Development of an Algebraic Stress/Two-Layer Model for Calculating Thrust Chamber Flow Fields

By

C. P. Chen, H. M. Shang and J. Huang

Department of Chemical Engineering
University of Alabama in Huntsville
Huntsville, AL 35899

ABSTRACT

Following the consensus of a workshop in Turbulence Modeling for Liquid Rocket Thrust Chambers, the current effort was undertaken to study the effects of second-order closure on the predictions of thermochemical flow fields. To reduce the instability and computational intensity of the full second-order Reynolds Stress Model, an Algebraic Stress Model (ASM) coupled with a two-layer near wall treatment was developed. Various test problems, including the compressible boundary layer with adiabatic and cooled walls, recirculating flows, swirling flows and the entire SSME nozzle flow were studied to assess the performance of the current model. Detailed calculations for the SSME exit wall flow around the nozzle manifold were executed. As to the overall flow predictions, the ASM removes another assumption for appropriate comparison with experimental data, to account for the non-isotropic turbulence effects.

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11th Workshop for CFD Applications in Rocket Propulsion
April 20-22, 1993
NASA- Marshall Space Flight center

- **Improve Predictive Capabilities of Turbulent Transport in Thrust Chamber**
- **Non-Isotropic and Compressibility Effects are the Focus of the Study**
- **Simplified Reynolds Stress Modeling**
- **Further Modeling in Turbulent Transport of Thermal Energy and Chemical Species -** $\overline{u_i'C'}$ **and** $\overline{u_i'T'}$ **etc.**

Motivation and Objective

- Higher Order Models Are Desirable For Calculating Thrust Chamber Flow Fields
  - --- 1991 Thrust Chamber Turbulence Modeling Workshop

- To Develop a Simplified 2nd-Order Turbulence Model For Thrust Chamber Flow Calculation
  - --- Near wall treatment
  - --- Efficiency and stability

APPROACH

- PDE's for Reynolds stress $\overline{u_i u_j}$ can be derived .
  Modeling any unknown in terms of Reynolds stress, the mean strain rate etc.

- Simplifications of the Differential Reynolds stresses Equations

— Algebraic Stress Model(ASM)

- Non-linear constitutive relations (Spezial)

APPROACH (DRS Equation)

- Differential Reynolds Stress Equation

$$\frac{D}{Dt}\overline{\rho u_i'' u_j''} = P_{ij} + D_{ij} + \pi_{ij} + C_{ij} - \varepsilon_{ij}$$

$$P_{ij} = -\overline{\rho}\,[\widetilde{u_i'' u_k''}\frac{\partial \tilde{u}_j}{\partial x_k} + \widetilde{u_j'' u_k''}\frac{\partial \tilde{u}_i}{\partial x_k}]$$ production

$$D_{ij} = \frac{\partial}{\partial x_k}[\overline{\rho}\,\widetilde{u_i'' u_j'' u_k''} + \delta_{ik}\overline{u_j'' p'} + \delta_{jk}\overline{u_i'' p'} - (\overline{\mu S_{ik} u_j''} + \overline{\mu S_{jk} u_i''})]$$ diffusion

$$\pi_{ij} = \overline{p'(\frac{\partial u_i''}{\partial x_j} + \frac{\partial u_j''}{\partial x_i})}$$ pressure-strain correlation

$$C_{ij} = -[\,\widetilde{u_i''}\frac{\partial \overline{P}}{\partial x_j} + \widetilde{u_j''}\frac{\partial \overline{P}}{\partial x_i}]$$ compressibility

$$\varepsilon_{ij} = \overline{\mu\,[S_{ik}\frac{\partial u_j''}{\partial x_k} + S_{jk}\frac{\partial u_i''}{\partial x_k}]}$$ dissipation

APPROACH --- ASM

- Similitude Principle (Mellor and Yamada)

$$P_{ij} - \frac{2}{3}\delta_{ij} P_k + \phi_{ij} + C_{ij} \cong 0$$

- Algebraic Reynolds Stress Model of Rodi

$$\frac{D}{Dt}\overline{\rho u_i'' u_j''} - D_{ij} \cong \frac{\widetilde{u_i'' u_j''}}{k}\left[\frac{Dk}{Dt} - D_k \right]$$

$$\Rightarrow \quad [P_{ij} + \phi_{ij} + C_{ij} - \varepsilon_{ij}] = \frac{\widetilde{u_i'' u_j''}}{k}[P_k - \varepsilon]$$

APPROACH ---Pressure-Strain Term

$$\pi_{ij} = \overline{p'\left(\frac{\partial u_i''}{\partial x_j} + \frac{\partial u_j''}{\partial x_i}\right)}$$

return to isotropy	$= -C_1 \frac{\varepsilon}{k} (\overline{\rho u_i'' u_j''} - \frac{2}{3} \delta_{ij} k)$	Rotta Model
rapid term	$-C_2 (P_{ij} - \frac{2}{3} \delta_{ij} P_k)$	IP Model
wall damping term	$+\pi_{ijw}$	Lumped with the two-layer model
in which	$P_k = \frac{1}{2} P_{ii}$	

- $$\frac{\overline{\rho u_i'' u_j''}}{k} = \frac{(1-C_2)(P_{ij} - \frac{2}{3}\delta_{ij}P_k) - \frac{2}{3}\delta_{ij}C_k + C_{ij}}{P_k + \varepsilon(C_1 - 1) + C_k} + \frac{2}{3}\delta_{ij}$$

where

$$P_{ij} = -\bar{\rho}\,[\widetilde{u_i'' u_k''}\frac{\partial \tilde{u}_j}{\partial x_k} + \widetilde{u_j'' u_k''}\frac{\partial \tilde{u}_i}{\partial x_k}]$$

$$P_k = \frac{1}{2}P_{ii}$$

k- ε Equations

- $$\frac{\partial \rho k}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_j k) = \frac{\partial}{\partial x_j}\left(C_k \rho \frac{k}{\varepsilon} \overline{u_j'' u_l''} \frac{\partial k}{\partial x_l}\right) + \mu_t G - \rho \varepsilon$$

- $$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial}{\partial x_j}(\rho U_j \varepsilon) = \frac{\partial}{\partial x_j}\left(C_\varepsilon \rho \frac{k}{\varepsilon} \overline{u_j'' u_l''} \frac{\partial \varepsilon}{\partial x_l}\right) + \frac{\varepsilon}{k}(C_{\varepsilon 1} \mu_t G - C_{\varepsilon 2} \rho \varepsilon)$$

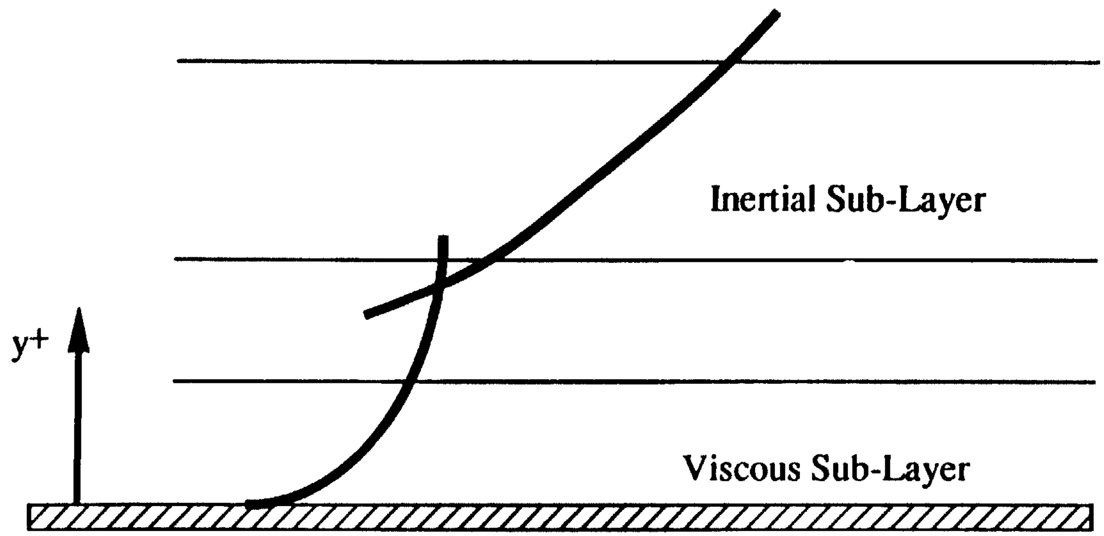
where the production term $\mu_t G$ takes the form

$$\mu_t G = -\rho \overline{u_i'' u_j''} \frac{\partial U_i}{\partial x_j} - \frac{\mu_t}{\rho^2} \frac{\partial \rho}{\partial x_j} \frac{\partial P}{\partial x_j}$$

Turbulence model costants

$C_{\epsilon 1}$	$C_{\epsilon 2}$	C_k	C_ϵ	C_1	C_2
1.45	**1.92**	**0.22**	**0.15**	**2.5**	**0.5**

Two-Layer Wall Treatment



Outer Layer ---
resolved by
ASM

Inner Layer ---
Patched with a One-Equation
Model

Matching at

$$R_k = \frac{k^{1/2} y}{\mu} = 200$$

METHODOLOGIES

$$\nu_t = u' l' = C_\mu \, \kappa^{1/2} l_\mu = C_\mu \frac{\kappa^2}{\varepsilon} = C_\mu \frac{l_\varepsilon}{\kappa^{3/2}} \kappa^2$$

within Intertia Sublayer $\varepsilon = C_\mu^{3/4} \frac{\kappa^{3/2}}{l_\mu} = \frac{\kappa^{3/2}}{l_\varepsilon}$

$l_\mu = C_l y \left[1 - \exp\left(-\frac{R_\kappa}{A_\mu} \right) \right]$ ⟶ to be used in Eddy Viscosity

$l_\varepsilon = C_l y \left[1 - \exp\left(-\frac{R_\kappa}{A_\varepsilon} \right) \right]$ ⟶ to be used in k -equation

$$C_l = \kappa C_\mu^{-3/4}$$

Matching at $R_\kappa = \frac{\kappa^{1/2} y}{\nu}$

IMPLEMENTATIONS

- Implemented into MAST-2D
- Non-Staggered Grids, Sequential Solver
- Chakravarthy-Osher TVD Scheme
- PISO-C Algorithm
- Conjugate Gradient Matrix Solver
- Time Marching

Validations

- Incompressible & Compressible Flat Plate
  Cooled & Heated Wall, Up To Mach 10
- Incompressible & Compressible Recirculating Flows
- Incompressible Swirling Flows
- Thrust Chamber Flows

Compressible Flat Plate Flow

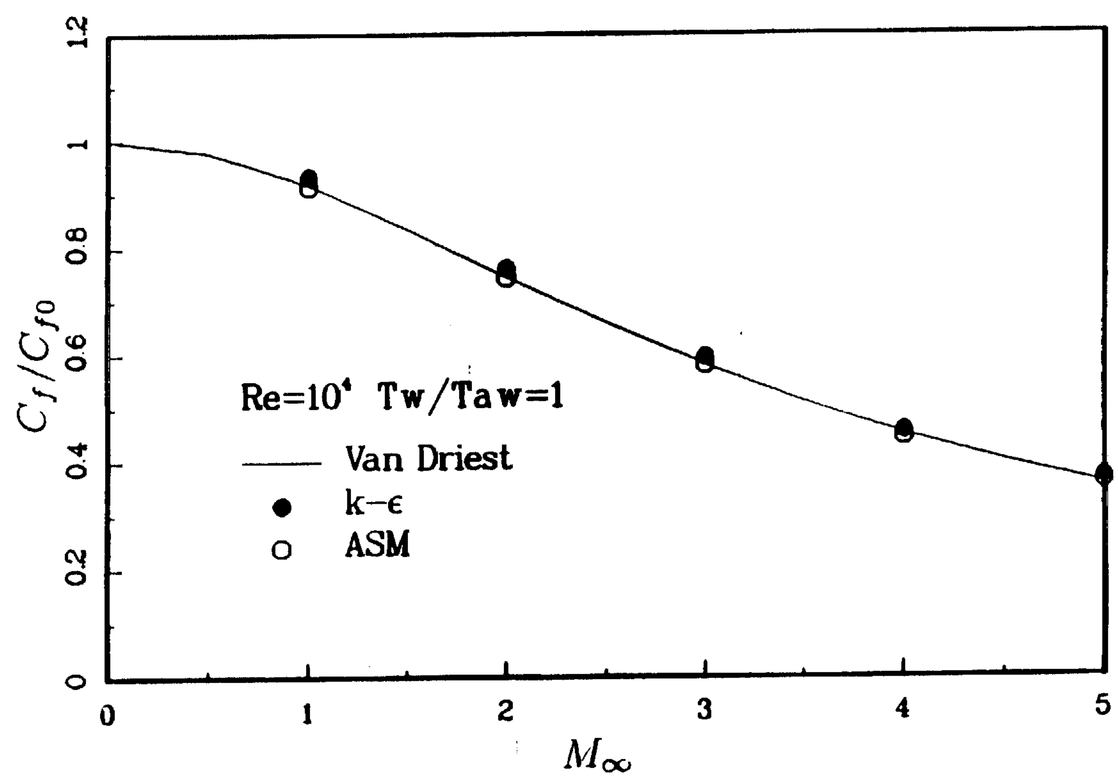


Fig. 1 Variation of C_f/C_{f0} with M_∞ for adiabatic wall boundary condition.

Compressible Flat Plate Flow

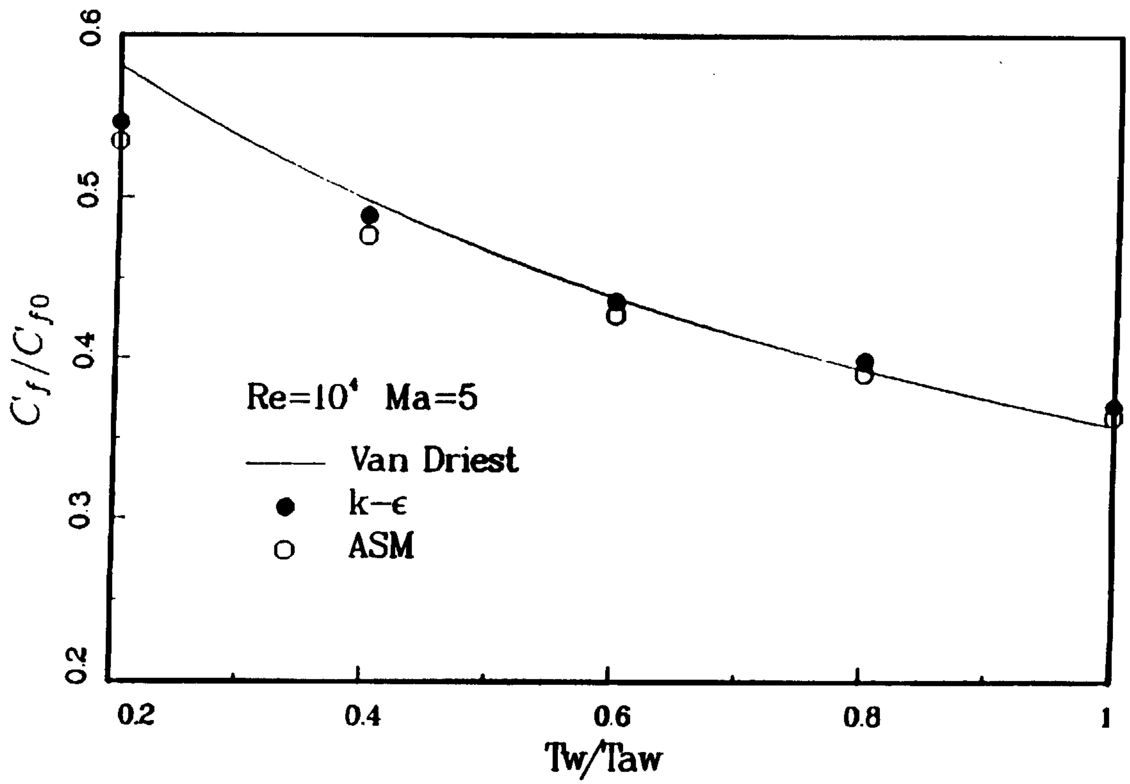


Fig. 2 Variation of C_f/C_{f0} with T_w/T_{aw} for $M_\infty = 5.0$.

Compressible Flat Plate Flow

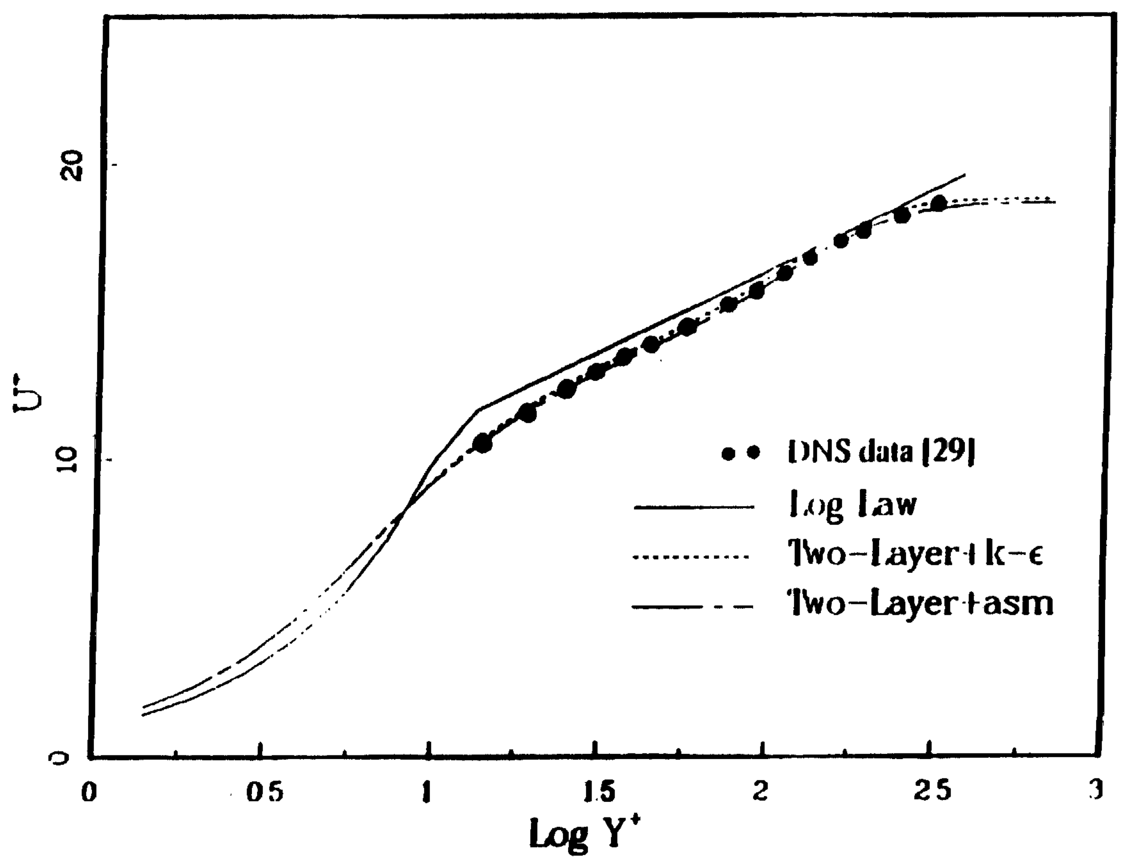
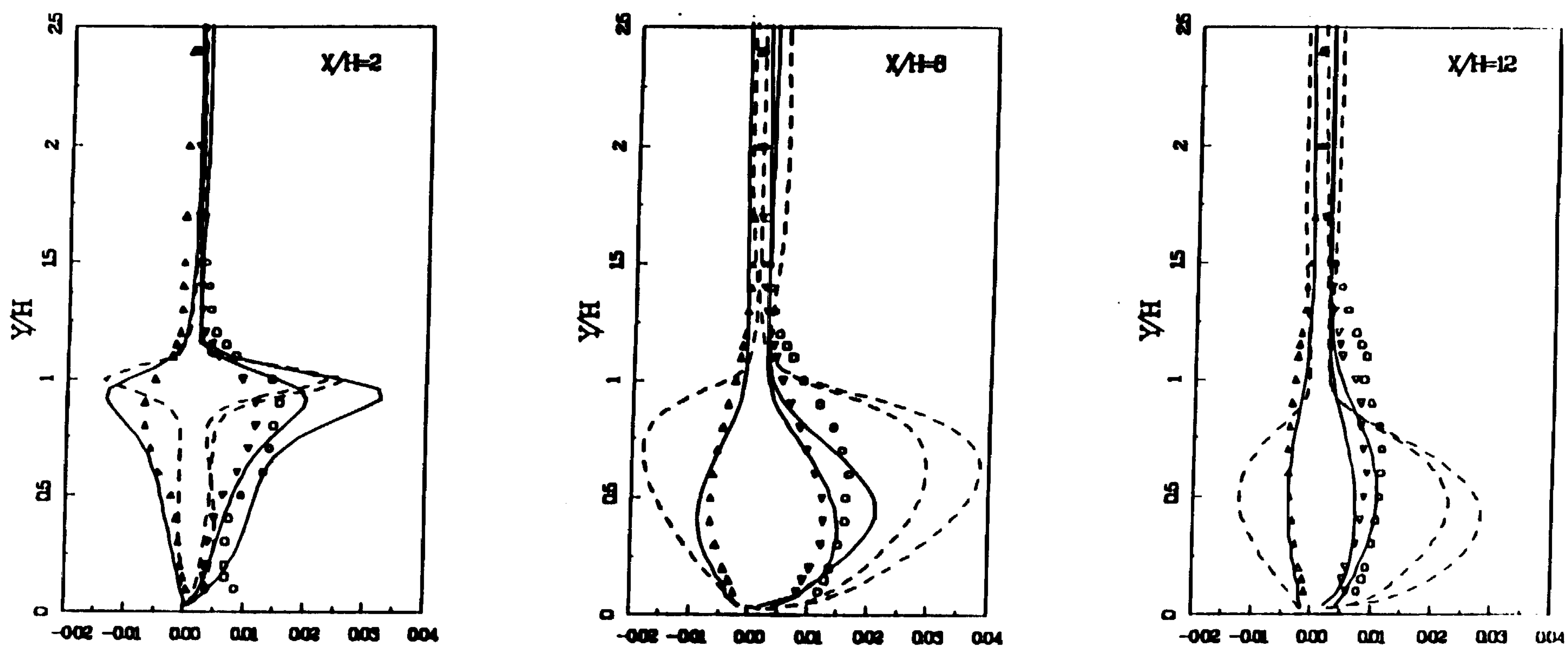


Fig. 3 Semi-log plots of u_c^+ for adiabatic wall boundary condition.

Backward Facing Step



ŧeattachment length: k-ε 4.67H
ASM 5.94H
EXP. ~6H

----- k − ε —— k − ε/ASM data: O $\overline{u_1u_1}$ ∇ $\overline{u_2u_2}$ Δ $\overline{u_1u_2}$

Fig. 5 Reynolds stress profiles for the backward-facing step turbulent flow (9:1), with data from [31]

Confined Swirling Flows

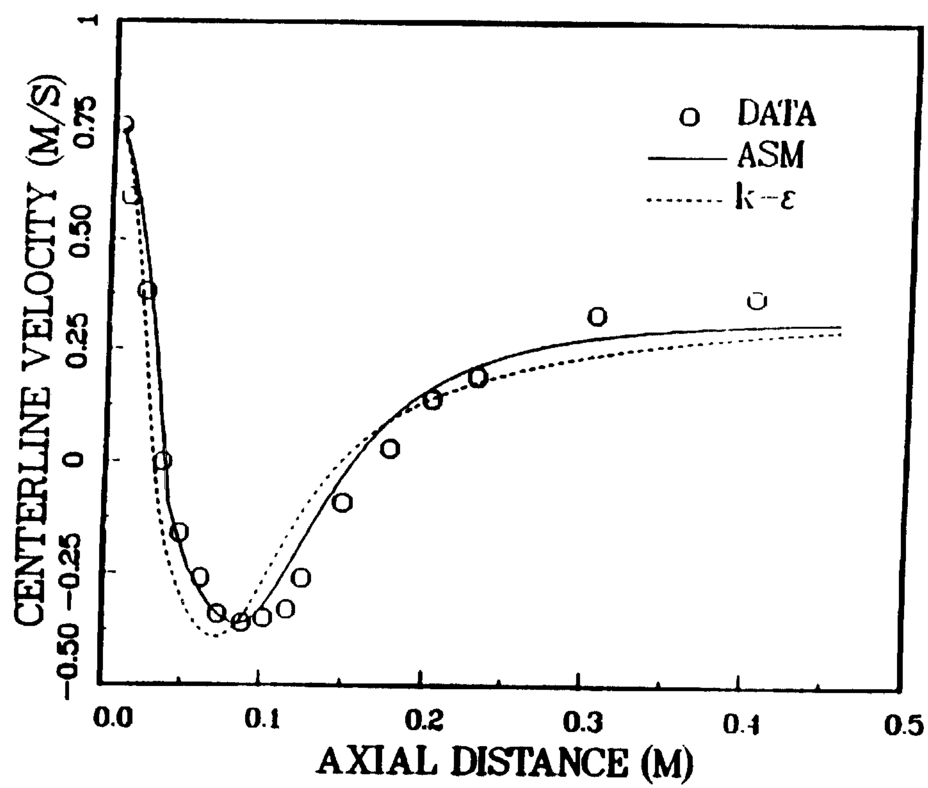


Fig. 6 Decay of mean axial centerline velocity.

Confined Swirling Flows

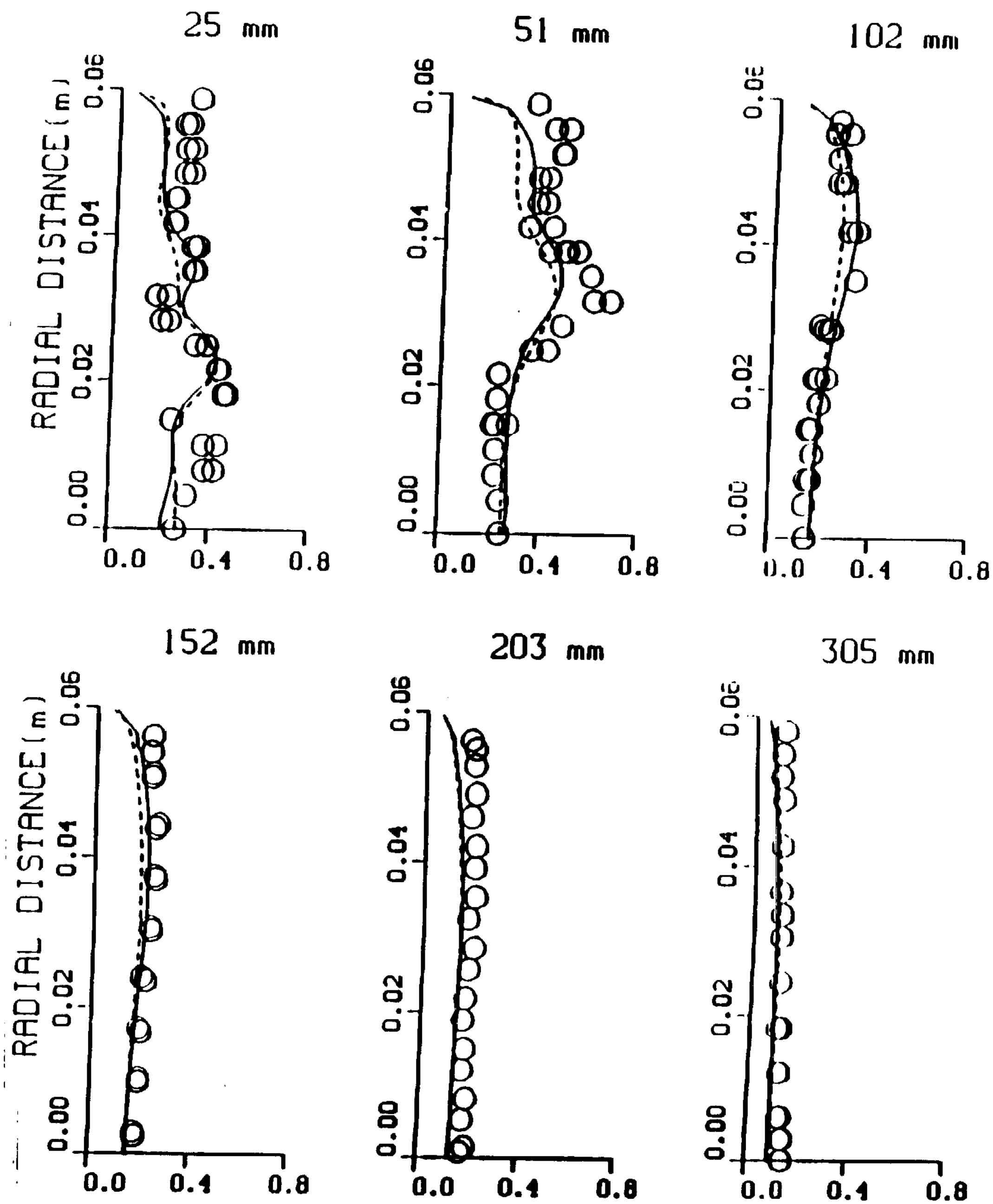


Fig. 8a Radial profiles of turbulent intensity($\sqrt{\overline{u'^2}}$)

Confined Swirling Flows

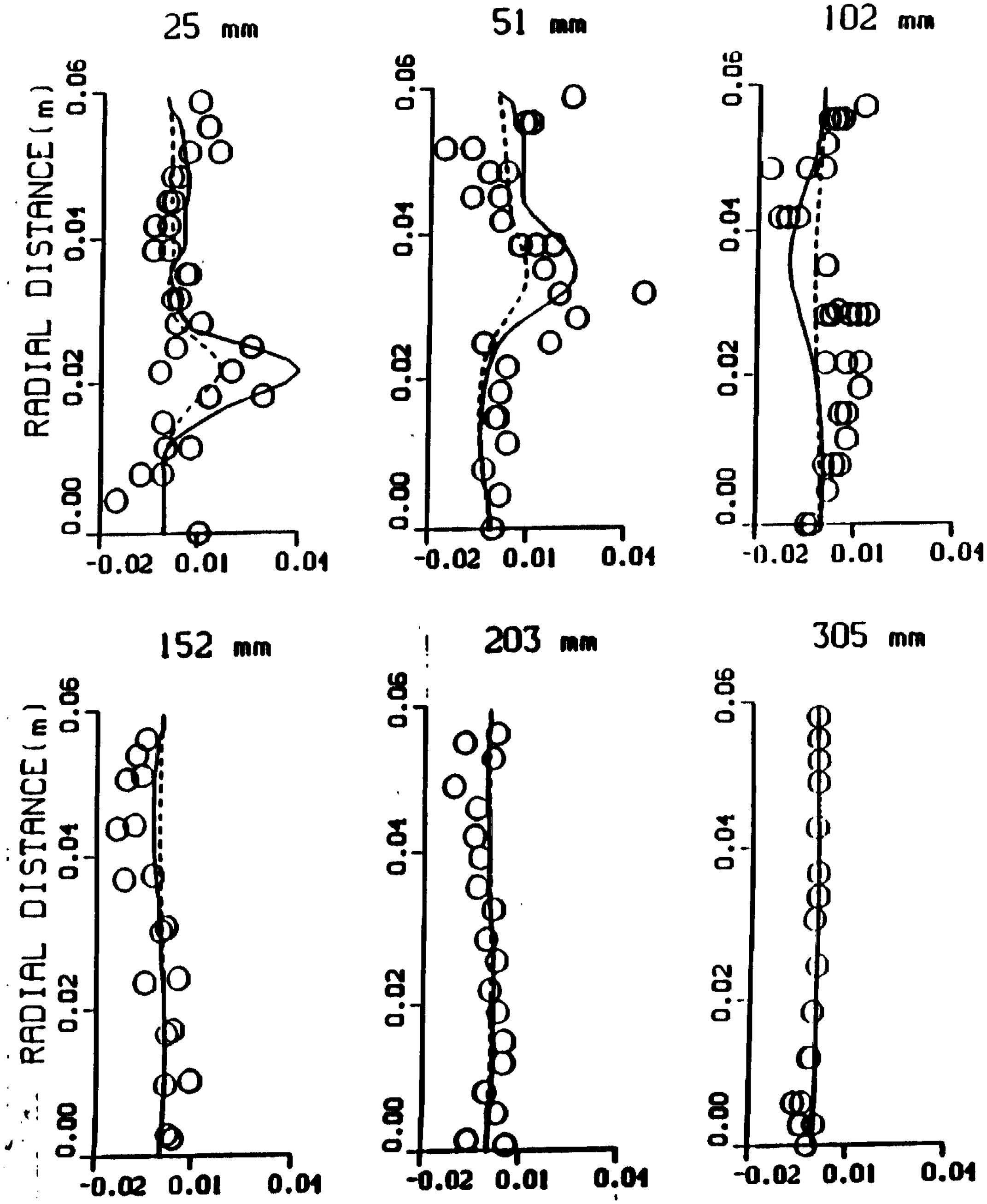


Fig. 8b Radial profiles of Reynolds stress($\overline{u'w'}$)

SSME Nozzles

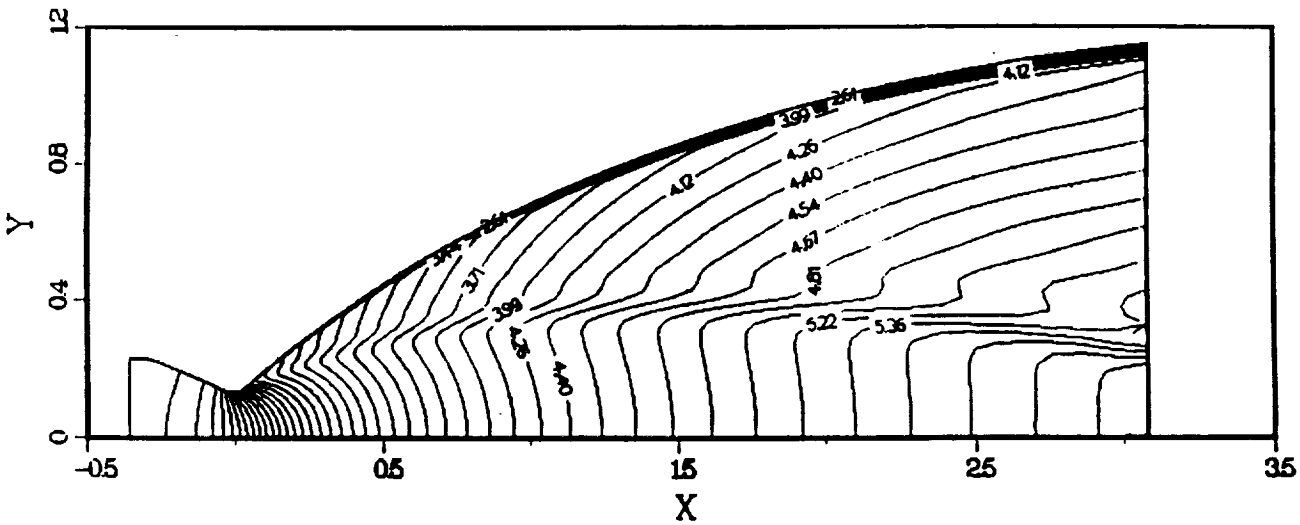


Fig. 9a Contour of Mach number for k-ε with two-layer model.

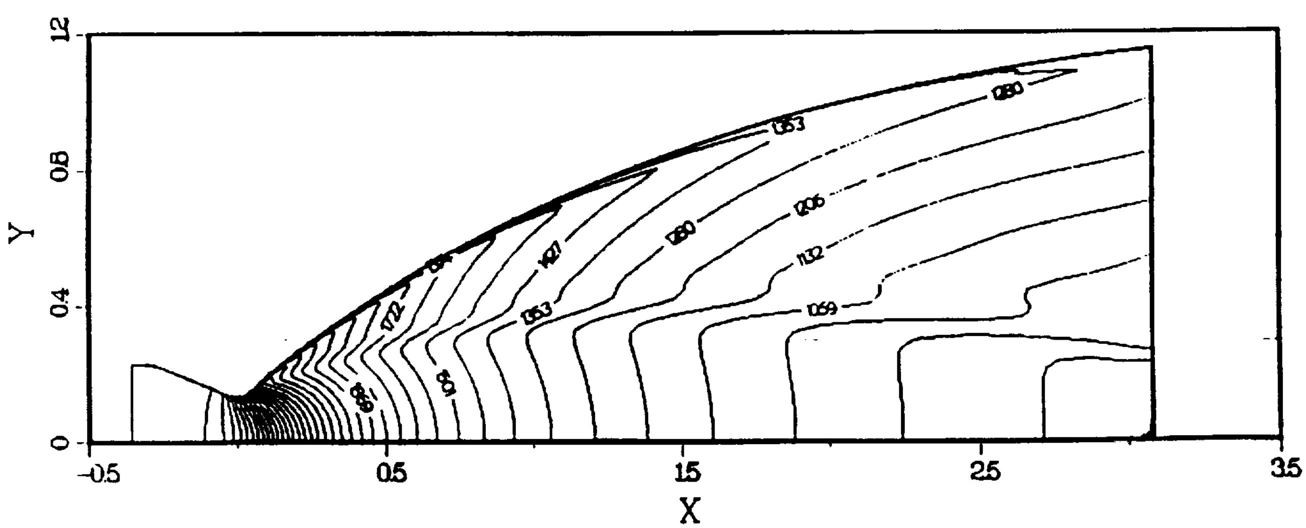


Fig. 10a Contour of temperature for k-ε with two-layer model.

SSME Nozzles

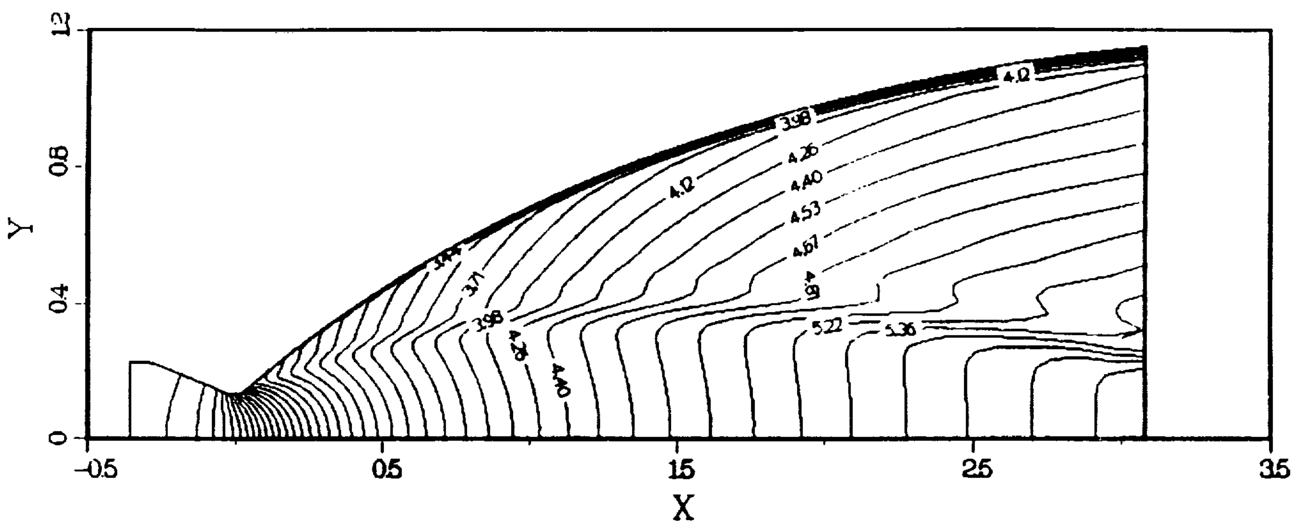


Fig. 9b Contour of Mach number for ASM with two-layer model.

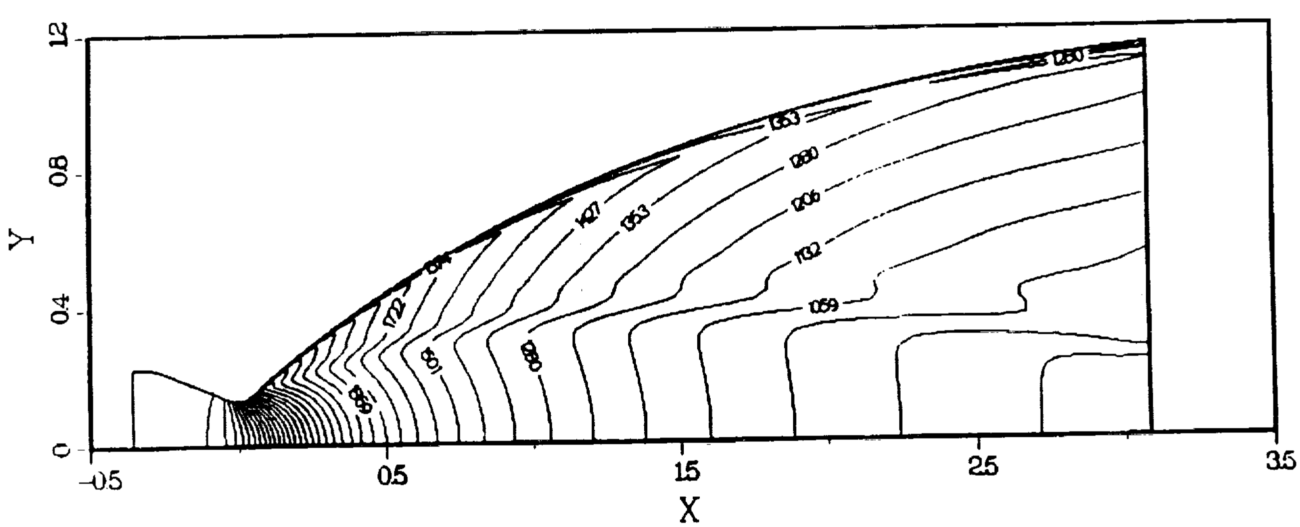


Fig. 10b Contour of temperature for ASM with two-layer model.

8 - STEP REACTIONS

		A	N	E
$M + O_2$	==== $O + O$	0.72000E+19	-1.0000	117908
$M + H_2$	==== $H + H$	0.55000E+19	-1.0000	103298
$M + H_2O$	==== $H + OH$	0.52000E+22	-1.5000	118000
$O + H$	==== OH	0.71000E+19	-1.0000	0.
$H_2O + OH$	==== $H_2O + H$	0.58000E+14	0.0000	18000
$H_2 + OH$	==== $H_2O + H$	0.20000E+14	0.0000	5166
$O_2 + H$	==== $OH + O$	0.22000E+15	0.0000	16800
$H_2 + O$	==== $OH + H$	0.75000E+14	0.0000	11099

$$k = AT^N \exp(-E/RT)$$

with k in $cm^3 \cdot mole^{-1} \cdot s^{-1}$ and E in $cal \cdot mole^{-1}$

From R.C.Rogers and Chinitz, ' Using a global hydrogen-air model in turbulent flow calcultions ', AIAA J., vol. 21, pp. 586-592, 1983.

SSME Nozzles

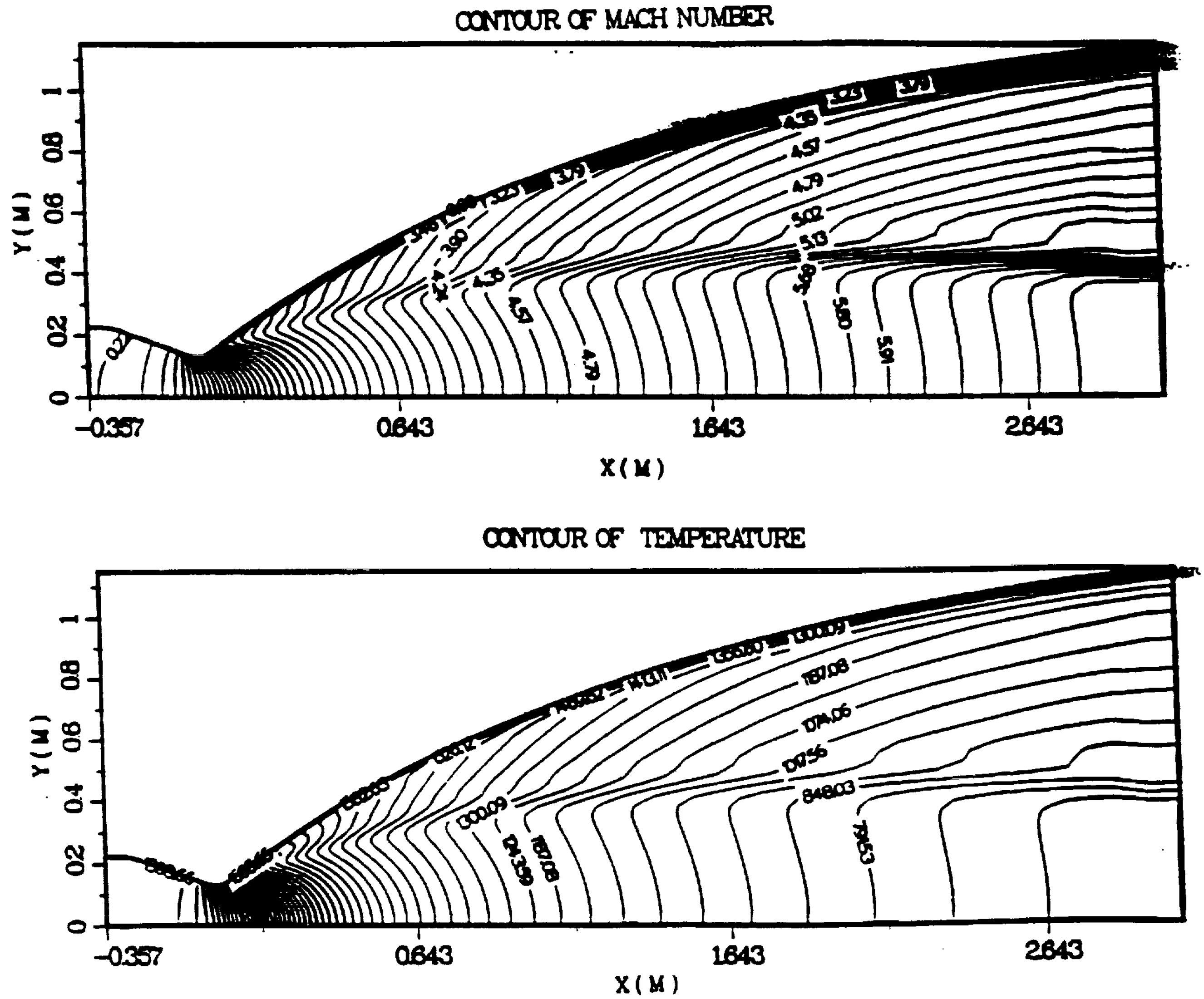


Figure A.5 Sample SSME Nozzle Flow Inputs and Results --- Turbulent, 8-Step Kinetics.
ISP = 452.78 sec. Exp. ISP=453.3sec

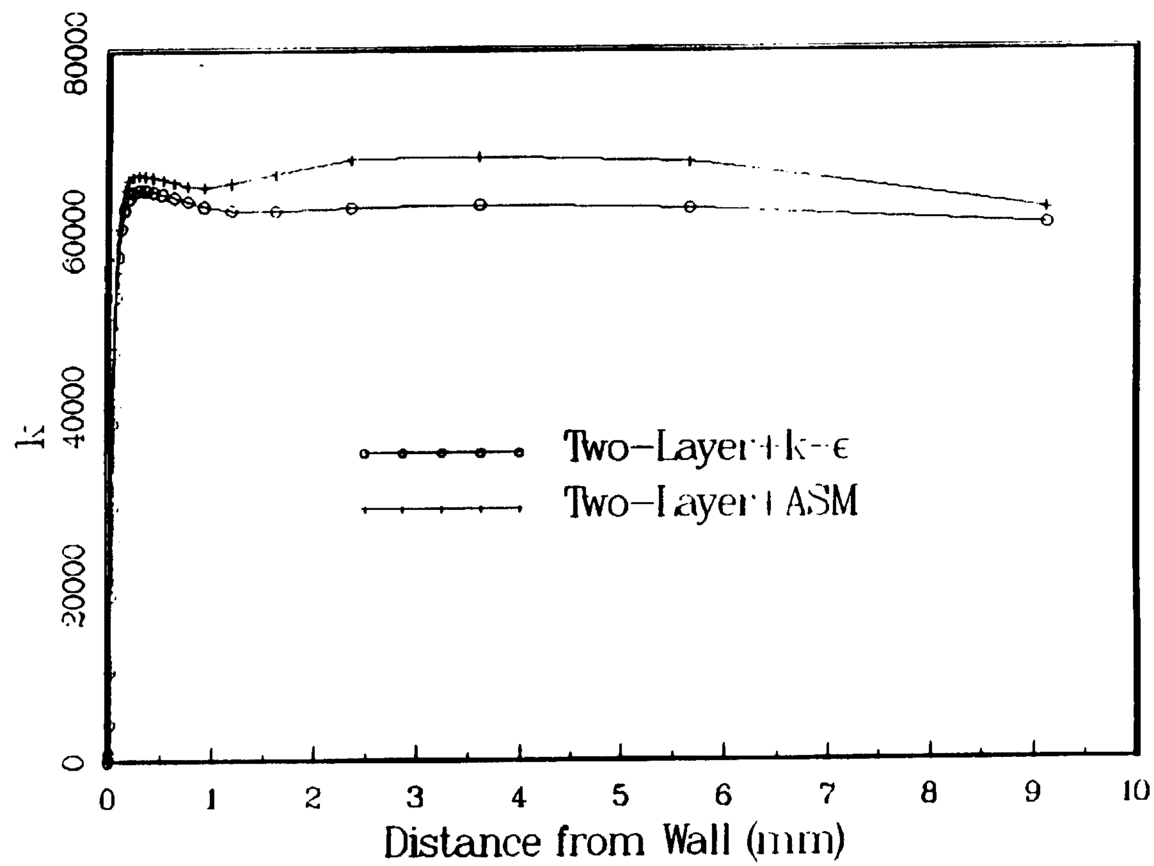


Fig. 11 Behavior of near wall TKE at nozzle exit.

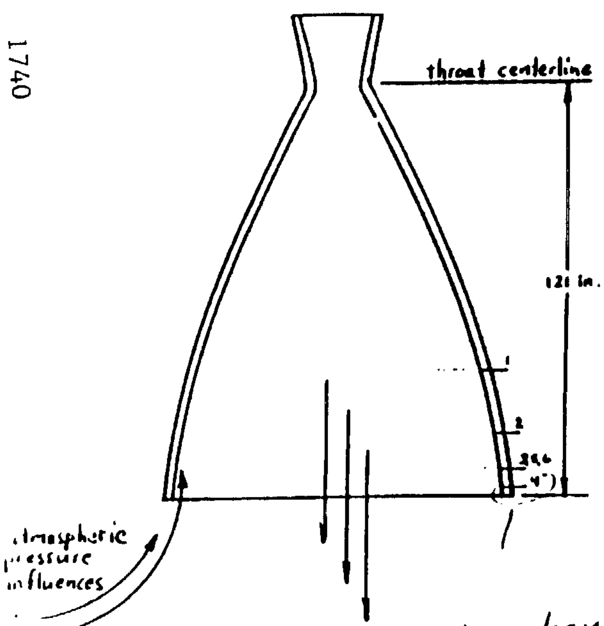
George C. Marshall Space Flight Center
Science and Engineering Directorate

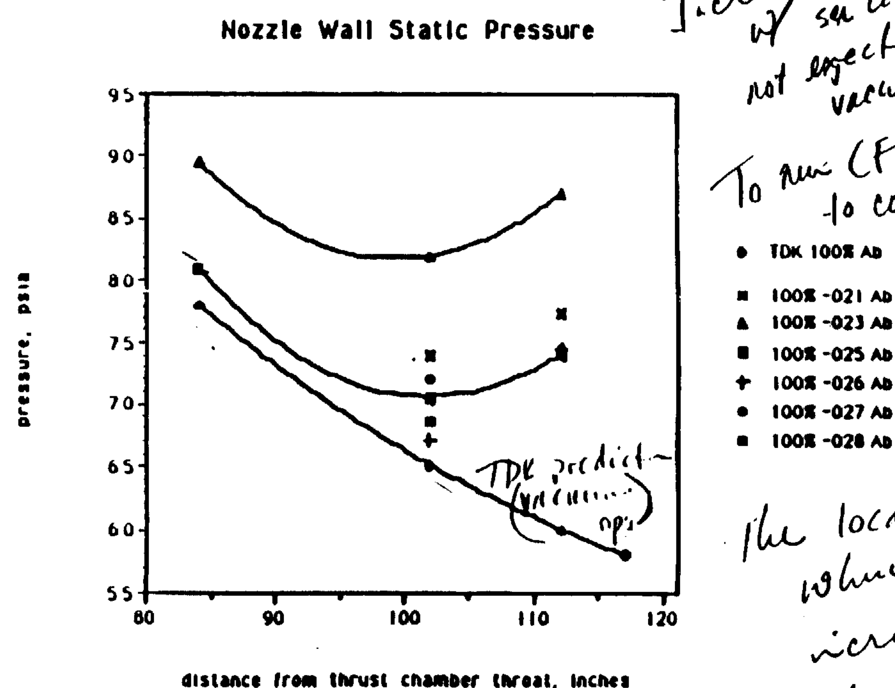
Propulsion Laboratory
Turbomachinery and Combustion
Devices Branch/EP62

Testbed Engine 3001 Summary Review
Combustion Devices (Tests 020-028)

National Aeronautics and Space Administration

Nozzle Wall Static Pressure

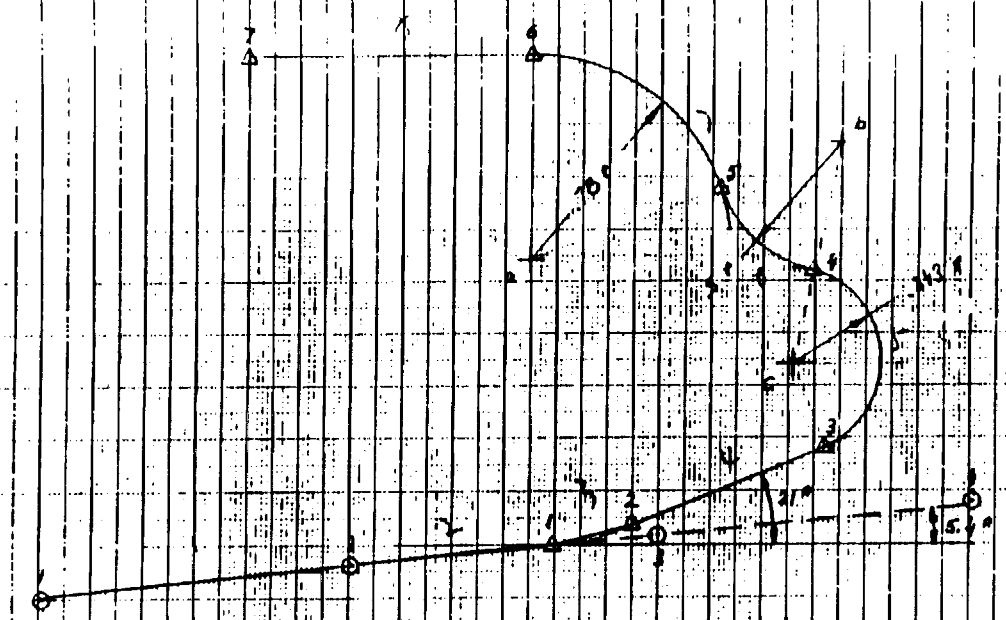




Per Klaus:
Increase associated w/ sea level.
not expected at vacuum conds.
To run CFD calcs to compare

The location where P increases is a surprise. It is farther upstream than expected

SSME Wall Contour at Exit



$R_T = 5.1527$ [in]

	x [in]	R [in]	x/R_T	R/R_T
1 ○	8.7348	45.008	22.8246	8.7348
2 ○	8.7583	45.129	23.0577	8.7583
3 ○	8.7812	45.247	23.2903	8.7812
4 ○	8.8034	45.361	23.5292	8.8034
a +	119.478	46.287	23.1875	8.9831
b +	120.7	46.73	23.4046	9.0690
c +	120.518	45.887	23.3893	8.9054

	x [in]	R [in]	x/R_T	R/R_T
1 △	119.528	45.207	23.2069	8.7735
2 △	119.94	45.28	23.2694	8.7816
3 △	120.64	45.06	23.4130	8.8032
4 △	120.60	46.24	23.4052	8.9739
5 △	120.24	46.56	23.3353	9.0360
6 △	119.50	47.07	23.1917	9.1350
7 △	118.40	47.07	22.9782	9.1350

117 118 119 120 121 122 123 x [in]

Fig. 12, SSME wall contour and geometry at exit

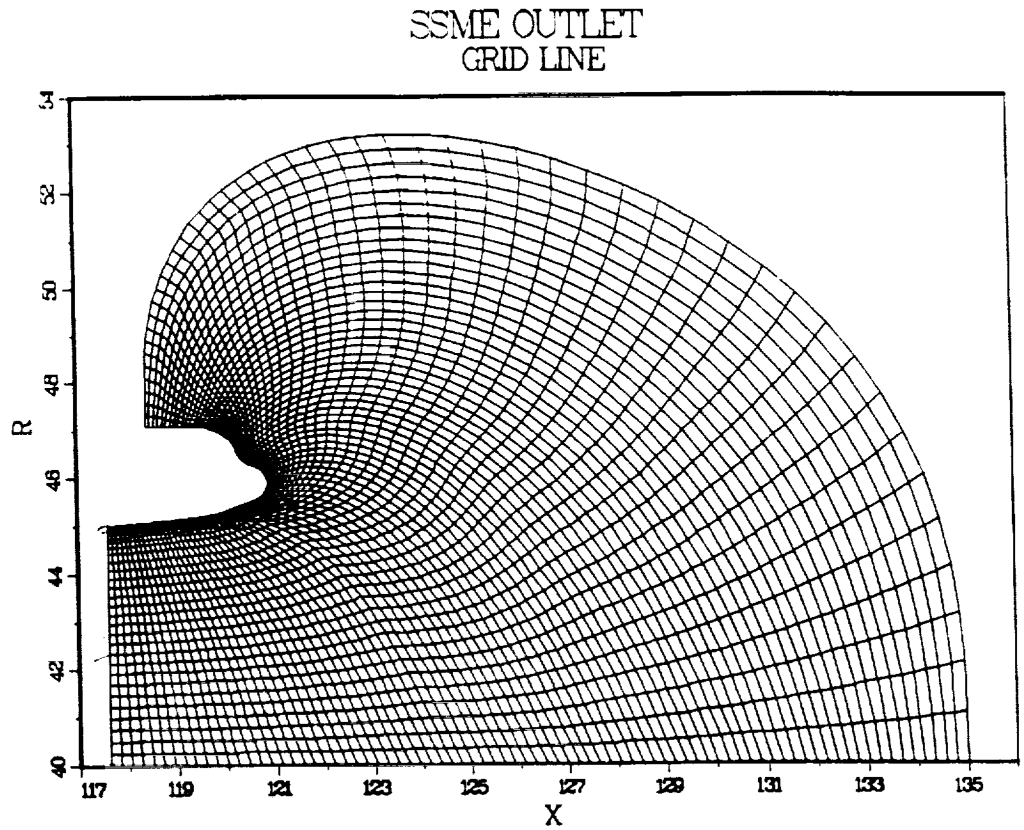


Fig. 13(a), Grid configurations for SSME nozzle exit manifold

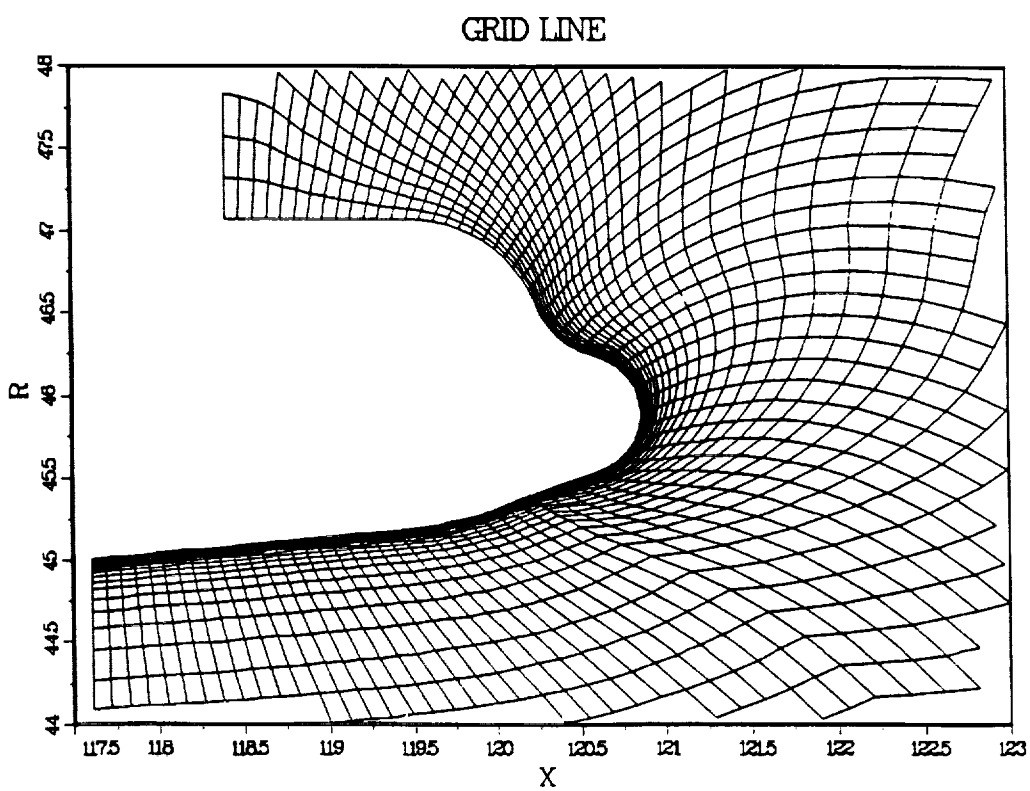
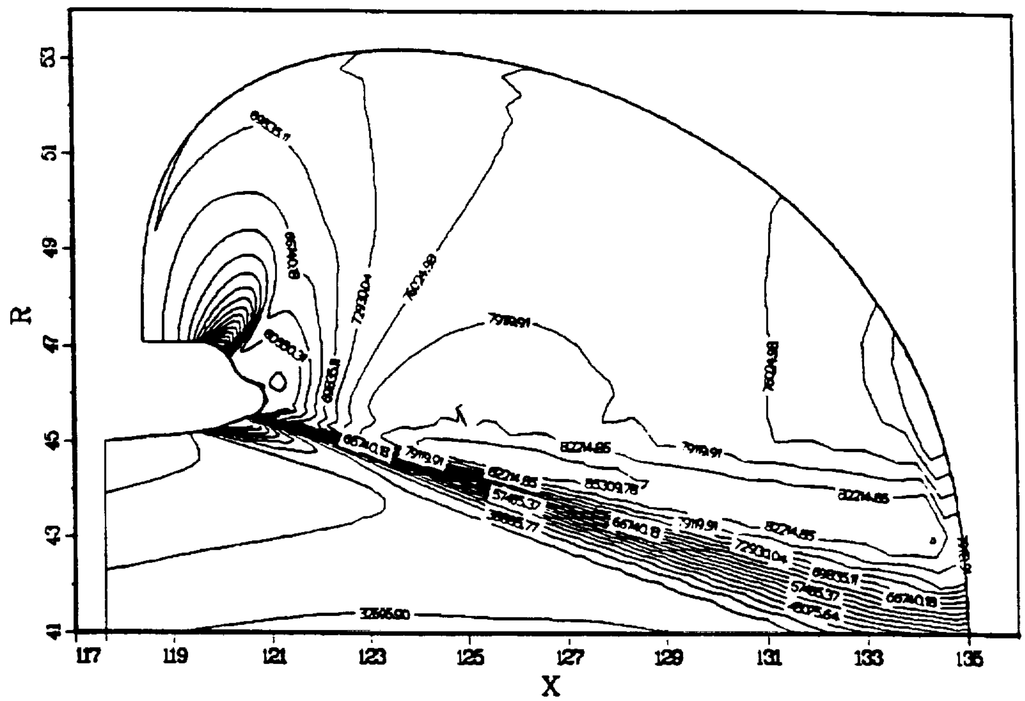
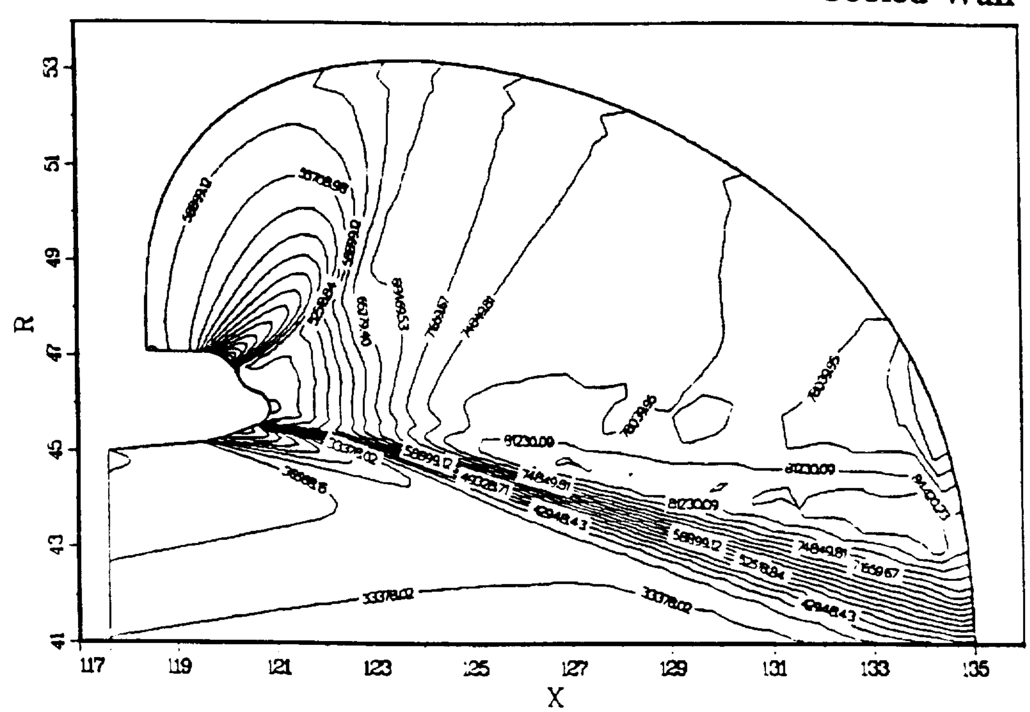


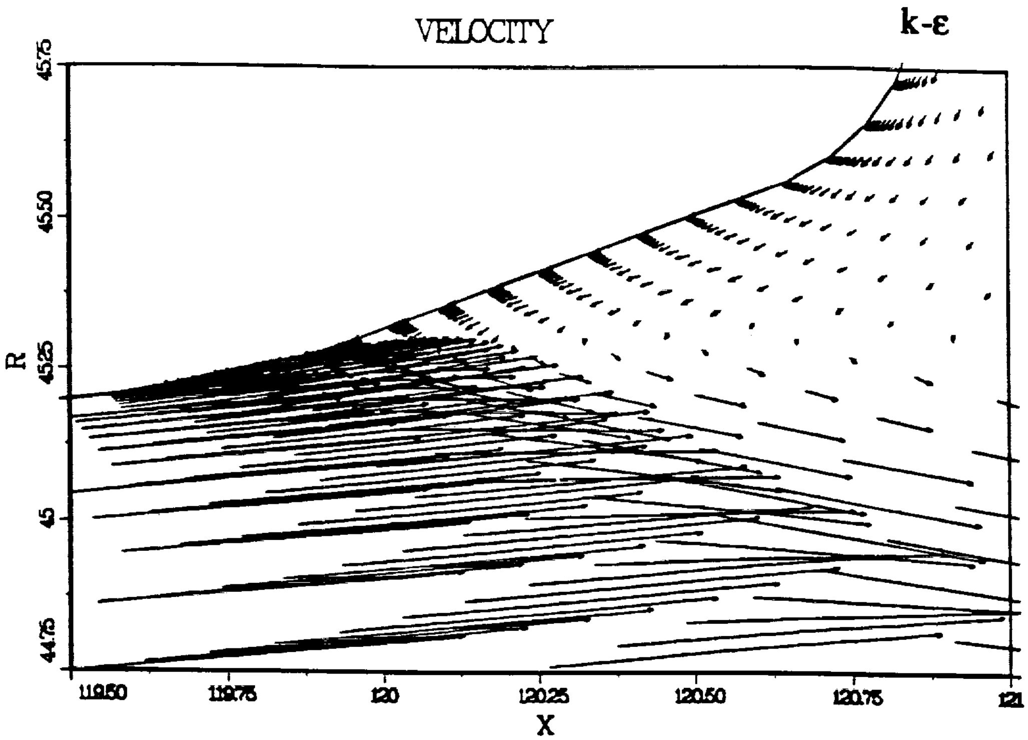
Fig. 13(b), Close-up grids for Figure 13(a)

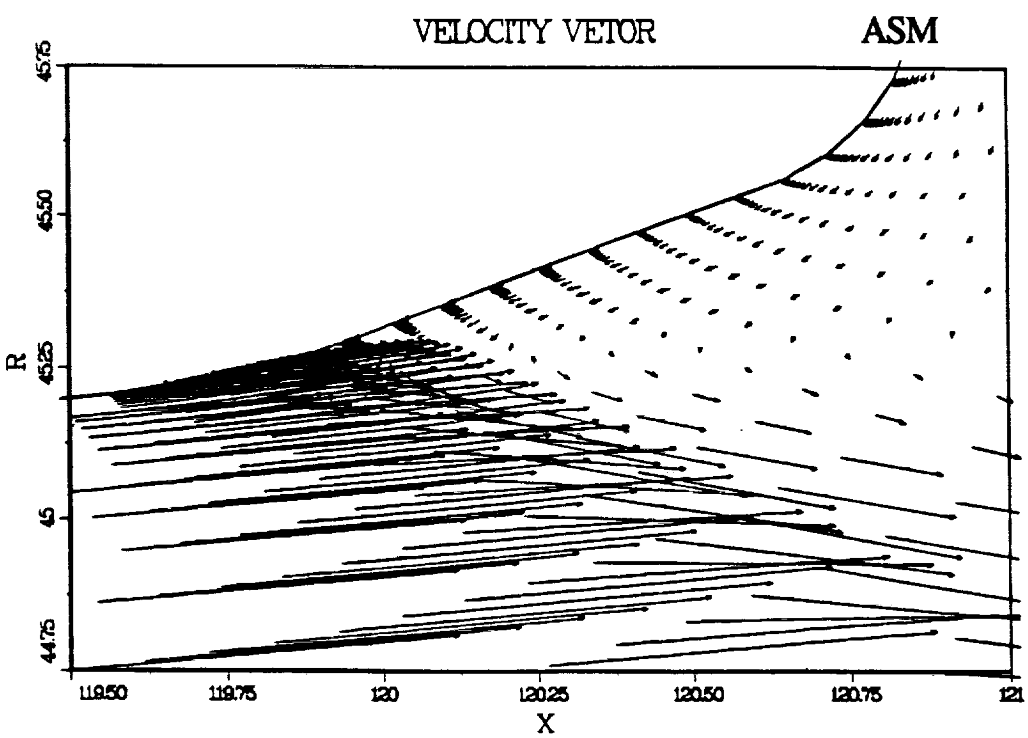
SSME OUTLET
CONTOUR OF PRSSURE
Uncooled-Wall



SSME OUTLET
CONTOUR OF PRSSURE
Cooled-Wall




VELOCITY
k-ε
R
45.75
45.50
45.25
45
44.75
119.50
119.75
120
120.25
120.50
120.75
121
X


VELOCITY VETOR
ASM
R
45.75
45.50
45.25
45
44.75
119.50
119.75
120
120.25
120.50
120.75
121
X

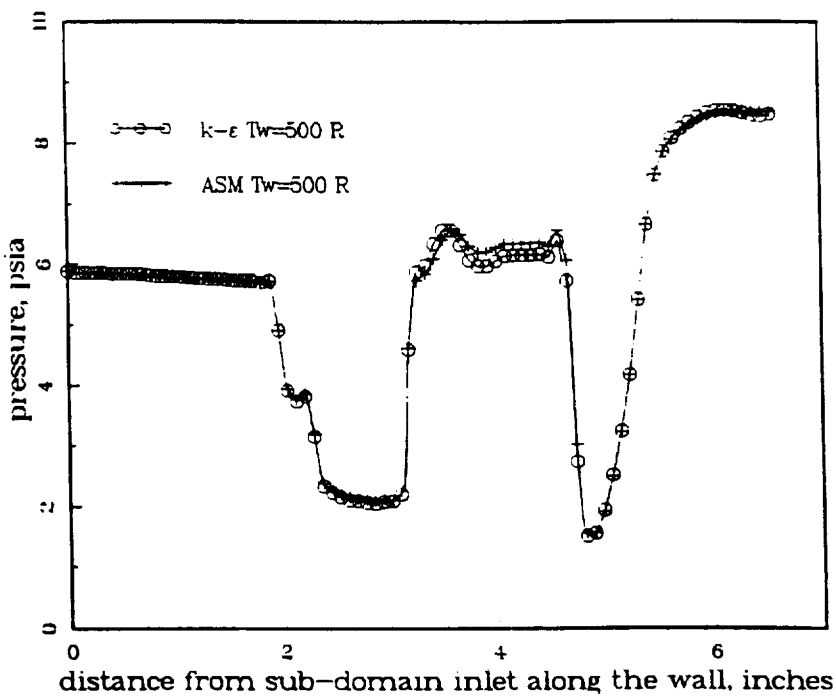


Fig. 17(a), Pressure levels along the wall near the nozzle exit using the ASM and $k-\varepsilon$ models

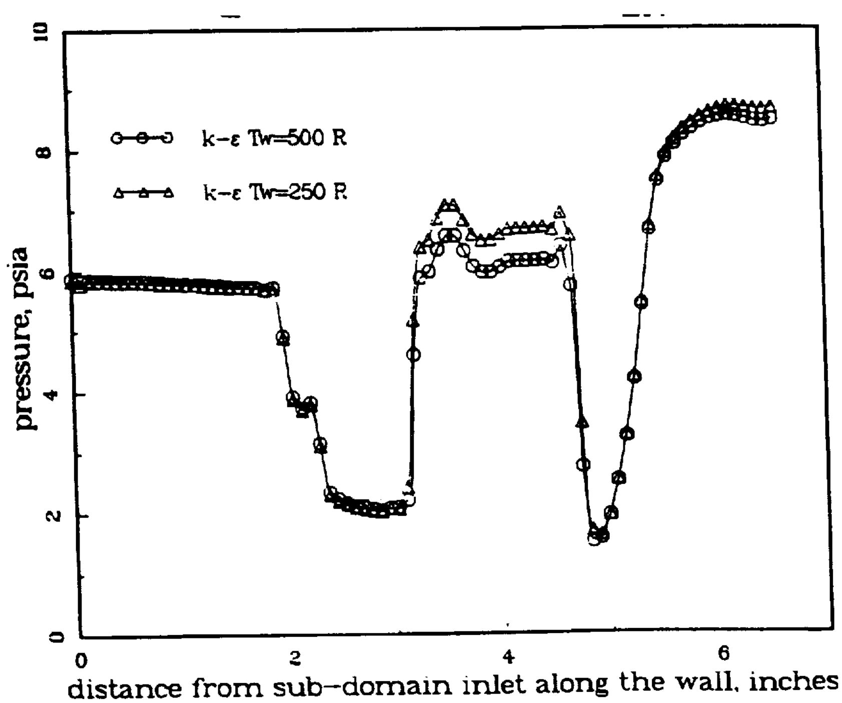


Fig. 17(b), Effects of wall temperature on the wall pressure

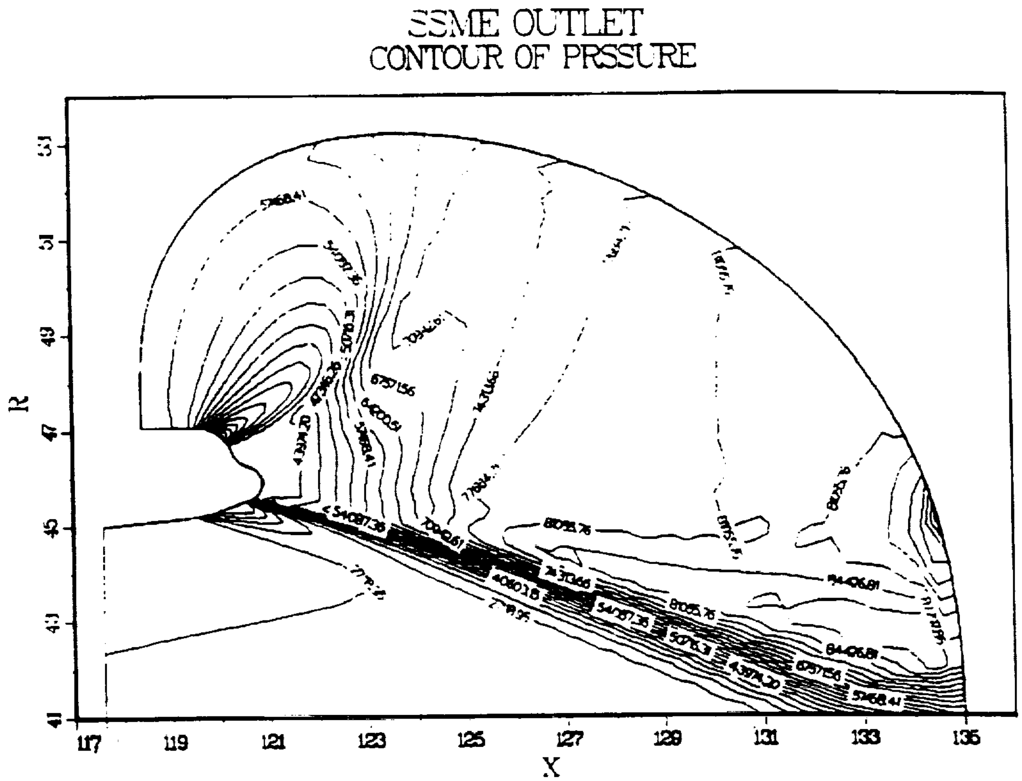


Fig. 18, Contour of pressure using 75% of the chamber pressure level

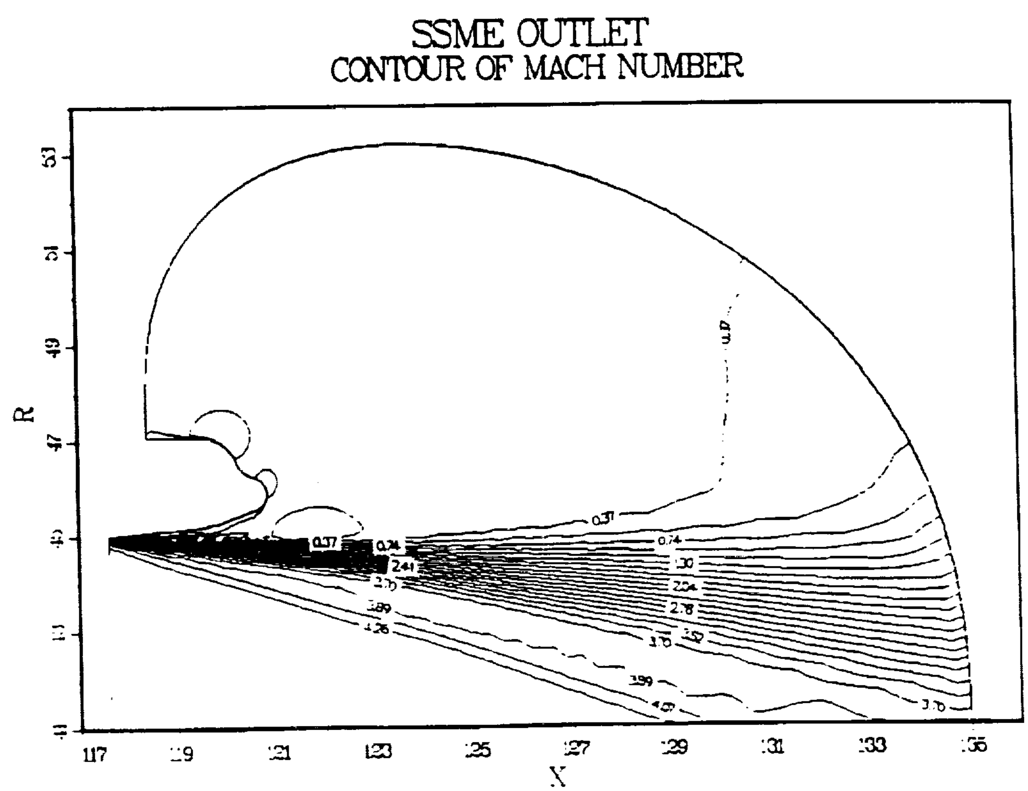


Fig. 19, Laminar flow calculations of the SSME exit flow

Summaries

- The Algebraic Stress Model Removes the Isotropic Turbulence Assumption for the Eddy Viscosity Type Models

- Improved On the Reynolds Stresses Predictions

- The ASM Does Not Improve Too Much On SSME Nozzle & Outlet Flows

  ---RSM

  ---Other Mechanisms

  Shock- Boundary Layer Interactions

  Entrainment Issues

- 3-D Calculations Are Desirable

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