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STATISTICS OF MULTI-LOOK AIRSAR IMAGERY: A COMPARISON OF THEORY WITH MEASUREMENTS

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1. INTRODUCTION

The intensity and amplitude statistics of SAR images, such as L-Band HH for SEASAT and SIR-B, and C-Band VV for ERS-1 have been extensively investigated for various terrain, ground cover and ocean surfaces. Less well-known are the statistics between multiple channels of polarimetric or interferometric SARs, especially for the multi-look processed data. In this paper, we investigate the probability density functions (PDFs) of phase differences, the magnitude of complex products and the amplitude ratios, between polarization channels (i.e. HH,HV, and VV) using 1-look and 4-look AIRSAR polarimetric data. Measured histograms are compared with theoretical PDFs which were recently derived based on a complex Gaussian model (Lee et al., 1993).

NASA/JPL 1-look and 4-look AIRSAR data of Howland Forest and San Francisco were used for comparison. Histograms from 1-look SAR data agreed with theoretical PDFs. However, discrepancies were found when matching the 4-look polarimetric data with the 4-look PDFs. Instead, We found that the 3-look PDFs matched better. The problem was traced to the averaging of correlated 1-look pixels. We also verified these theoretical PDFs for forest, ocean surfaces, park areas and city blocks. This indicates that ground cover and terrain with different scattering mechanisms can be represented with this statistical model.

2. THE COMPLEX GAUSSIAN MODEL

A polarimetric radar measures the complete scattering matrix S of a medium at a given incidence angle. For a reciprocal medium, the three unique complex elements are S_{hh}, S_{hv}, and S_{vv}. Circular Gaussian Conditions (Goodman, 1985) are assumed. The circular Gaussian assumption has been verified by Sarabandi(1992) using 1-look polarimetric SAR data.

The multi-look AIRSAR processor compresses polarimetric data by averaging Mueller matrices of 1-look pixels in the azimuth direction. Due to oversampling, the neighboring 1-look pixels of AIRSAR data are somewhat correlated. For mathematical simplicity, the PDFs for multi-look phase difference, etc. were derived under the assumption of statistical independence.

3. MULTI-LOOK PHASE DIFFERENCE DISTRIBUTION

Let n be the number of looks, and $S_1(k)$ and $S_2(k)$ be the *kth* 1-look samples of any two components of the scattering matrix. For polarimetric and interferometric radars, the multi-look phase difference is obtained by

$$\psi = Arg[\frac{1}{n}\sum_{k=1}^{n} S_1(k)\, S_2^*(k)] \qquad (1)$$

Under the circular Gaussian assumption, the multi-look phase distribution were derived by multiple integrations of special functions (Lee et al., 1993). The multi-look phase difference PDF is

$$p(\psi) = \frac{\Gamma(n+1/2)\,(1-|\rho_c|^2)^n \beta}{2\sqrt{\pi}\,\Gamma(n)\,(1-\beta^2)^{n+1/2}} + \frac{(1-|\rho_c|^2)^n}{2\pi} F(n,1;1/2;\beta^2), \qquad (2)$$

with

$$\beta = |\rho_c| \, Cos(\psi-\theta), \qquad -\pi < \psi \le \pi \qquad (3)$$

where $F(n, 1; 1/2; \beta^2)$ is a Gauss hypergeometric function, and θ and $|\rho_c|$ are the phase and the magnitude of the complex correlation coefficient defined as

$$\rho_c = \frac{E[S_1 S_2^*]}{\sqrt{E[|S_1|^2]\,E[|S_2|^2]}} = |\rho_c|\, e^{i\theta} \qquad (4)$$

The 1-look (n=1) PDF can be obtained by applying mathematical identities of the hypergeometrical function (Lee et al.,1993). The 1-look PDF obtained is identical to that of Kong (1988) and Sarabandi (1992). The PDF of Eq. (2) depends only on the number of looks and the complex correlation coefficient. The peak of the distribution is located at $\psi=\theta$. A plot of standard deviation versus $|\rho_c|$ is given in Fig. 1, which verifies a similar but less accurate figure (Zebker, 1992) computed with interferometric radar data. As shown in Fig. 1, multi-look processing effectively reduces the phase error, especially when n=16 and 32.

4. DISTRIBUTION OF THE MAGNITUDE OF THE NORMALIZED MULTI-LOOK COMPLEX PRODUCT

The magnitude of product of S_1 and S_2 is an important measure in polarimetric SAR, and it is the magnitude of interferogram from an interferometric SAR. The normalized magnitude is defined as

$$\xi = \frac{|\frac{1}{n}\sum_{k=1}^{n} S_1(k)\, S_2^*(k)|}{\sqrt{E[|S_1|^2]\,E[|S_2|^2]}} \qquad (5)$$

The PDF of ξ (Lee et al.,1993) is

$$p(\xi) = \frac{4\,n^{n+1}\xi^n}{\Gamma(n)\,(1-|\rho_c|^2)}\, I_0\!\left(\frac{2|\rho_c| n\xi}{1-|\rho_c|^2}\right) K_{n-1}\!\left(\frac{2n\xi}{1-|\rho_c|^2}\right) \qquad (6)$$

where $I_0(\ )$ and $K_n(\ )$ are modified Bessel functions. The PDF for the unnormalized magnitude can be easily obtained from Eq.(6) by using Eq. (5). The standard deviations of this PDF is plotted versus the correlation coefficient in Fig. 2. This plot indicates that the magnitude of complex product can be used as a discriminator, especially for large n.

5. MULTI-LOOK AMPLITUDE RATIO DISTRIBUTIONS

The amplitude ratio between S_{hh} and S_{vv} has been an important discriminator in the study of polarimetric radar returns. Let the normalized ratio

$$v=\sqrt{\left(\sum_{k=1}^{n}|S_1(k)|^2/E[|S_1|^2]\right)/\left(\sum_{k=1}^{n}|S_2(k)|^2/E[|S_2|^2]\right)} \qquad (7)$$

The multi-look amplitude ratio PDF (Lee et al., 1993) is

$$p(v)=\frac{2\Gamma(2n)\,(1-|\rho_c|^2)^n\,(1+v^2)\,v^{2n-1}}{\Gamma(n)\,\Gamma(n)\,[(1-v^2)^2-4|\rho_c|^2v^2]^{n+1/2}}\,,\quad 0\le v<\infty \qquad (8)$$

For n=1, we have the 1-look amplitude distribution, which is identical to the results of Kong(1988), when his PDF is properly normalized.

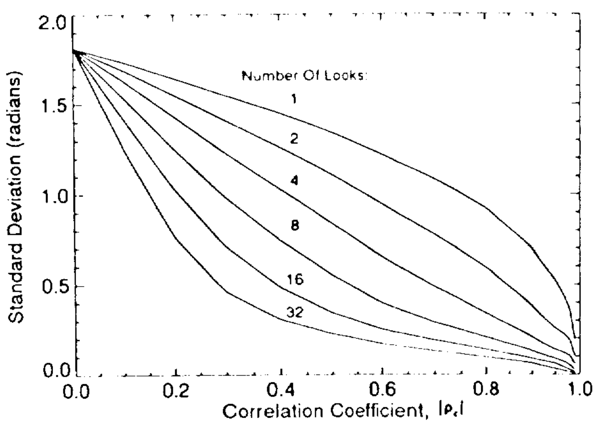

Fig. 1 Phase difference standard deviations versus the correlation coefficient, $|\rho_c|$, for the number of look from n= 1, 2, 4, 8, 16 and 32. The value of θ=0.

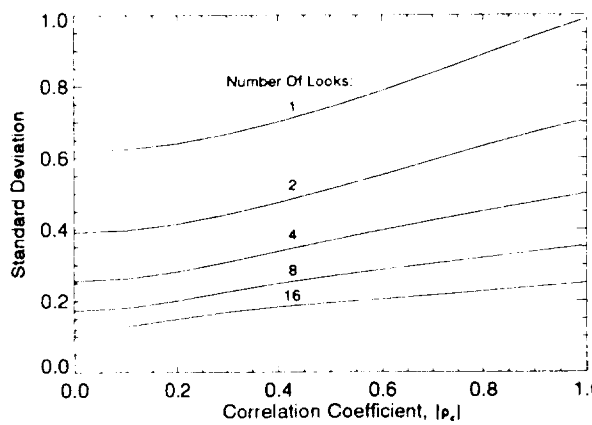

Fig. 2 Standard deviations of Normalized magnitude of multi-look interferogram versus correlation coefficient, $|\rho_c|$, for the number of look from n= 1, 2, 4, 8 and 16.

6. COMPARISON OF THEORETICAL PDFs WITH AIRSAR MEASUREMENTS

In this section, the AIRSAR polarimetric data was used to compute histograms of phase differences, normalized products and amplitude ratios to verify this complex Gaussian model. Homogeneous regions of forest were selected, and the complex correlation coefficient (i.e., θ and $|\rho_c|$) and histograms were computed. We have checked the 1-look C-Band and L-Band data, and found that the agreements between histograms and their corresponding PDFs are good. However, discrepancies exist in the 4-look data. Using 4-look Howland Forest data (CM1084), the match between the histogram and the 4-look phase difference PDF are not as good (Fig. 3A). A better match was found with a 3-look PDF (Fig. 3B). Histograms of amplitude and normalized product are also shown good agreements with 3-look PDFs. To save space, only the case of HH to VV ratio is shown in Fig. 3C.

The problem was traced to the averaging of correlated 1-look neighboring pixels during the multi-look processing. To verify it, we use 1-look data of Howland Forest (HR1084C) and perform the 4-look processing by averaging four pixels separated by two pixels in the azimuth direction. Since the correlation between every other pixels is much less than that between its immediate neighbors, statistical independence is assured. The results are shown in Fig.4 for all three variables under study. The agreement with 4-look PDFs is very good. We can conclude that due to the correlation of 1-look data, the 4-look AIRSAR data has the characteristics of a 3-look.

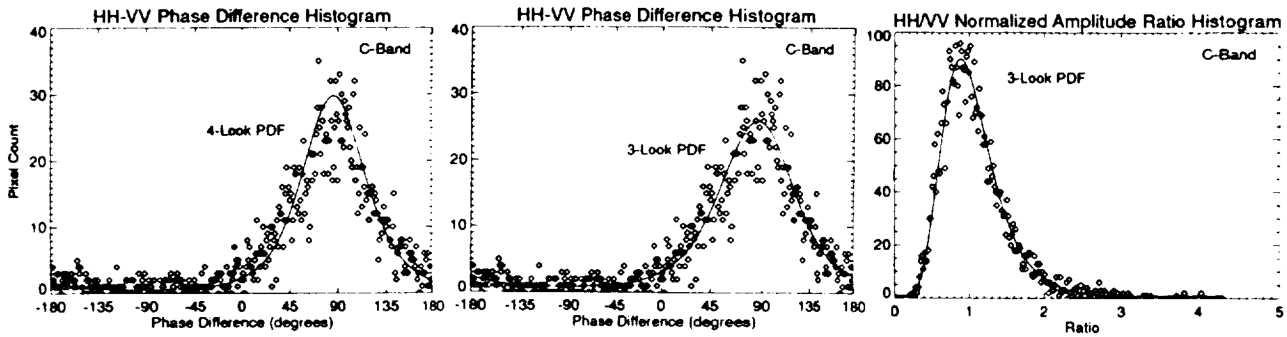

Fig. 3 Experimental results using 4-look AIRSAR data of Howland Forest (CM1804) with $|\rho_c|=0.491$ and $\theta=84.9°$. (A) The histogram of phase difference between HH and VV and the theoretical 4-look PDF. (B) The same histogram but with a 3-look PDF. The match is better. (C) The histogram of normalized amplitude ratio, HH/VV and the 3-look PDF.

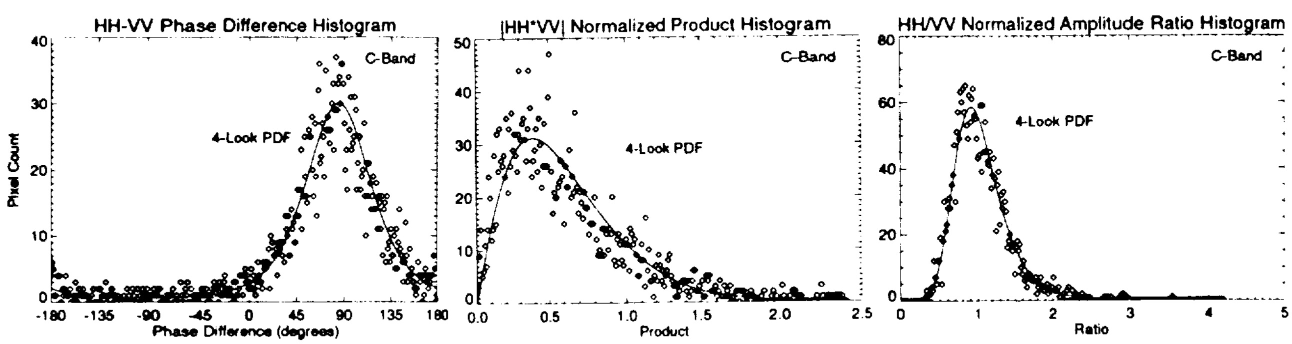

Fig. 4 Using 1-look AIRSAR data (HR1084C), the 4-look data were computed by averaging pixels separated by a distance of two pixels. (A) HH-VV phase difference histogram and the 4-look PDF. The agreement is good. (B) The histogram of normalized $|HH^{*}VV|$ and the 4-look PDF. (C) The histogram of normalized $|HH|/|VV|$ and the 4-look PDF.

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