Convergence Acceleration of Implicit Schemes in the Presence of High Aspect Ratio Grid Cells

P. E. O. Buelow, S. Venkateswaran and C. L. Merkle
Propulsion Engineering Research Center
The Department of Mechanical Engineering
The Pennsylvania State University
University Park, PA 16802.

The performance of Navier-Stokes codes are influenced by several phenomena. For example, the robustness of the code may be compromised by the lack of grid resolution, by a need for more precise initial conditions or because all or part of the flowfield lies outside the flow regime in which the algorithm converges efficiently. A primary example of the latter effect is the presence of extended low Mach number and/or low Reynolds number regions which cause convergence deterioration of time marching algorithms. Recent research into this problem by several workers including the present authors has largely negated this difficulty through the introduction of time-derivative preconditioning. In the present paper, we employ the preconditioned algorithm to address convergence difficulties arising from sensitivity to grid stretching and high aspect ratio grid cells.

Strong grid stretching is particularly characteristic of turbulent flow calculations where the grid must be refined very tightly in the dimension normal to the wall, without a similar refinement in the tangential direction. High aspect ratio grid cells also arise in problems that involve high aspect ratio domains such as combustor coolant channels. In both situations, the high aspect ratio cells can lead to extreme deterioration in convergence. It is the purpose of the present paper to address the reasons for this adverse response to grid stretching and to suggest methods for enhancing convergence under such circumstances.

Numerical algorithms typically possess a maximum allowable or optimum value for the time step size, expressed in non-dimensional terms as a CFL number or von-Neumann number (VNN). In the presence of high aspect ratio cells, the smallest dimension of the grid cell controls the time step size causing it to be extremely small, which in turn results in the deterioration of convergence behaviour. For explicit schemes, this time step limitation cannot be exceeded without violating stability restrictions of the scheme. On the other hand, for implicit schemes, which are typically unconditionally stable, there appears to be room for improvement through careful tailoring of the time-step definition based on results of linear stability analyses. In the present paper, we focus on the central-differenced alternating direction implicit (ADI) scheme. The understanding garnered from this analyses can then be applied to other implicit schemes.

In order to systematically study the effects of aspect ratio and the methods of mitigating the associated problems, we use a two pronged approach. We use stability analyses as a tool for predicting numerical convergence behavior and numerical experiments on simple model problems to verify predicted trends. Based on these analyses, we determine that efficient convergence may be obtained at all aspect ratios by getting a combination of things right. Primary among these are the proper definition of the time step size, proper selection of viscous preconditioner and the precise treatment of boundary conditions. These algorithmic improvements are then applied to a variety of test cases to demonstrate uniform convergence at all aspect ratios.

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Philosophy of Grid Aspect Ratio Study

- Assessment of High Aspect Ratio Problem
  — Disparate propagation speeds in X and Y
- Stability Theory
  — Scalar Convection-Diffusion Equation
  — Euler Equations
  — Navier-Stokes Equations
- Numerical Convergence Studies
  — Simple Model Problems
  — Realistic Flow Problems
- Improved Algorithm to Provide Aspect Ratio Control
  — Precise Time-Step Definition
  — Viscous Preconditioning
  — Boundary Condition Implementation

The Navier-Stokes Equations

$$\Gamma\frac{\partial Q_v}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = H + L(Q_v)$$

- Solution Vector

$$Q_v = (\ p,\ u,\ v,\ T\ )^T$$

- Preconditioning Matrix

$$\Gamma = \begin{pmatrix} 1/\epsilon c^2 & 0 & 0 & 0 \\ u/\epsilon c^2 & \rho & 0 & 0 \\ v/\epsilon c^2 & 0 & \rho & 0 \\ \frac{h+\frac{1}{2}(u^2+v^2)}{\epsilon c^2} - 1 & \rho u & \rho v & \rho C_p \end{pmatrix}$$

- Parameter ϵ
  - — Activates Inviscid and Viscous Preconditioning
  - — Value Depends on Local Mach Number and Cell Reynolds Number

Numerical Solution Procedure

- Central-Differenced ADI Algorithm

$$\left[S + \frac{\partial A}{\partial x} - \frac{\partial}{\partial x}R_{xx}\right] S^{-1} \left[S + \frac{\partial B}{\partial y} - \frac{\partial}{\partial y}R_{yy}\right] \Delta Q_v = -\mathcal{R}^n$$

- Approximate factorization errors control convergence behavior.

- Optimum CFL_{u+c} is typically between 1 and 10.
  — Other inviscid and viscous time scales are optimized by the preconditioning matrix.

Time-Step Definition

- Local Time-Stepping or Constant CFL Condition

$$Max\ (\ CFL_x,\ CFL_y\ ) = CFL$$

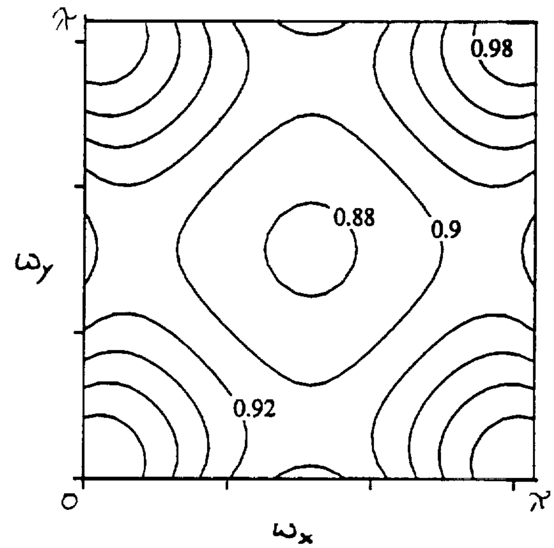
- For high aspect ratios, CFL_x and CFL_y become disparate

$$CFL_y = CFL, \qquad CFL_x = CFL/AR$$

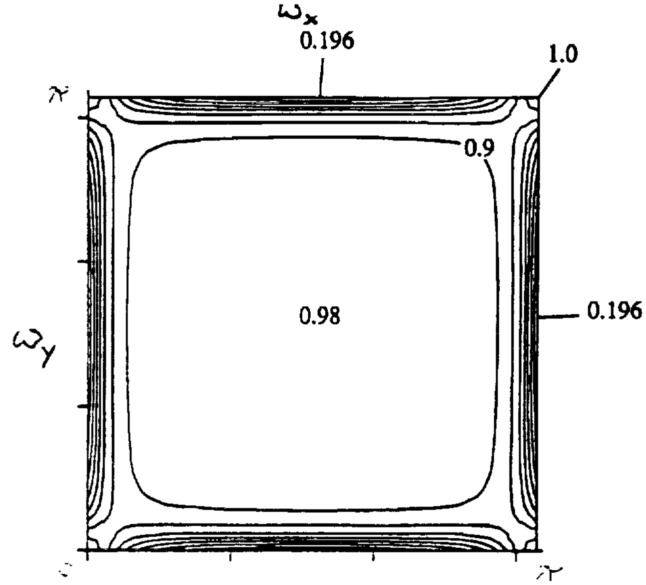
Euler Stability Analysis

- Aspect Ratio (AR) of Unity

$CFL = 1$



$CFL = 10$

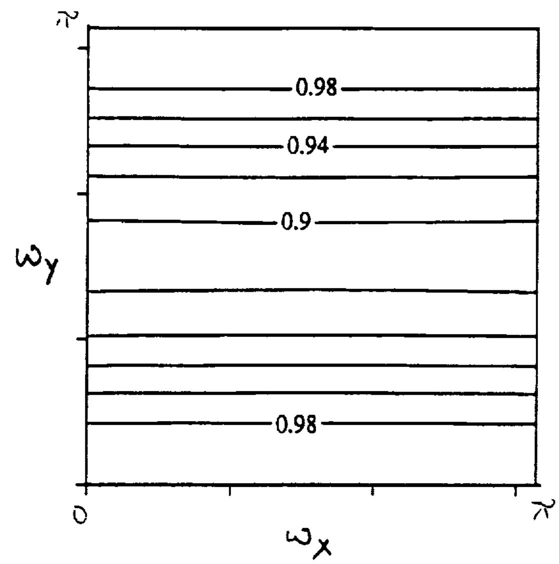


Euler Stability Analysis

- Aspect Ratio (AR) of 100

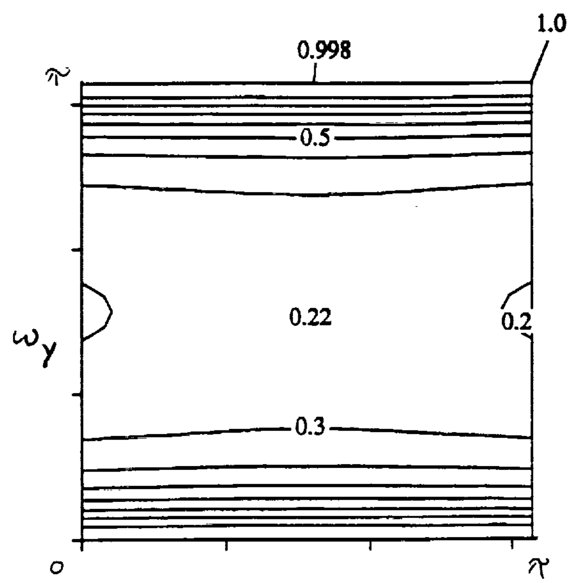
$CFL_y = 1$

$CFL_x = 0.01$



$CFL_y = 10$

$CFL_x = 0.1$

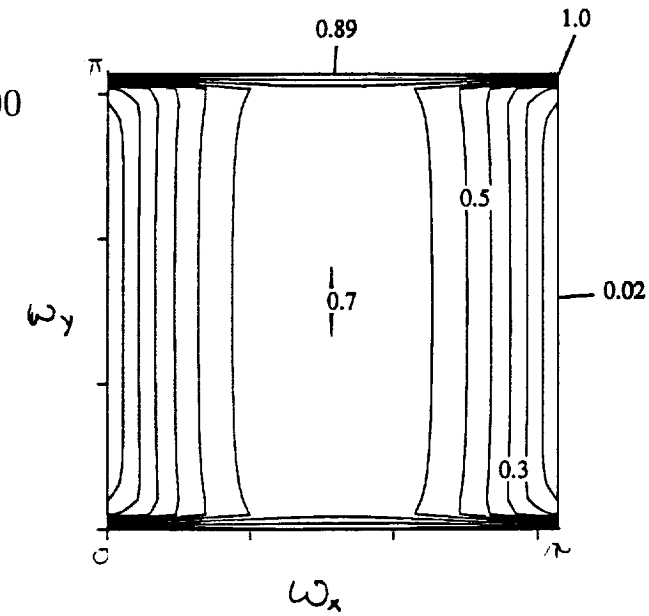


Euler Stability Analysis

- Aspect Ratio (AR) of 100

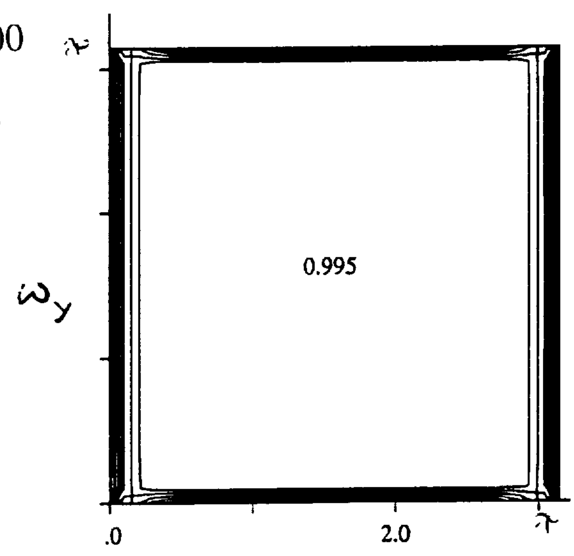
$CFL_y = 100$

$CFLx = 1$



$CFL_y = 1000$

$CFL_x = 10.$



New Time-Step Definition

- Conclusions from Stability Analysis:
  - — Min-CFL Preferable to Max-CFL
  - — Efficient Convergence at all AR
- Minimum-CFL Definition

$$Min\ (\ CFL_x,\ CFL_y\ ) = CFL$$

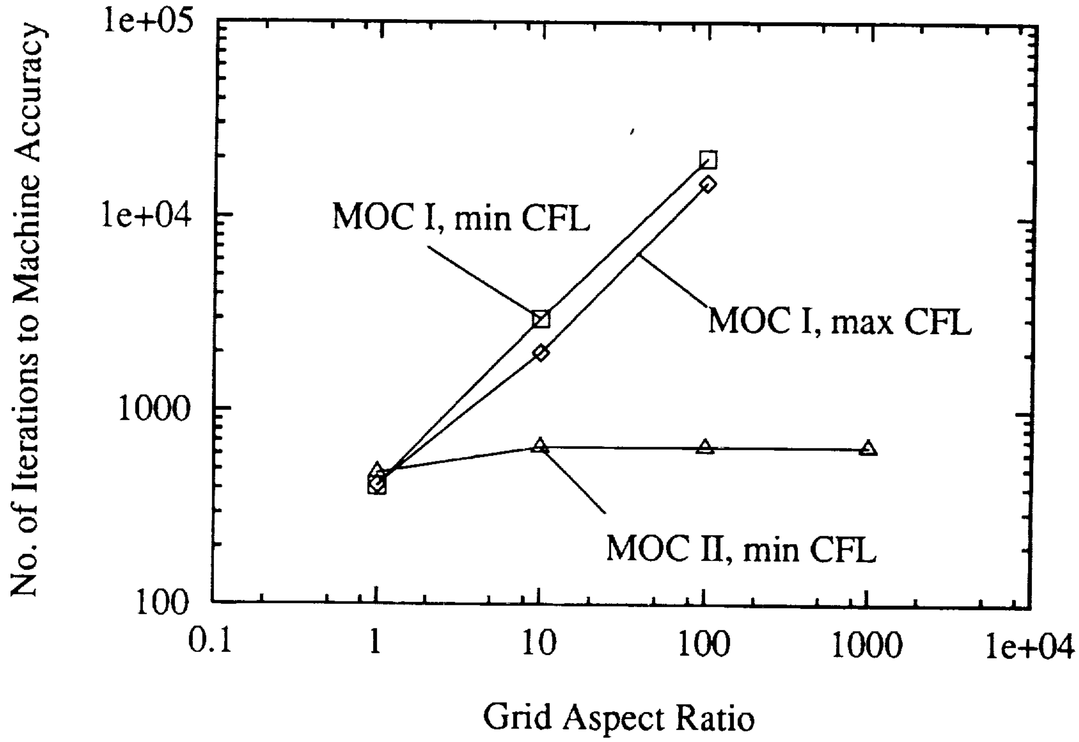
- For high aspect ratios,

$$CFL_y = CFL * AR, \qquad CFL_x = CFL$$

Implementation of Boundary Conditions

- Extrapolation vs Characteristic
  - Both work well for small CFL's
  - Characteristics usually superior at high CFL's
- Proper MOC Implementation:
  - Implicit procedures
  - Boundary conditions applied before approximate factorization
  - Consistent order of accuracy: LHS / RHS

High Aspect Ratio Convergence—Inviscid Duct



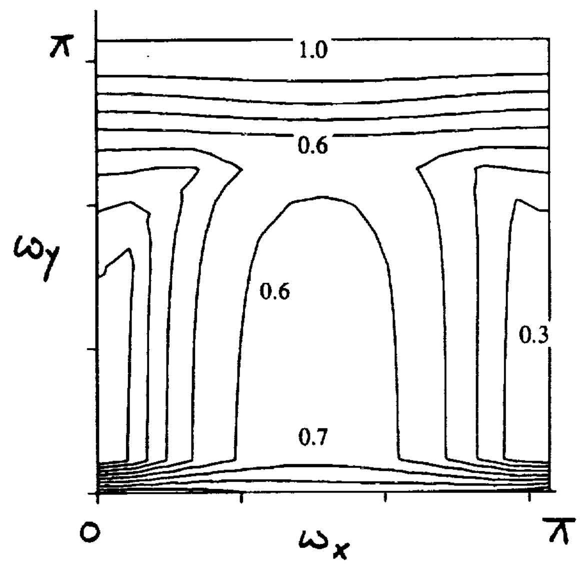
Navier-Stokes Analysis

- Parameter ϵ controls low Re number convergence
  - Viscous terms limit time step at high AR
  - ϵ chosen to optimize inviscid and viscous modes simultaneously
- Obvious choice: $\epsilon = f$ (Max-CFL, Max-VNN)
- Scalar Stability Results: $\epsilon = f$ (Min-CFL, Min-VNN)

Navier-Stokes Analysis

- Min-CFL, Min-VNN Stability Result

$AR = 1000$



$CFL_x = 1, \; CFL_y = 1000$

$VNN_x = 1, \; VNN_y = 1 \times 10^6$

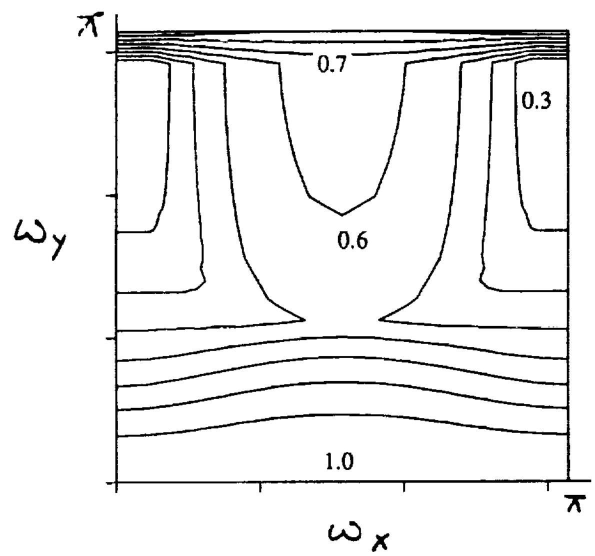
Navier-Stokes Analysis

- Conclusions from Stability Results:
  - — Vector system different from scalar equation
  - — Approximate factorization error $CFL_x * VNN_y$ limits convergence
- Viscous preconditioner, $\epsilon = f$ (Min-CFL,Max-VNN)
  - — Maintains Min-CFL for 'inviscid' modes
  - — Uses traditional Max-VNN definition for 'viscous' modes

Navier-Stokes Analysis

- Min-CFL, Max-VNN Stability Result

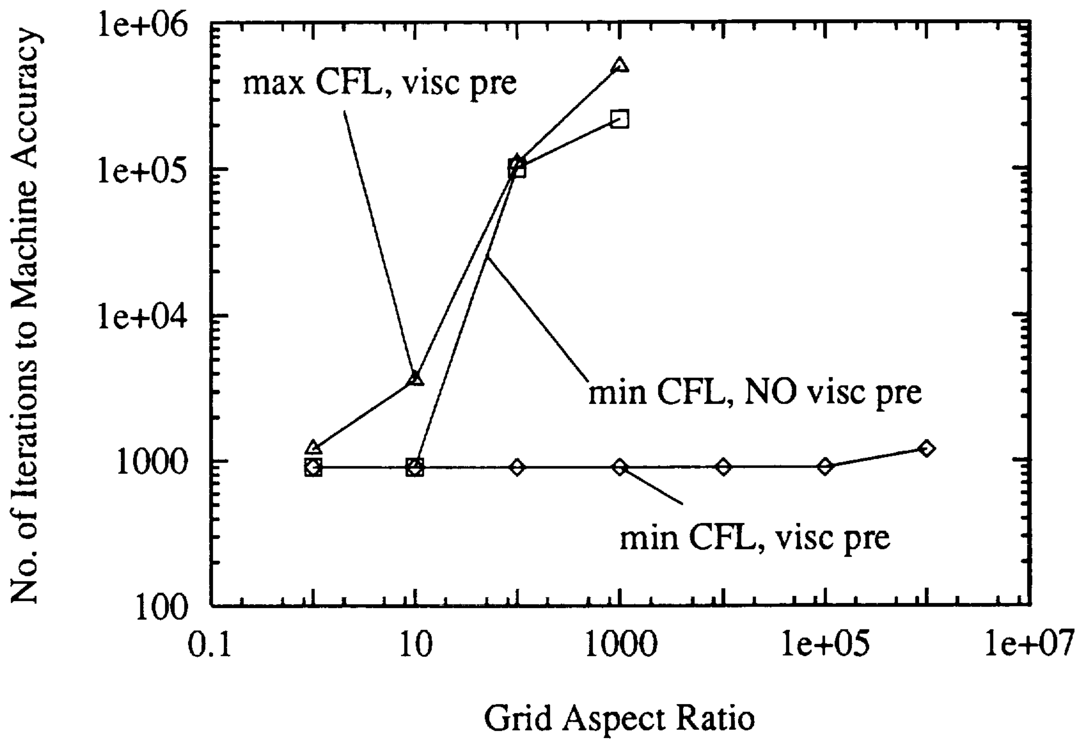
  $AR = 1000$



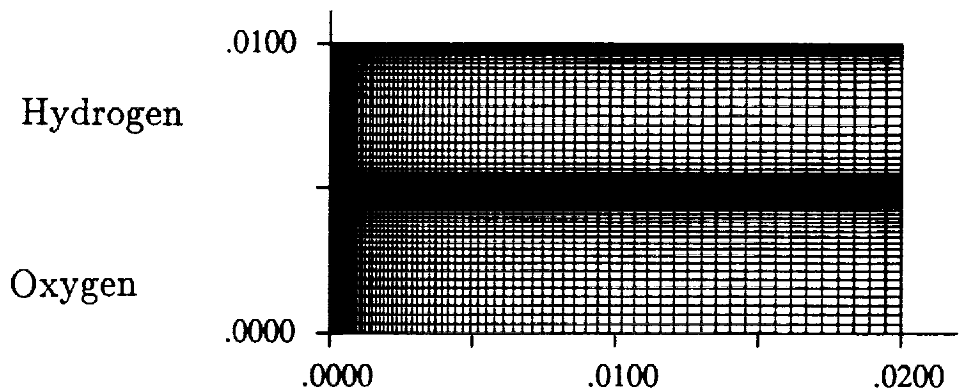
$CFL_x = 1, \; CFL_y = 1000$

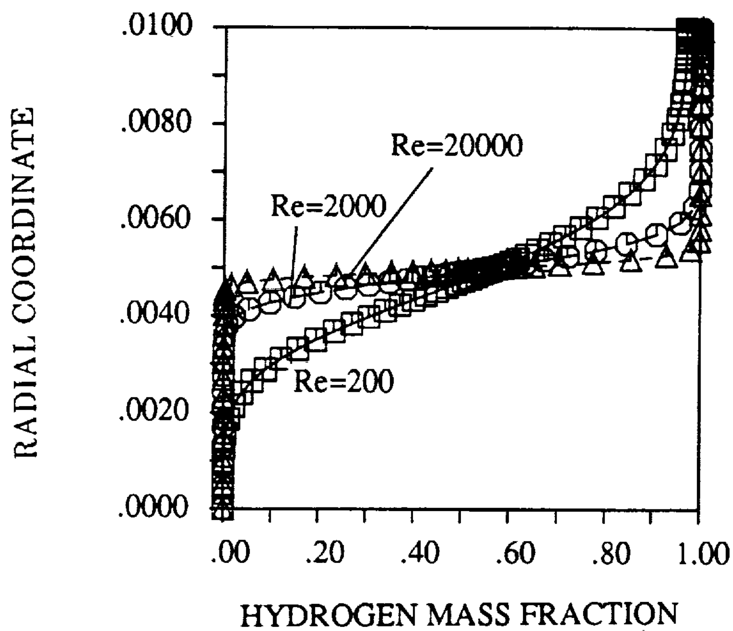
$VNN_x = 1 \times 10^{-6}, \; VNN_y = 1$

High Aspect Ratio Convergence—Viscous Duct

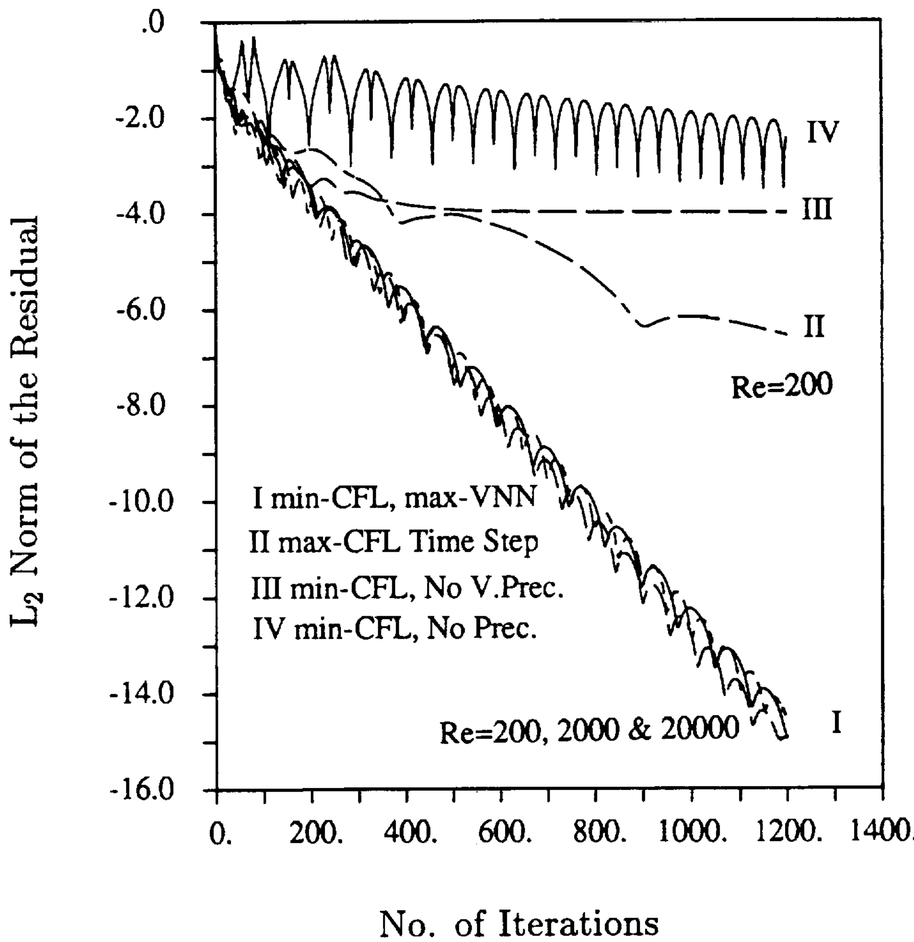


Hydrogen/Oxygen Shear-Layer Stretched Grid and Solution

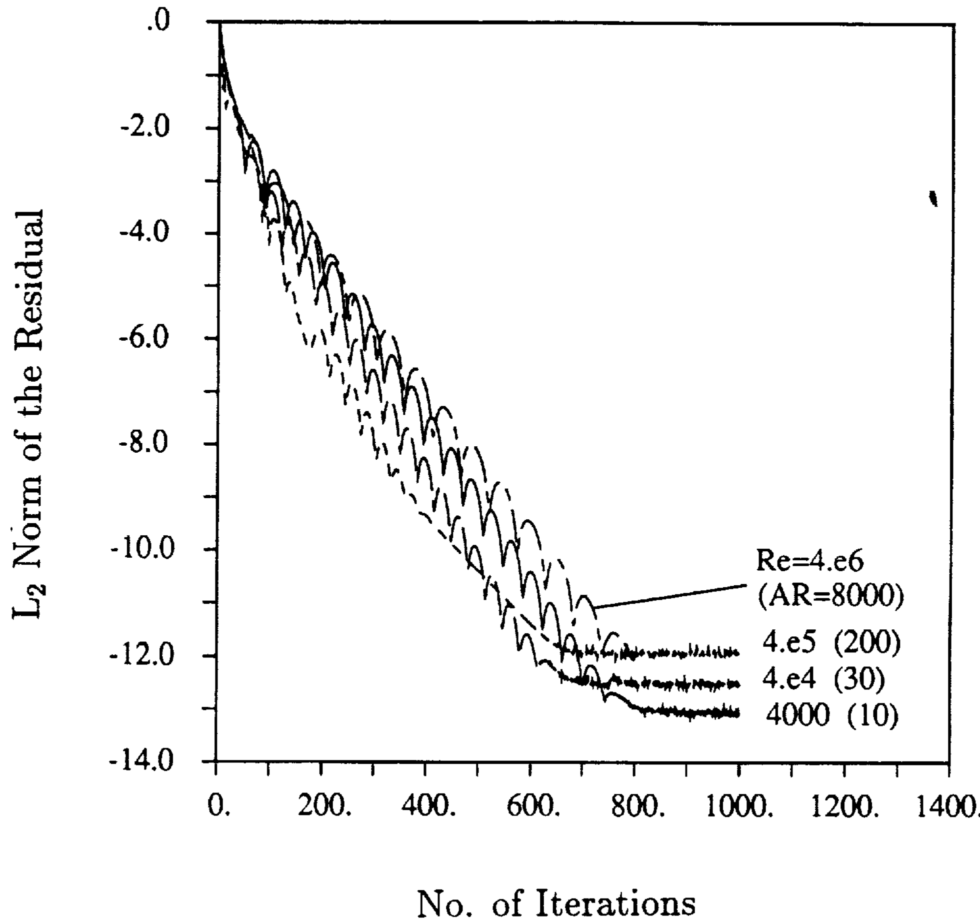




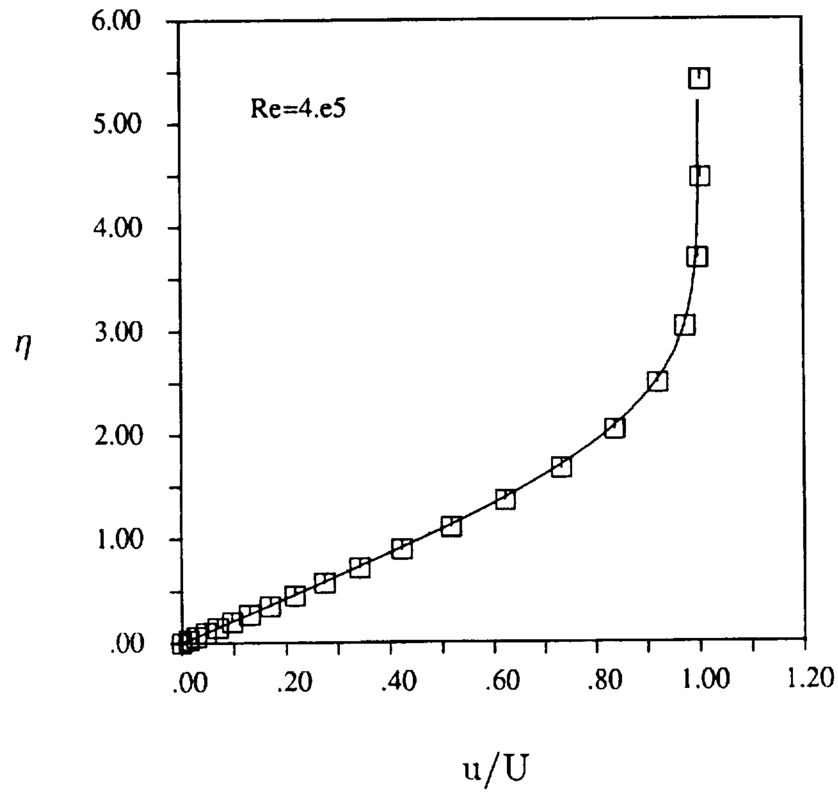
Hydrogen/Oxygen Shear-Layer Convergence



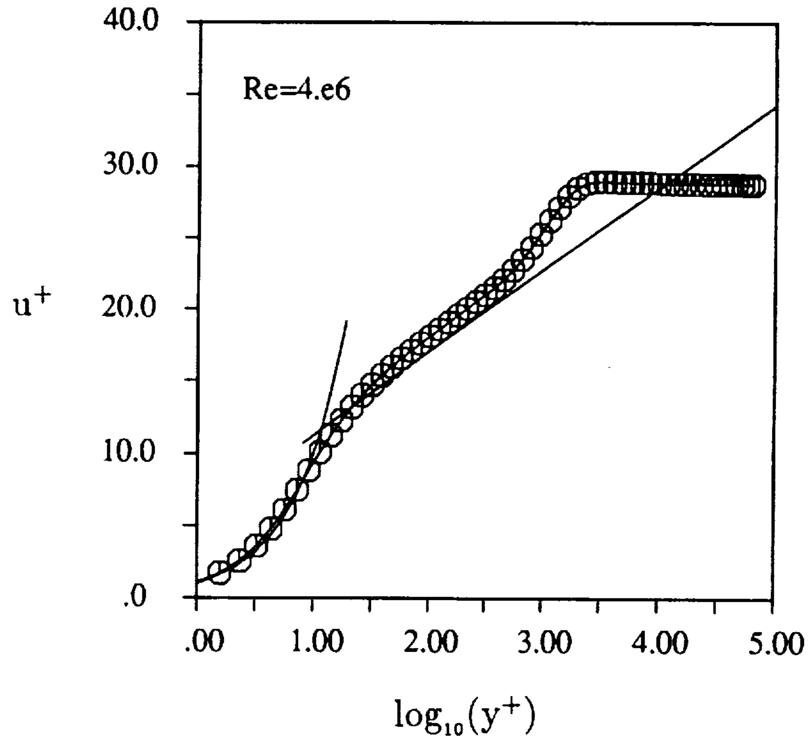
High Reynolds Number Boundary Layer Convergence—Stretched Grid



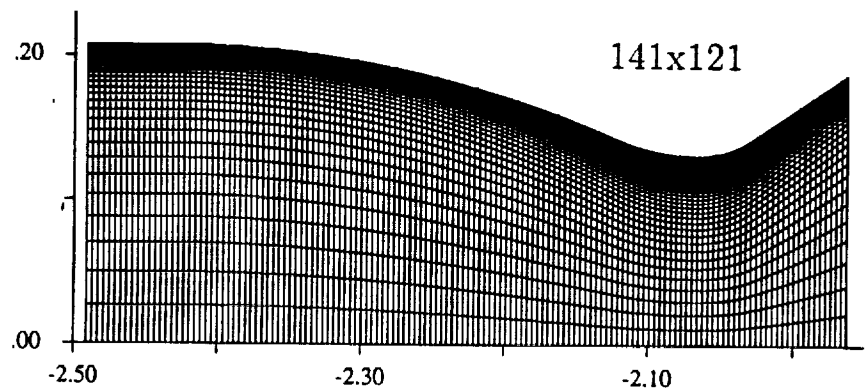
High Reynolds Number Boundary Layer
Blasius Solution

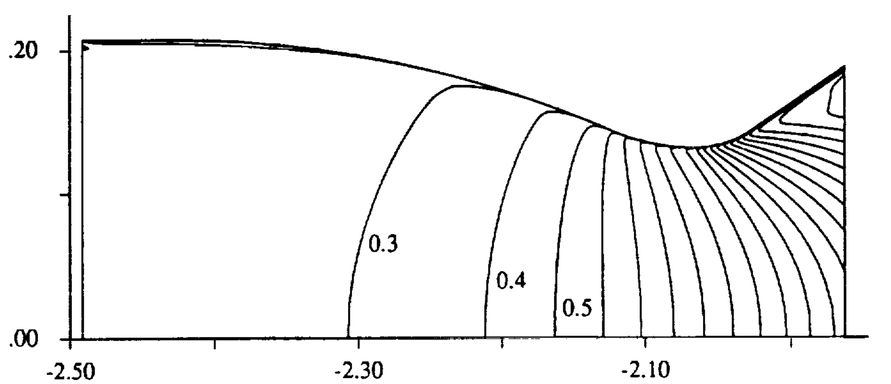


High Reynolds Number Boundary Layer Turbulent Velocity Profile

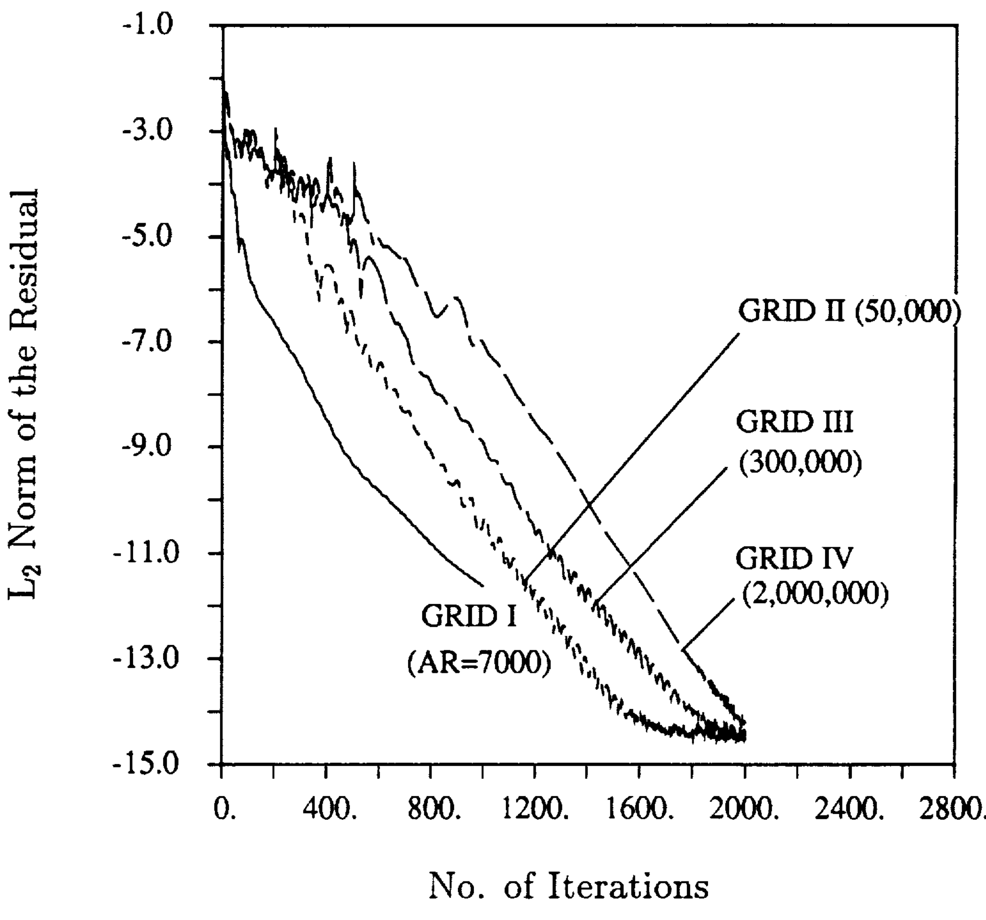


Turbulent Nozzle Computation
Stretched Grid and Solution

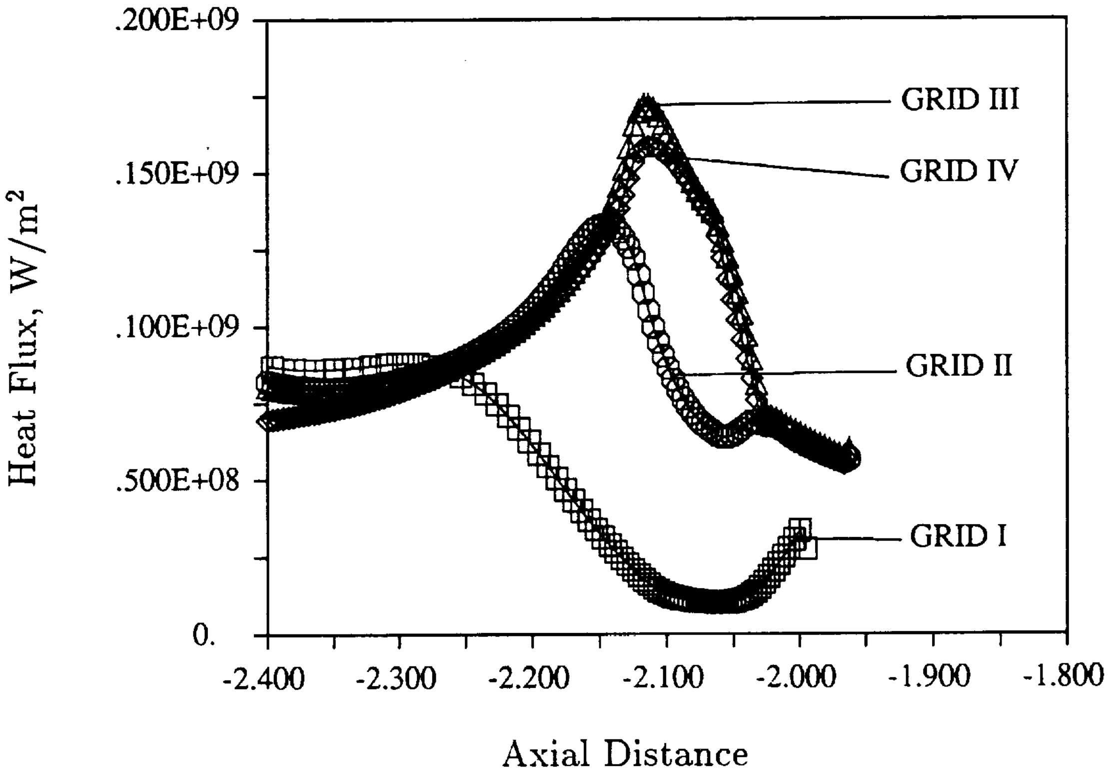




Turbulent Nozzle Computation Convergence



Turbulent Nozzle Computation
Wall Heat Flux



Conclusions—High Aspect Ratio Study

- High Aspect Ratio Analysis
  - Stability Theory
  - Numerical Convergence Studies
- Convergence Control:
  - Min-CFL Time Step
  - Max-VNN Viscous Preconditioner
  - Correct implementation of boundary conditions
- Uniform Convergence Demonstrated for All AR's
  - Above issues addressed in combination
  - Efficient convergence for variety of test cases

Conclusions (Contd.)

- Present results are for two-dimensional central-differenced *ADI* scheme
- Explicit Schemes:
  - — Optimum time step causes poor convergence at high *AR*'s
- Upwind Schemes Also Suffer at High *AR*'s
  - — Present improvements may be incorporated
  - — Rich variety of approximate factorization methods
- Three-dimensional computations:
  - — *ADI* scheme is conditionally stable
  - — Two kinds of high aspect ratio grids
  - — Algorithmic improvements appear promising