1995121363 N95-27784

A Modified Proportional Navigation Scheme for Rendezvous and Docking with Tumbling Targets: The Planar Case

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A two-phase proportional navigation scheme is developed for the case of two rigid bodies engaged in a rendezvous/docking maneuver. The target vehicle is nonmaneuvering, but does have constant nonzero angular and linear velocities. Under these conditions, it is shown that previously obtained solutions are not applicable. Analytical solutions are obtained leading to relationships between the transverse and LOS navigation constants. It is shown that the transverse navigation constant for the second phase of the maneuver must be 2. Also, initial conditions necessary for rendezvous are presented.

Introduction

The concepts of proportional navigation have been widely developed for the terminal phase of intercept problems [1-4]. To a lesser extent, some form of proportional navigation has found application in rendezvous problems [5-8]. Regardless of the application (i.e., intercept or rendezvous), these studies have all used point mass bodies for both the target and chase vehicles. For intercept type problems, this is not an issue since one is only interested in two points occupying the same location in space at the same time. However, for rendezvous and docking type problems, this is a significant issue. For example, it is possible for two points other that the berthing mechanism points to come in contact with each leading to conditions which may not be favorable for the overall mission.

In a recent paper [8], true proportional navigation was modified to include a commanded acceleration proportional to the centripetal acceleration (i.e., along the LOS). It was shown that this modification resulted in a simultaneous zeroing of the range-to-go and range-to-go rate, a condition necessary for rendezvous. The short comings of this analysis is that this is applicable only to rendezvous maneuvers between point masses. If this procedure is applied to rigid bodies (i.e., bodies with finite dimensions), then contact between the bodies may occur prior to rendezvous as is depicted in Fig. 1. In this paper, further modifications of the guidance scheme of Ref. 8 are presented and discussed. These modifications specifically address (i) the issue of contact prior to docking and (ii) targets which are both translating and rotating. Both these issues arise due to the consideration of rigid bodies as opposed to point masses.

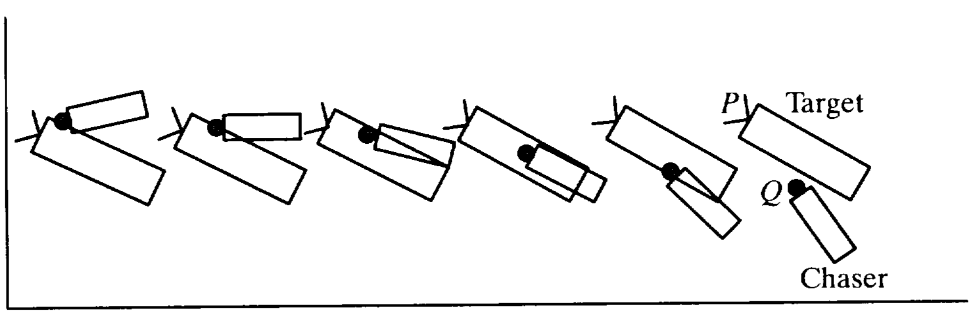

Fig. 1 Interference Associated with finite dimension bodies

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Kinematic Equations

Consider the planar rendezvous scenario as depicted in Fig. 2. The target vehicle translates with constant velocity $\vec{V}_T$ and rotates with constant angular velocity $\vec{\omega}_T$. The target vehicle has a berthing mechanism at point P which is located with respect to the vehicle's center of mass (c.m.) by $\vec{r}_P$. The chase vehicle has its berthing mechanism at point Q which is located with respect to the vehicle's c.m. by $\vec{r}_Q$. For convenience, the docking and the grapple mechanisms are both assumed to be aligned with their respective position vectors.

Let $\mathfrak{F}_C$, $\mathfrak{F}_G$, $\mathfrak{F}_N$, and $\mathfrak{F}_T$ be respectively, coordinate frames attached to the chase vehicle, the LOS vector, the inertial space, and the target vehicle. For motion in the xy-plane, denote the attitude and angular velocity of $\mathfrak{F}_T$ relative to $\mathfrak{F}_N$ by ϕ and $\vec{\omega}_T = \dot{\phi}\hat{\vec{k}}$, respectively. Similarly, denote the attitude and angular velocity of $\mathfrak{F}_C$ relative to $\mathfrak{F}_G$ by β and $\vec{\omega}_C = \dot{\beta}\hat{\vec{k}}$. Finally, define the LOS frame $\mathfrak{F}_G$ in such a manner that the LOS vector is $\vec{r} = -r\hat{\vec{j}}_G$ and denote the attitude and angular velocity of this frame relative to $\mathfrak{F}_T$ by θ and $\vec{\omega}_G = \dot{\theta}\hat{\vec{k}}$, respectively.

The inertial acceleration of the c.m. C_C of the chase vehicle is

$$\ddot{\vec{R}}_C = \vec{\Omega}_T \times (\vec{\Omega}_T \times \vec{r}_p) + \overset{\circ\circ}{\vec{r}} + \overset{\circ}{\vec{\Omega}}_G \times \vec{r} + \vec{\Omega}_G \times (\vec{\Omega}_G \times \vec{r}) + 2\vec{\Omega}_G \times \overset{\circ}{\vec{r}}$$
$$- \overset{\circ}{\vec{\Omega}}_C \times \vec{r}_Q - \vec{\Omega}_C \times (\vec{\Omega}_C \times \vec{r}_Q) \tag{1}$$

where $(\overset{\circ}{\vec{\ }})$ denotes the time derivative of the components of a vector and $\vec{\Omega}_T \triangleq \vec{\omega}_T$, $\vec{\Omega}_G \triangleq \vec{\omega}_T + \vec{\omega}_G$, and $\vec{\Omega}_C \triangleq \vec{\omega}_T + \vec{\omega}_G + \vec{\omega}_C$ are the absolute angular velocities of $\mathfrak{F}_T$, $\mathfrak{F}_G$, and $\mathfrak{F}_C$, respectively. Coordinatizing Eq. (1) in $\mathfrak{F}_G$ (i.e., the LOS frame) results in

$$\ddot{\vec{R}}_c = a_{LOS\perp}\, \vec{i}_G + a_{LOS}\, \vec{j}_G \tag{2}$$

where

$$a_{LOS\perp} = \varrho\ddot{\delta} + 2\dot{\varrho}\dot{\delta} + \ddot{\beta}[r_{Qx}\sin\beta + r_{Qy}\cos\beta] - \dot{\phi}^2[r_{Px}\cos\theta + r_{Py}\sin\theta]$$
$$+ (\dot{\delta}^2 + \dot{\beta}^2)[r_{Qx}\cos\beta - r_{Qy}\sin\beta] \tag{3}$$

and

$$a_{LOS} = -\ddot{\varrho} + \varrho\dot{\delta}^2 + \dot{\phi}^2[r_{Px}\sin\theta - r_{Py}\cos\theta] - \ddot{\delta}[r_{Qx}\cos\beta - r_{Qy}\sin\beta]$$
$$+ 2\dot{\delta}\dot{\beta}[r_{Qx}\sin\beta + r_{Qy}\cos\beta] \tag{4}$$

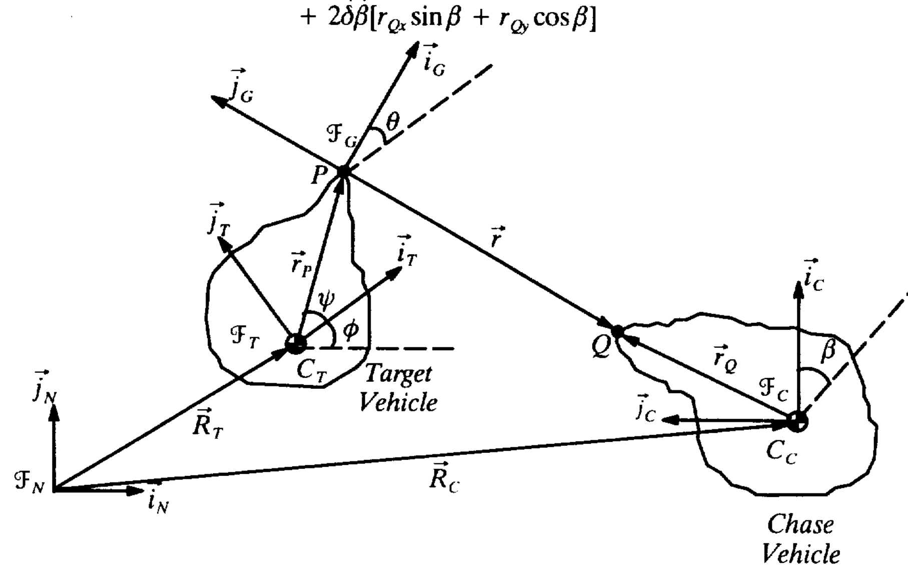

Fig. 2 Rendezvous Geometry

For convenience, the following definitions were utilized:

$$\dot{\delta} \triangleq \dot{\theta} + \dot{\phi} \tag{5}$$

and

$$\varrho \triangleq r + r_{Qx}\sin\beta + r_{Qy}\cos\beta \tag{6}$$

where $\dot{\delta}$ represents the absolute angular velocity of $\mathcal{F}_G$ and ϱ represents the projection of the vector from point P to C_c onto the LOS direction; we will refer to this quantity as the "pseudo-range-to-go" to distinguish it from the true range-to-go r.

Navigation Scheme

The proposed navigation acceleration are defined as

$$a_{LOS\perp_C} \triangleq \lambda_{LOS\perp}\dot{\varrho}\dot{\delta} - \dot{\phi}^2[r_{Px}\cos\theta + r_{Py}\sin\theta] + \ddot{\beta}[r_{Qx}\sin\beta + r_{Qy}\cos\beta] + (\dot{\delta}^2 + \dot{\beta}^2)[r_{Qx}\cos\beta - r_{Qy}\sin\beta] \tag{7}$$

$$a_{LOS_C} \triangleq -\lambda_{LOS}\varrho\dot{\delta}^2 + \dot{\phi}^2[r_{Px}\sin\theta - r_{Py}\cos\theta] - \ddot{\delta}[r_{Qx}\cos\beta - r_{Qy}\sin\beta] + 2\dot{\delta}\dot{\beta}[r_{Qx}\sin\beta + r_{Qy}\cos\beta] \tag{8}$$

where $\lambda_{LOS\perp}$ and λ_{LOS} are respectively the transverse and LOS navigation constants. The first terms of Eqs. (7) and (8) are the "classical" proportional terms proposed by Yaun and Hsu [8]. The remaining terms of Eqs. (7) and (8) are required to compensate for the angular velocity of the target and the angular velocity and angular acceleration of the chase vehicle

Equating the kinematic accelerations with the proposed navigation accelerations, the equations of relative motion can be developed as

$$\varrho\ddot{\delta} + 2\dot{\varrho}\dot{\delta} = \lambda_{LOS\perp}\dot{\varrho}\dot{\delta} \tag{9}$$

$$\ddot{\varrho} - \varrho\dot{\delta}^2 = \lambda_{LOS}\varrho\dot{\delta}^2 \tag{10}$$

which are similar to those developed in Ref. 8. It was shown that these equations have solutions

$$\dot{\delta} = \dot{\delta}_0\left(\frac{\varrho}{\varrho_0}\right)^{a-1} \tag{11}$$

$$\dot{\varrho}^2 = \frac{\lambda_{LOS} + 1}{a}\varrho_0^2\dot{\delta}_0^2\left[\left(\frac{\varrho}{\varrho_0}\right)^{2a} - 1\right] + \dot{\varrho}_0^2 \tag{12}$$

where $a \equiv \lambda_{LOS\perp} - 1$. By specifying ϱ and $\dot{\varrho}$ at some point in the trajectory (e.g., at rendezvous), Eq. (12) becomes a constraint relationship between $\lambda_{LOS\perp}$ and λ_{LOS}. In Ref. 8, Yaun and Hsu used the conditions that the range-to-go and range-to-go rate are zero at rendezvous to develop a specific relationship between $\lambda_{LOS\perp}$ and λ_{LOS}. In terms of the "pseudo" quantities, these conditions can be restated as: $\varrho \to \varrho_f$ (since $r \to 0$ implies $\varrho = r_{Qx}\sin\beta_f + r_{Qy}\cos\beta_f$) and $\dot{\varrho} \to 0$ (assuming $\dot{\beta} = 0$). These conditions are no longer valid since we are concerned with rigid bodies rather than point masses. In fact, if one uses these conditions, the scenario depicted in Fig. 1 is generated. To avoid any contact prior to docking, we divide the maneuver into two guidance phases. In phase 1, the chase vehicle aligns itself with the target vehicle and acquires the angular velocity of the target vehicle. In phase 2, the chase vehicle maintains the angular velocity of the target and simultaneously reduces the range-to-go and range-to-go rate to zero. In both phases of flight, the following conditions are required for the chase vehicle:

- The berthing mechanism is aligned with the LOS (i.e., $\beta_{f_1} \approx 0$).
- The angular velocity of the vehicle relative to the LOS is small (i.e., $\dot{\beta}_{f_1} \approx 0$).

These conditions are can be enforced with an attitude controller onboard the vehicle.

The desired terminal conditions at the end of each maneuver phase are as follows.

Phase 1

- The LOS vector is parallel to $\vec{r}_P$ (i.e., $\theta_{f1} \cong 2m\pi + \frac{\pi}{2} + \psi$ where m is the number of revolutions of the chase vehicle with respect to the target frame and ψ is the angle between the vector $\vec{r}_p$ and the $\vec{i}_T$ axis).
- The LOS relative rotation rate is zero (i.e., $\dot{\theta}_{f1} = 0$).
- The two vehicles must be separated by some distance ϱ_{f_1} which must be greater than or equal to the sum of the largest dimension of each vehicle in order to avoid contact (i.e., $\varrho_{f2} = r_1 + r_{Qx}\sin\beta_{f1} + r_{Qy}\cos\beta_{f1} > 0$; see Fig 3)
- The chase vehicle has some residual closing velocity as ϱ_{f1} (i.e., $\dot{\varrho}_{f1} = \dot{r}_1 + \dot{\beta}_{f1}(r_{Qx}\cos\beta_{f1} - r_{Qy}\sin\beta_{f1}) < 0$)

Phase 2

- The LOS rotation rate is maintained at zero throughout this phase thereby ensuring proper alignment of the vehicles (i.e., $\dot{\theta}_{f1} = 0$ and $\theta_{f1} \cong 2m\pi + \frac{\pi}{2} + \psi$).
- The grappling mechanism of the chase vehicle approaches the docking port of the target vehicle (i.e., $\varrho_{f2} = r_{Qx}\sin\beta_{f2} + r_{Qy}\cos\beta_{f2}$).
- The range-to-go rate approaches zero as $\varrho \rightarrow \varrho_{f_2}$ (i.e., $\dot{\varrho}_{f2} = \dot{\beta}_{f2}(r_{Qx}\cos\beta_{f2} - r_{Qy}\sin\beta_{f2})$)

These requirements place constraints on the initial conditions of the chase vehicle at the beginning of the rendezvous maneuver. The initial conditions constraints are addressed below.

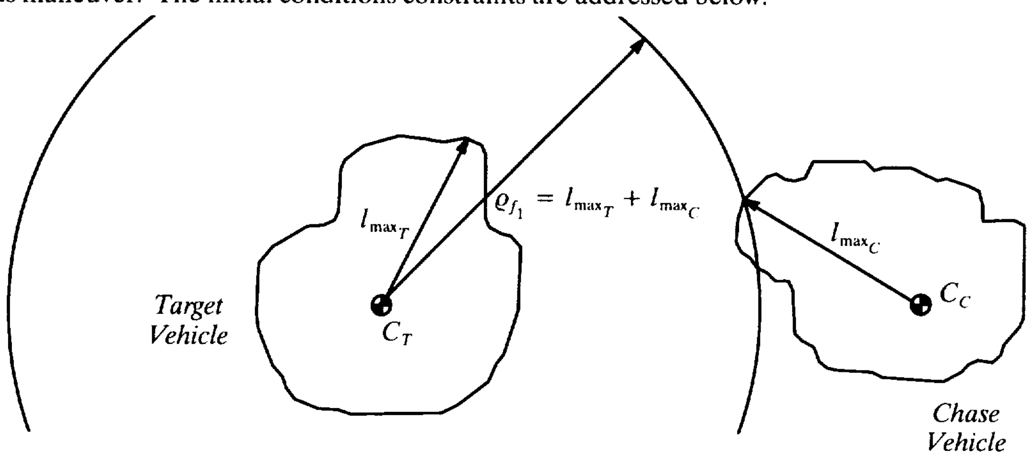

Fig. 3 Radius for noninterference

Since the initial and terminal conditions are different for each phase, the relationship between the guidance constants will also be different for each phase. We now develop these relationship, first for phase 1 and then phase 2.

Relationship between $\lambda_{LOS\perp}$ and λ_{LOS}

By substituting the appropriate initial and terminal conditions for phase 1 into Eq. (12), the following relationship between $\lambda_{LOS\perp}$ and λ_{LOS} is

$$\lambda_{LOS_1} = \frac{\dot{\varrho}_{f1}^2 - \dot{\varrho}_{01}^2}{\varrho_{01}^2\dot{\delta}_{01}^2}\frac{\alpha_1}{(\frac{\varrho_{f1}}{\varrho_{01}})^{2\alpha_1} - 1} - 1 \tag{13}$$

where $\alpha_1 = \lambda_{LOS\perp_1} - 1$. This constraint relationship between $\lambda_{LOS\perp_1}$ and λ_{LOS_1} assures a specific closing velocity $\dot{\varrho}_{f_1}$ at the end of the first stage. As is expected, the relationship developed in Ref. 8 is obtained when $\varrho_{f_1} = \dot{\varrho}_{f_1} \equiv 0$.

Similarly, substitution of the appropriate initial and terminal conditions for phase 2 results in the following relationship between the guidance constants where $\alpha_2 = \lambda_{LOS\perp_2} - 1$. Note that the initial conditions of phase 2 are the terminal conditions of phase 1.

$$\lambda_{LOS_2} = -\frac{\dot{\varrho}_{f1}^2}{\varrho_{f1}^2\dot{\delta}_{f1}^2}\frac{a_2}{(\frac{\varrho_{f2}}{\varrho_{f1}})^{2a_2} - 1} - 1 \tag{14}$$

Phase 1 Solution

We now develop an expression for the absolute attitude of LOS frame, δ, as function of ϱ. First, we rewrite Eq. (11) as

$$\mathrm{d}\delta = \dot{\delta}_0\left(\frac{\varrho}{\varrho_0}\right)^{a-1}\mathrm{d}t = \dot{\delta}_0\left(\frac{\varrho}{\varrho_0}\right)^{a-1}\frac{\mathrm{d}\varrho}{\dot{\varrho}} \tag{15}$$

where $\dot{\varrho}$ is obtained from Eq. (12). For the first phase of flight, Eq. (12) can be written as

$$\dot{\varrho} = -\frac{1}{k^{1/2}}(c_2 - c_1\varrho^{2a})^{1/2} = -\frac{1}{(-k)^{1/2}}(c_1\varrho^{2a} - c_2)^{1/2} \tag{16}$$

where the negative sign is chosen to ensure $\dot{\varrho} < 0$ and

$$k \triangleq \varrho_{f1}^{2a_1} - \varrho_{01}^{2a_1} \tag{17}$$

$$c_1 \triangleq \dot{\varrho}_{01}^2 - \dot{\varrho}_{f1}^2 > 0 \tag{18}$$

$$c_2 \triangleq \dot{\varrho}_{01}^2\varrho_{f1}^{2a_1} - \dot{\varrho}_{f1}^2\varrho_{01}^{2a_1} \tag{19}$$

From the definition of k, we observe that $a_1 < 0$ or $a_1 > 1/2$ which implies that $\lambda_{LOS\perp_1} < 1$ or $\lambda_{LOS\perp_1} > 3/2$. The case of $a_1 < 0$, both k and c_2 are positive. However, for the case of $a_1 > 1/2$, k is negative and c_2 is sign indefinite. Since the solution of Eq. (15) is dependent on the sign of c_2, for the remainder of this paper, we consider the case of $a_1 < 0$.

Substituting Eq. (16) into Eq. (15) and integrating from the initial conditions to the terminal conditions of stage 1 yields

$$\delta_{f1} - \delta_{01} = \frac{\dot{\delta}_{01}}{a_1\varrho_{01}^{(a_1-1)}}\left[\frac{k}{c_1}\right]^{1/2}\left\{\sin^{-1}\left[\frac{\dot{\varrho}_{f1}^2 k}{c_2}\right]^{1/2} - \sin^{-1}\left[\frac{\dot{\varrho}_{01}^2 k}{c_2}\right]^{1/2}\right\} \tag{20}$$

Now, an expression for the time-of-flight can be obtained from

$$T = \int_{t_{0i}}^{t_{fi}} dt = \int_{\varrho_{0i}}^{\varrho_{fi}} \frac{d\varrho}{\dot{\varrho}} \tag{21}$$

Substitution of $\dot{\varrho}$ from Eq. (16) results in

$$T_1 = -k^{1/2}\int_{\varrho_{01}}^{\varrho_{f1}} \frac{d\varrho}{(c_2 - c_1\varrho^{2a_1})^{1/2}} \tag{22}$$

In general, for arbitrary values of $a_1 < 0$, Eq. (22) has no known anti-derivative and therefore must be evaluated numerically. (Solutions are available for integer values of a_1)

We now use Eq. (20) together with Eq. (22) to calculate the initial condition of $\dot{\varrho}_{01}$ that will provide the desirable LOS attitude at the end of stage one.

Phase 1 Initial Conditions

First, the requirement that $\dot{\theta}_{f1} = 0$ at the end of the first phase places a constraint on the initial relative attitude rate of the guidance frame. Combining Eqs. (5) and Eq. (11) with the condition $\dot{\theta}_{f1} = 0$ we observe

$$\dot{\theta}_{01} = \dot{\phi}\left[\left(\frac{\varrho_{01}}{\varrho_{f1}}\right)^{a_1-1} - 1\right] \tag{23}$$

where ϱ_{f1} replaces ϱ. Thus, the initial LOS rate is defined by the initial and final pseudo-range-to-go and the transverse navigation constant of phase 1.

Second, the requirement that $\theta_{f1} = 2m\pi + \frac{\pi}{2} + \psi$ places a constraint on the initial value of the pseudo-range-to-go rate. To evaluate this constraint, we integrating Eq. (5) with $\dot{\phi} = \omega_T$ over the entire time-of-flight T_1 to get

$$\delta_{f1} - \delta_{01} = \theta_{f1} - \theta_{01} + \dot{\phi}T_1 \tag{24}$$

Substituting the right hand side of Eq. (20) for $\delta_{f1} - \delta_{01}$, the right hand side of Eq. (22) for T_1, and the requirement $\theta_{f1} = 2m\pi + \frac{\pi}{2} + \psi$, Eq. (24) becomes

$$\frac{4m+1}{2k^{1/2}}\pi + \frac{\psi - \theta_{01}}{k^{1/2}} = \frac{\dot{\delta}_{01}}{\alpha_1 \varrho_{01}^{(\alpha_1 - 1)}} \frac{1}{c_1^{1/2}} \left\{ \sin^{-1}\left[\frac{\varrho_{f1}^2 k}{c_2}\right]^{1/2} - \sin^{-1}\left[\frac{\varrho_{01}^2 k}{c_2}\right]^{1/2} \right\} + \int_{\varrho_{01}}^{\varrho_{f1}} \frac{\dot{\phi}\, d\varrho}{(c_2 - c_1 \varrho^{2\alpha_1})^{1/2}} \tag{25}$$

An analytical solution for $\dot{\varrho}_{01}$ cannot be obtained since $c_1 = c_1(\dot{\varrho}_{01})$ and $c_2 = c_2(\dot{\varrho}_{01})$ (see Eqs. (18) and (19)) and the integral in Eq. (25) does not have an explicit anti-derivative. However, a solution for $\dot{\varrho}_{01}$ can be obtained from Eq. (25) using an iterative scheme. In this paper, Newton's method was used. For the cases analyzed, we assumed $m = 0$, implying the chase vehicle makes no complete revolution in phase 1.

Stage 2 Solution

The primary function of the guidance algorithm in this phase of the maneuver is to reduce both the range-to-go and the range-to-go rate to zero while maintaining the alignment of the berthing mechanisms. That is, throughout the entire second stage, we require $\dot{\theta}=0$. This is equivalent to (see Eq. (5))

$$\dot{\delta}_2 = \dot{\delta}_{0_2} = \dot{\phi} \tag{26}$$

Substituting this relationship into Eq. (11), the general solution for $\dot{\delta}$, yields

$$\dot{\phi} = \dot{\phi}\left(\frac{\varrho}{\varrho_{02}}\right)^{(\alpha_2 - 1)} \tag{27}$$

Since ϱ varies throughout the process, the above equality holds if and only if $\alpha_2 = 1$ which implies $\lambda_{LOS\perp_2} = 2$. With $\alpha_2 = 1$ and λ_{LOS_2} as defined in Eq. (14), $\dot{\varrho}$ can be determined from Eq. (12) as

$$\dot{\varrho} = \dot{\varrho}_{02}\left[\frac{\varrho_{f2}^2 - \varrho^2}{\varrho_{f2}^2 - \varrho_{02}^2}\right]^{1/2} \tag{28}$$

which is zero when $\varrho = \varrho_{f_2}$.

This completes the analysis. We now apply the results of the analysis to a typical rendezvous and docking scenario.

Numerical Simulation

The simulated results were obtained by numerically integrating the relative motion equations written in terms of the range-to-go r instead of the pseudo-range-to-go ϱ. In doing so, we are able to include the dynamics associated with the chase vehicle (i.e., the effects of its attitude controller). The state vector used was $[r\ \delta\ \beta\ \dot{r}\ \dot{\delta}\ \dot{\beta}]$. A fixed step size ($\Delta t = 0.001$), fourth-order Runge-Kutta integration algorithm was used to numerically integrate the differential equations.

Conditions for a typical scenario are shown in Table 1. For this scenario, the initial rendezvous conditions determined from Eqs. (23) and (25) are $\dot{\theta}_{01} = -0.4918$ and $\dot{\varrho}_{01} = -237.54$, respectively. An initial misalignment of $\beta = 0.05$ and an initial alignment rate of $\dot{\beta} = 0.04$ were used for the chase vehicle.

Table 1. System Configuration

r_{01}	1000	ψ	0
V_T	300	θ_{01}	3.77(1.2π)
$\dot{\phi}$	0.5	α_{01}	4.712(1.5π)
r_{Px}	10	$\lambda_{LOS\perp_1}$	0.95
r_{Py}	0	r_{f1}	15
r_{Qx}	0	$\dot{r}_{f1}$	−10
r_{Qy}	5	$\lambda_{LOS\perp_2}$	2

Figures 4- 8 show the simulation results for the scenario depicted in Table 1. Parts (c) and (d) of Figs. 4 and are enlargements of the terminal phase of flight depicted in parts (a) and (b). Figure 4 shows the flight data for the first stage. Notice that the range-to-go rate $\dot{r}$ goes to the designated value of -10 as range-to-go r goes to the specified value of 15. Also the relative LOS attitude θ goes to the desirable value of 90° (1.57 rad.) as the relative LOS attitude rate $\dot{\theta}$ goes to approximately zero.

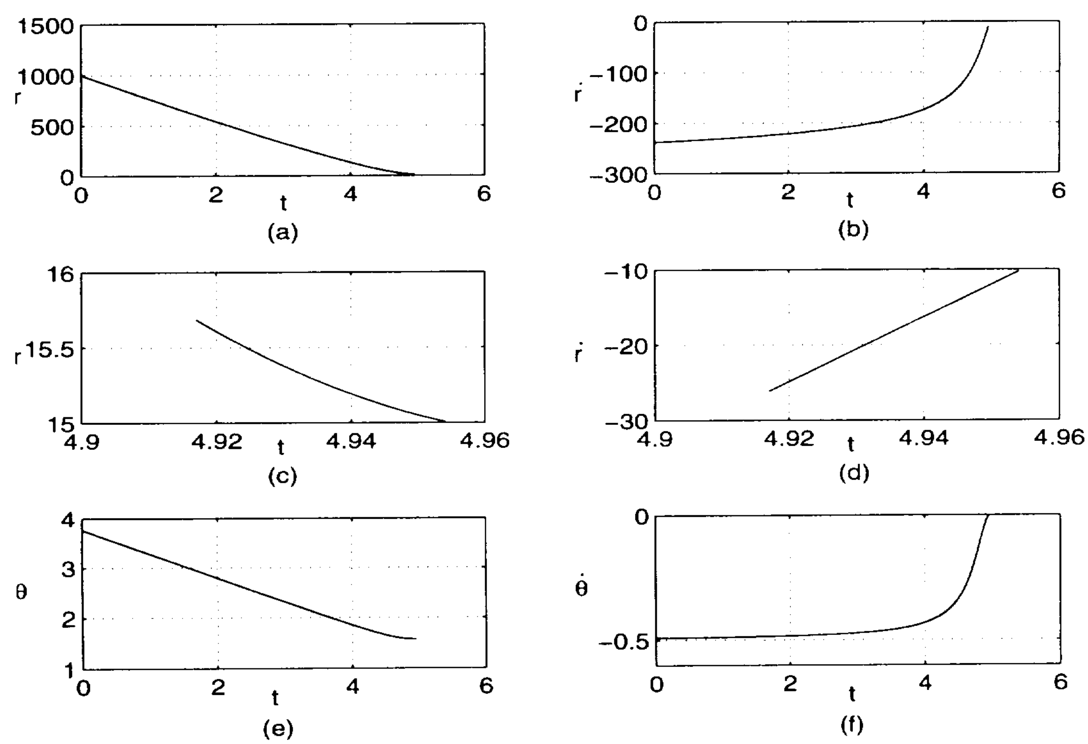

Fig. 4 Phase 1 flight data

Figure 5 shows the flight data for the second stage. Notice that the range-to-go rate $\dot{r}$ and the range-to-go r simultaneously approach zero. Also the relative LOS attitude θ is maintained within the proximity of the desirable value of 90° (1.57 rad.). The slight variations are due to the relative LOS attitude rate $\dot{\theta}$ being close to zero but not exactly zero throughout the second stage. This is not a consequence of the numerics, but rather a consequence of the fact that an actual attitude controller was modelled for the chase vehicle.

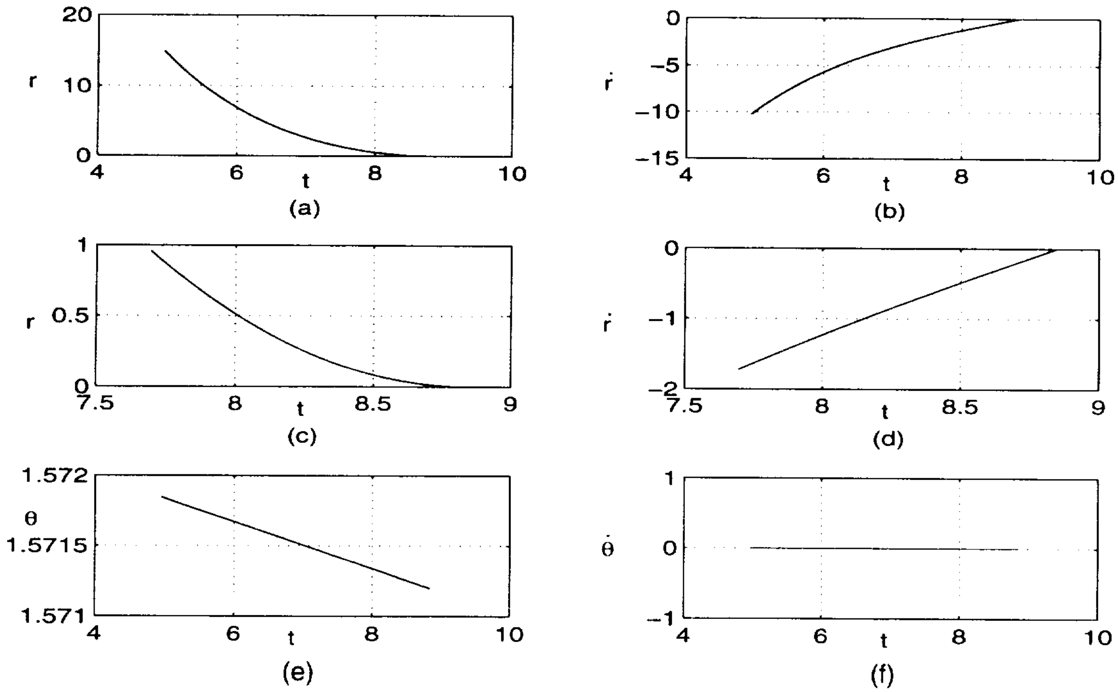

Fig. 5 Phase 2 flight data

Figure 6 shows the accelerations commanded throughout the maneuver. The curves on the left half of the plot are the acceleration commands for phase 1 and the curves on the right half of the plot are for phase 2. Notice that since $\vec{r} = -r\,\hat{\vec{j}}_{G'}$, then the a_{LOS} accelerations shown represent accelerations in the positive r-direction.

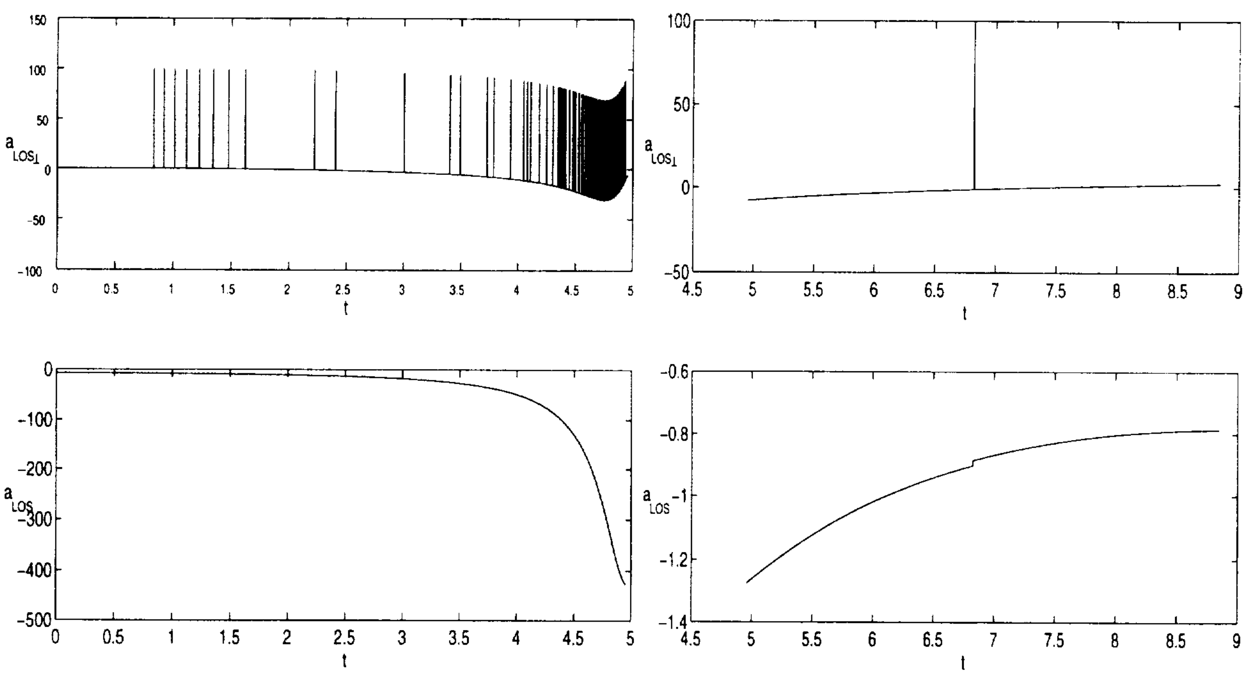

Fig. 6 Profiles of commanded accelerations

Figures 7 and 8 show the terminal portions of phases 1 and 2, respectively. Figure 7 shows the chase vehicle approaching the target from above and acquiring properly alignment with the target at the end of the first stage.

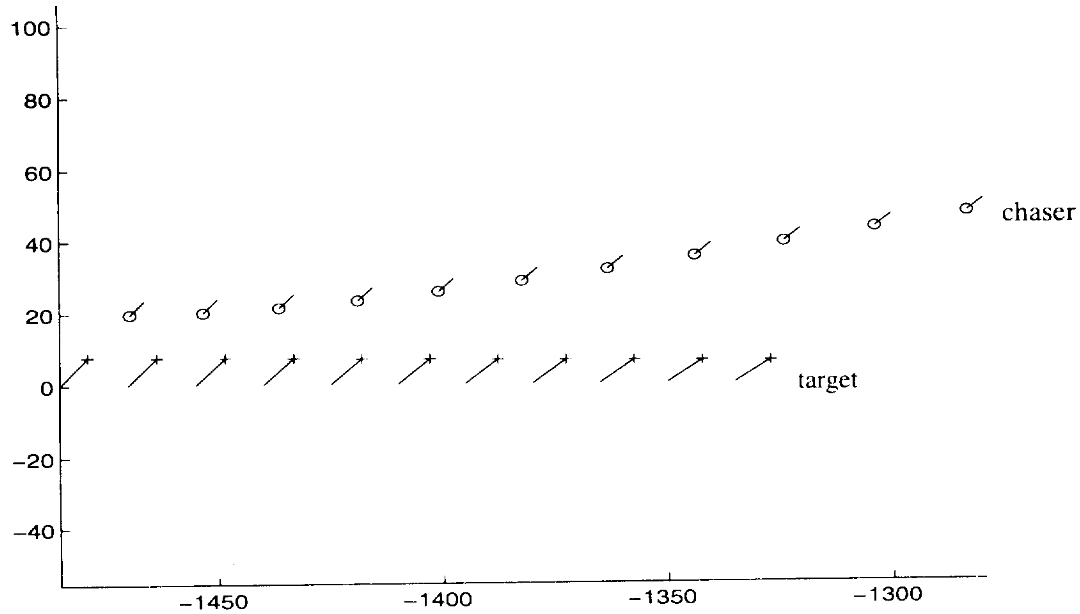

Fig. 7 Alignment of chase and target vehicles at the end of phase 1

Figure 8 shows that, during the second stage, the chase vehicle is able to maintain it's attitude with respect to the target while it nulls the distance between the grappling arm and the docking port. It also shows that, at the end of the second stage, the chase vehicle docks with the target with the proper relative attitude as specified.

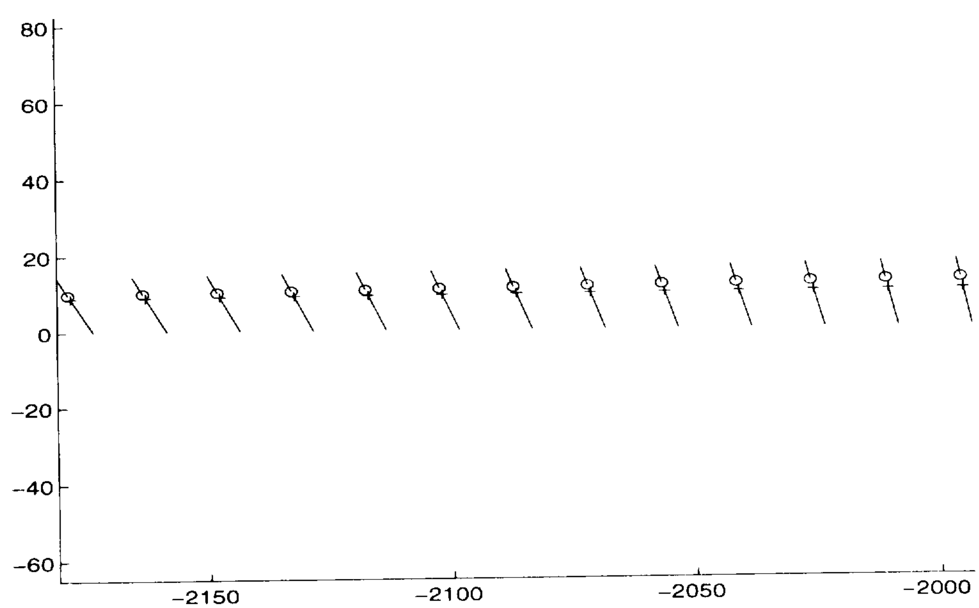

Fig. 8 Docking at the end of phase 2

Conclusion

Solutions to the equations of motion of two rigid bodies engaged in a planar rendezvous and docking maneuver are obtained. Some solutions are analytic whereas others are pseudo-analytic. It was shown that rendezvous and dock of rigid bodies require a two phases maneuver. In the first phase, the LOS rotation rate is driven to zero while aligning the berthing mechanisms of the two vehicles, and in the second phase, the LOS rate is maintained at zero while the range-to-go and range-to-go rates are simultaneously driven to zero. The second phase of the maneuver requires a transverse navigation constant of 2. An illustrative example is presented.

Future work involves the integration of appropriate sensor and actuator models and a proof-of-concept demonstration.

Acknowledgement

The authors wish to acknowledge the financial support of NASA grant NAG8-280 sponsored through Marshall Space Flight Center..

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