43 to 54 degrees, and turning angles are 9 to 20 degrees; as the Mach number increases, the angle of turn also increases.

It appears that in a turbine, after separating from the inlet corner, the flow reattaches on the blade tip and an oblique shock is formed at 0.4-1.4 tip gap heights into the clearance gap. The resulting shock-boundary layer interaction may contribute to further enhancement of already high heat transfer to the blade tip in this region. This in turn could lead to higher blade temperatures and adversely affect blade life and turbine efficiency.

Introduction

Tip leakage flow through the clearance gaps of unshrouded turbomachinery blades is known to cause reductions in efficiency and performance [Roelke, 1973, Hourmouziadis and Albrecht, 1987]. It is also known that enhanced heat transfer to the tips of turbine rotor blades, resulting from the separation and reattachment of the leakage flow, can be a major factor in determining blade life in high temperature gas turbines [Moore et al., 1989]. There have therefore been many recent studies of flow and heat transfer in tip gaps [Bindon, 1987, Metzger and Bunker, 1989, Metzger, Dunn, and Hah, 1991, Yaras et al., 1989]. But most of

these studies have considered only incompressible flow, and features of compressible flow such as overexpansion to high supersonic Mach numbers, shock formation within the tip gap, and shock boundary layer interaction have received little attention. It is important that this compressible flow physics be understood sufficiently well to be included in the turbine design process.

Consider, for example, turbine rotor blades in gas turbines operating with transonic flow. Around the airfoil, the flow accelerates to supersonic Mach numbers near the suction surface in regions of low static pressure. Similarly flow passing through the tip clearance gap will accelerate as the pressure falls. But its path is not as smooth as that around the airfoil profile. Efforts are made to reduce the leakage flow by using sharp corners, for example on the pressure side, and cavities. The flow may, therefore, overexpand locally and exhibit regions of supersonic flow and complex compressible flow structure.

In an attempt to shed some light on compressible flow development in tip gaps, Henry and Moore [Moore et al., 1989] made a preliminary study using a water table flow simulation. Using the hydraulic analogy between free surface liquid flow and two-dimensional compressible flow, they gained some insight into possible Mach number distributions and shock patterns to be found in turbine tip clearance gaps. It appeared that overexpansion of flow around the pressure surface corner leads to an oblique shock wave which extends from the inlet corner region to the shroud wall at about two tip gap heights into the tip gap and then reflects back to the turbine blade tip. For tip gap exit pressures corresponding to blade suction surface Mach numbers greater than 1.0, they found local maximum Mach numbers within the tip gap in the range of about 1.5 to 1.8.

### Compressible Orifice Flows

Although little was found in the literature about compressible tip gap flows, much research has been done on compressible orifice flows. Benson and Pool [1965] describe early research in this area.

Benson and Pool numerically solved the equations for steady, isentropic flow of air through a two-dimensional slit. They then compared the results with Schlieren photographs and interferograms. Figure 1 shows the computed free streamlines for various back pressures. The flows range from incompressible flow  $p_b/p_0 \sim 1.0$ ,  $M \sim 0.0$ , to sonic flow,  $p_b/p_0 = 0.5283$ ,  $M = 1.0$ , to choked flow,  $p_b/p_0 = 0.0389$ ,  $M = 2.77$ .

Consider flow accelerating along the orifice wall to the sharp corner. If the back pressure is low enough to cause supersonic flow, this flow will undergo a Prandtl-Meyer expansion at the corner to the freestream Mach number. As a

result the flow turns through the corresponding expansion angle. The free streamline then continues to turn through a further ninety degrees to the point of its maximum (downward) slope. Subsequently the jet reaches a maximum width followed by a contraction (as the free streamline turns upward again).

For subsonic and sonic flow, the jet simply contracts to an asymptotic jet width. This width is  $\pi/(\pi+2) = 0.61$  of the slit width for incompressible flow and 0.74 of the slit width for sonic flow, as seen in Fig. 1. With supersonic flow, the jet width contracts and then expands. At a freestream Mach number of about 1.3 ( $p_b/p_0 \sim 0.36$ ), the maximum width of the jet is equal to the slit width. At choked flow, the maximum width is about five times the slit width.

Norwood [1961] performed a computational study similar to that of Benson and Pool. He concentrated mostly on flow near the slit. He also performed experimental work on jet reattachment in two-dimensional models of a flapper valve. The geometry of these was like that of the slit in Fig. 1 except that downstream of the minimum area a straight wall was inclined downwards at an angle of  $22\frac{1}{2}$  degrees to the horizontal. This effectively produced a tip gap with a sharp inlet corner and a linearly increasing height. Norwood visualized the flow with shadowgraph pictures.

At low upstream stagnation pressures ( $p_b/p_0 > \sim 0.23$ ), the jet in Norwood's models followed the horizontal wall like the subsonic and just supersonic flows in Figure 1. Further increase in the upstream pressure ( $p_b/p_0 < \sim 0.23$ ), however, caused the jet to jump to the inclined wall. Norwood noted that in his two-dimensional models the flow jumped at a relatively constant pressure ratio. This suggested that the only characteristic parameter for the flow development was the Mach number. It also indicated that the phenomenon of reattachment on the inclined wall was, initially at least, a compressibility effect depending on the Mach number rather than a frictional effect.

In a shadowgraph picture of flow along the inclined wall, Norwood observed two features of interest in the present study. The first was "a teardrop shaped region," or "bubble," at the edge of the orifice where the flow is rapidly accelerating and the streamlines are highly curved. Norwood argued that the pressure in the bubble was very low because of the entrainment of the air inside into the main stream. The second interesting feature was an oblique shock just downstream of the throat starting on the inclined wall at the end of the bubble. This is due to the change in direction the supersonic flow encounters when it contacts the wall.

### Present Contribution

The work of Moore and Elward [1992] was aimed at further understanding the mechanism of shock formation near the inlet of the tip clearance gap. The

flow structure was related to the development of compressible flow in sharp-edged orifices. Particular features of interest include the length scale of the formation process and the strength of the shocks produced. In this technical note, the findings of Moore and Elward are summarized.

### Model of shock formation

Figure 2 shows the flow model postulated by Moore and Elward for the shock formation. The flow separates from the corner at the tip gap inlet. Supersonic freestream flow with a Mach number  $M_1$  then overturns and intersects the blade tip at a distance  $x_i$  from the corner. Here the flow is abruptly turned through an angle  $\delta$ , giving rise to an oblique shock wave at an angle  $\sigma - \delta$  to the surface. This shock formation process is simply modelled as shock formation in supersonic flow over a wedge. The shock angles are then given by

$$\sin \sigma = \frac{1}{M_1} \sqrt{\frac{\frac{\rho_2}{\rho_1}}{\frac{k+1}{2} - \left[\frac{k-1}{2}\right] \frac{\rho_2}{\rho_1}}} \quad (1)$$

and

$$\tan(\sigma - \delta) = \frac{\tan \sigma}{\frac{\rho_2}{\rho_1}} \quad (2)$$

### Effective wall location of wave formation

Figure 3 shows the free streamlines calculated for sharp-edged orifice flows by Norwood.

From the free streamline results of Benson and Pool and of Norwood, Fig. 4 was constructed. This figure shows the distance,  $x_i$ , from the orifice entrance to the point of intersection of the free streamline with a line drawn from the orifice edge parallel to the orifice centerline, plotted against the freestream Mach number. This distance is plotted as  $x_i/w$ , or the distance in orifice half-widths. The figure shows that at higher Mach numbers, the free streamline intersects the "wall" closer

to the orifice edge. As the Mach number decreases, the free streamline intersects the wall farther and farther downstream. The free streamline becomes parallel to the wall at a pressure ratio of about 0.36 or  $M \sim 1.3$ , as suggested by the results of Benson and Pool.

Figure 4 also plots the data for five water table cases against the Mach number,  $M_1$ , from Moore and Elward. Two points are plotted for each case, the location of the intersection of the line of median heights with the channel wall, denoted by the symbol  $m$ , and the location of the intersection of the line of maximum heights with the channel wall, denoted by the symbol  $p$ . The intersections for the lines of maximum height agree well with the wall locations predicted from the calculated compressible flow free streamlines. This data lies in the range  $x_i/w = 0.9-1.2$ . The trend in the data follows the predicted variation with free stream Mach number.

The angles of turn,  $\delta$ , or equivalently the angles with which the free streamlines intersect the wall, are plotted against freestream Mach number in Fig. 5. The figure shows the turning angles from Moore and Elward and the free streamline angles of Benson and Pool and of Norwood. As was seen in Fig. 4, for Mach numbers above about 1.3, the free streamline intersects the wall. The angle of intersection then becomes larger as the Mach number increases.

The line of minimum Mach number required for an attached shock for a given  $\delta$  is also shown on Fig. 5. It appears that the shock formation is like that of an attached shock on a wedge of half-angle,  $\delta$ .

Another interpretation of the shock formation is that it is like turbulent reattachment in supersonic flow. Carriere [1970] has presented a correlation of experimental results for two-dimensional flow. Again Figure 5 shows that this is in reasonable agreement with the data, but the trend is toward somewhat lower turning angles, or later reattachment, at higher Mach numbers. This perhaps supports the argument by Norwood that the phenomenon of reattachment on his inclined wall was a compressibility effect rather than a frictional effect.

### Wave formation in compressible flows

The results in Figs. 4 and 5 may be used to predict oblique shock formation in compressible flows. For example, consider a flow with a free stream Mach number of 1.8, that is, a minimum pressure  $p_1/p_0 = 0.174$  (with  $k = 1.4$ ). Figures 4 and 5 give the wave location and turning angle as  $x_i/w = 0.96$  and  $\delta = 18.8$  degrees, respectively. Equations 1 and 2 may then be solved to get  $\sigma = 61.1$  degrees and  $\sigma - \delta = 42.3$  degrees. The resulting predicted flow is shown plotted in Fig. 6.

## Implications for jet engine heat transfer

The 2-D, incompressible turbulent flow calculations of Moore, et al. [1989] showed an area of enhanced heat transfer on the pressure side of a turbine blade tip as shown in Fig. 7. The heat transfer was enhanced by up to 1.8 times the downstream fully developed value in the first two to three tip gap heights. The estimated intersection of the free streamline with the sidewall of  $x_i/w$  between 0.4 and 1.4, observed in Fig. 4, would indicate a shock forming within this region of already enhanced heat transfer. The shock-boundary layer interaction could serve to further enhance the heat transfer. Increased heat transfer would lead to higher metal temperatures and increased rates of oxidation and material weight loss. This would reduce both the expected useful life of the turbine blade and the turbine efficiency.

## References

Benson, R. S., and Pool, D. E., 1965, "Compressible Flow Through a Two-Dimensional Slit," *Int. J. Mech. Sci.*, Vol. 7, pp. 315-336.

Bindon, J. P., 1987, "Measurement of Tip Clearance Flow Structure on the End-Wall and within the Clearance Gap of an Axial Turbine Cascade," *I. Mech. E.* 1987-6, pp. 43-52, Int. Conf. on "Turbomachinery--Efficiency Prediction and Improvement," Cambridge, England.

Carriere, P., 1970, "Analyse Theorique du Decollement et du Recollement Turbulents au Bord de Fuite d'un Aubage aux Vitesses Supersoniques," *Flow Research on Blading*, L. S. Dzung, Ed., Elsevier Pub. Co., New York, pp. 210-242.

Elward, K. M., 1989, "Shock Formation in Overexpanded Flow--A Study Using the Hydraulic Analogy," M. S. Thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, April.

Hourmouziadis, J., and Albrecht, G., 1987, "An Integrated Aero/Mechanical Performance Approach to High Technology Turbine Design," AGARD Conference Proceedings No. 421 on Advanced Technology for Aero Gas Turbine Components, Paris, France.

Metzger, D. E., and Bunker, R. S., 1989, "Cavity Heat Transfer on a Transverse Grooved Wall in a Narrow Flow Channel," *ASME J. of Heat Transfer*, Vol. 111, pp. 73-79.

Metzger, D. E., Dunn, M. G., and Hah, C., 1991, "Turbine Tip and Shroud Heat Transfer," *ASME J. of Turbomachinery*, Vol. 113, pp. 502-507.

Moore, J., and K. M. Elward, 1992, "Shock Formation in Overexpanded Tip Leakage Flow," ASME Paper No. 92-GT-1.

Moore, J., Moore, J. G., Henry, G. S., and Chaudhry, U., 1989, "Flow and Heat Transfer in Turbine Tip Gaps," *ASME J. of Turbomachinery*, Vol. 111, July, pp. 301-309.

Norwood, R. E., 1961, "Two Dimensional Transonic Gas Jets," *Sc. D. Thesis*, Massachusetts Institute of Technology, Cambridge, Massachusetts, June.

Roelke, R. J., 1973, "Turbine Design and Application," Glassman, A. J., Ed., NASA SP-290, pp. 125-131.

Yaras, M., Yingkang, Z., and Sjolander, S. A., 1989, "Flow Field in the Tip Gap of a Planar Cascade of Turbine Blades," *ASME J. of Turbomachinery*, Vol. 111, No. 3, pp. 276-283.

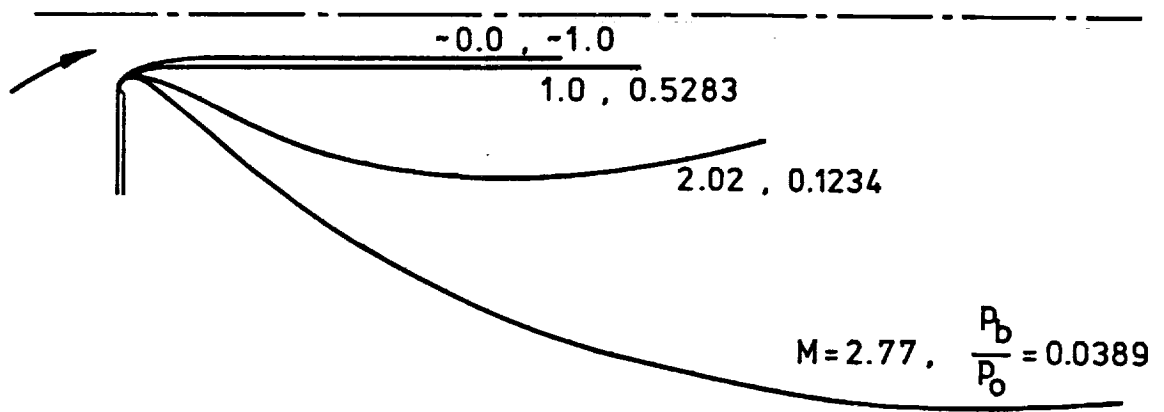


Figure 1. Free streamlines for inviscid flow of air from a half-slit with various back pressures; calculated by Benson and Pool

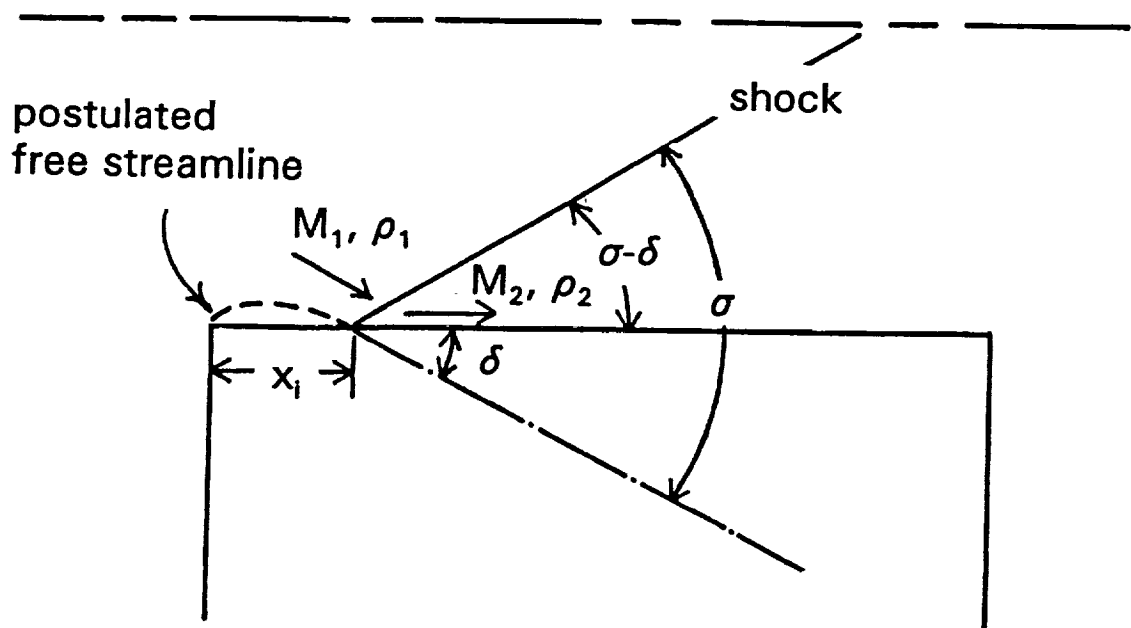


Figure 2. Model of shock formation



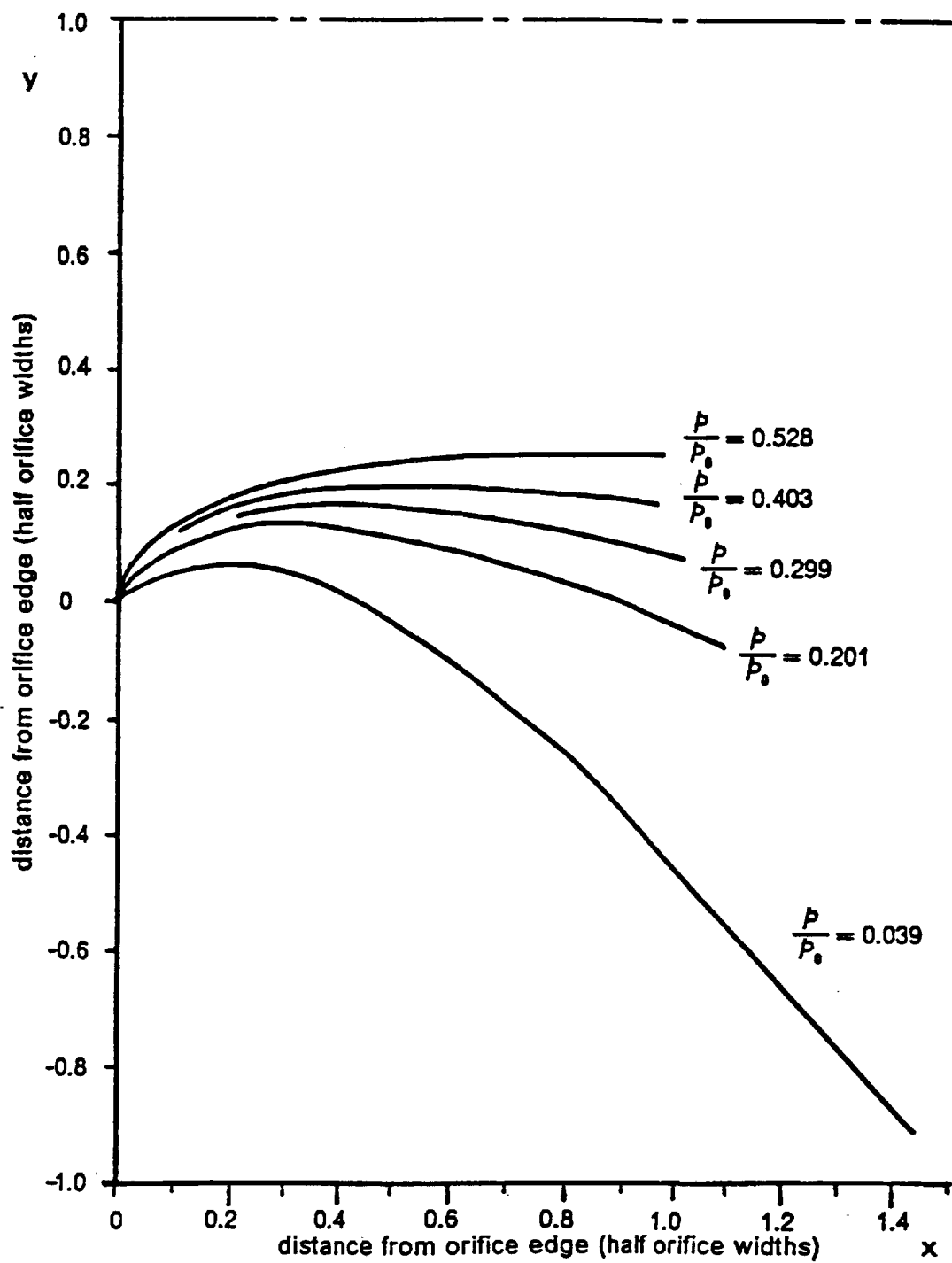


Figure 3. Norwood's calculated free streamlines from a half-slit

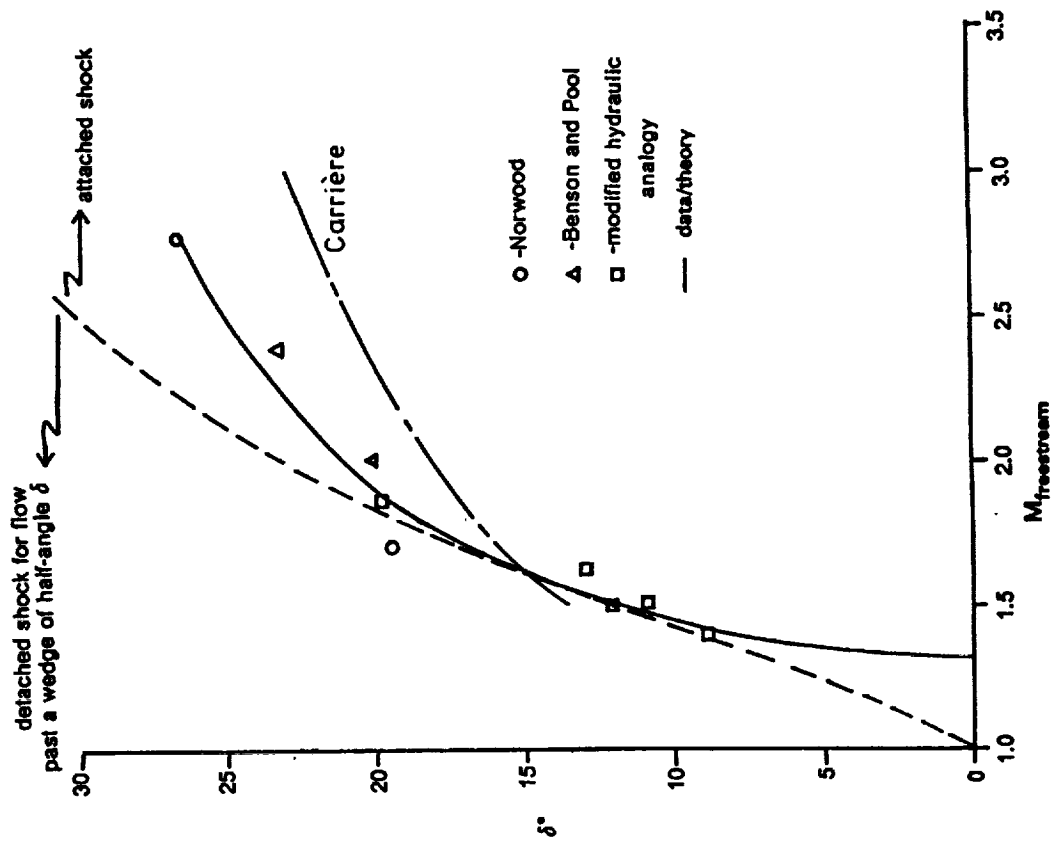


Figure 5. Compressible flow turning angles

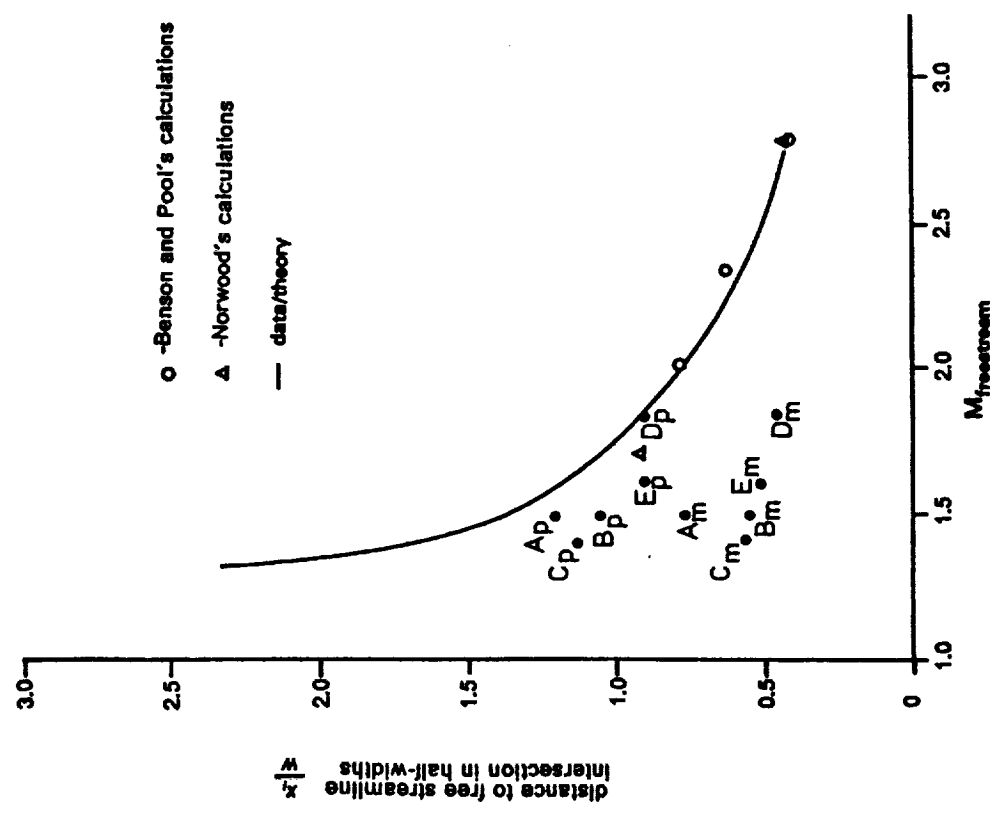
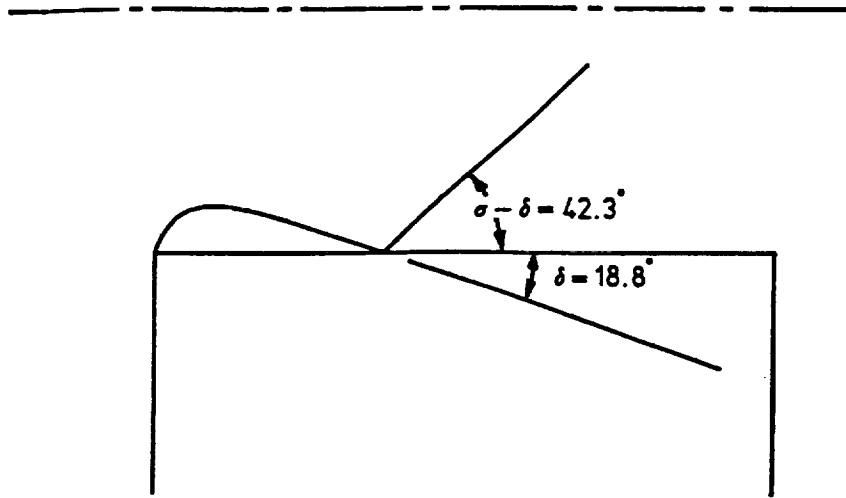
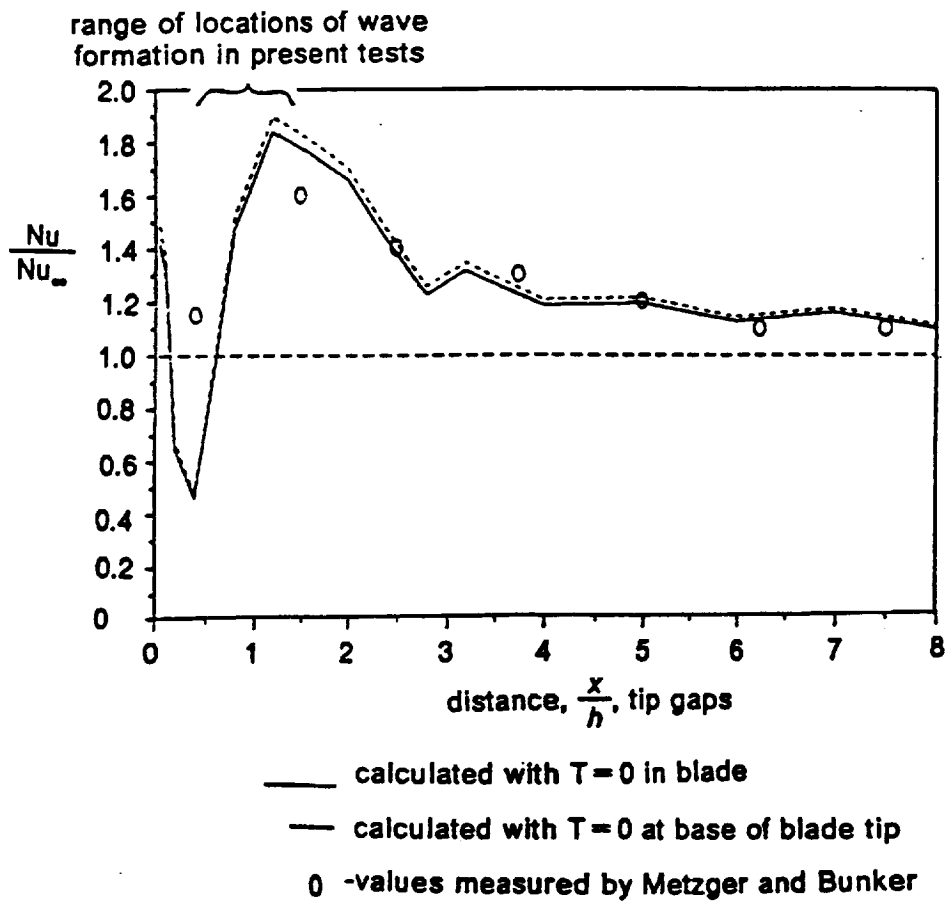


Figure 4. Comparison of free streamline intersection with channel sidewall



**Figure 6.** Predicted shock formation in compressible gas flow with  $M_{\text{freestream}} = 1.8$



**Figure 7.** Enhanced heat transfer to turbine blade tip

