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Telemetry Down-link Doppler as an Attitude Sensor for Spin Stabilized Spacecraft.*

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Abstract. The communications antenna on a spin stabilized spacecraft is seldom located on the spin axis, hence, the antenna is in motion relative to the center of mass of the spacecraft. The Doppler shift observed at the ground or space relay communications receivers will include oscillations whose frequency and amplitude are functions of the motion of the antenna and the attitude of the spacecraft relative to the line of sight (LOS). This functional dependence creates the possibility of estimating attitude parameters from Doppler measurements. This paper presents mathematical models of Doppler oscillations from spinning spacecraft, including the effects of nutation. Algorithms for estimating spin rate, attitude and nutation angle are described. Results of analysis of Doppler tracking of GOES-8 and WIND are also discussed.

Introduction. Doppler tracking measurements are usually treated as equivalent to measurements of the range-rate of the spacecraft center of mass. However, Doppler measurements actually measure the range rate of the antenna, which during attitude maneuvers or spin stabilization has its own motion relative to the center of mass. A simple model for a spinning spacecraft is to assume that the angular momentum and angular velocity vectors are collinear. Then, unless the antenna location is on the spin axis, the antenna will have uniform circular motion relative to the spacecraft center of mass, so that the range rate will vary sinusoidally. The frequency of this motion is the spin rate; the amplitude of the motion is a function of the angle between the LOS direction and the spin axis. Hence, Doppler data can be used to estimate the orientation of the spin axis. There are two major phenomena that may invalidate this simple model. One is nutation, where the angular velocity vector is rotating about the (inertially fixed) angular momentum vector. The other is multipath, where the propagation path from the spacecraft antenna to the tracker includes reflections from other parts of the spacecraft, parts which have different Doppler shifts than does the antenna. In addition, a spacecraft may have more than a single antenna. Multipath will not be discussed in this paper.

Apparent amplitude of Doppler oscillations. The amplitude of the oscillations in Doppler tracking caused by satellite rotation is a function of the antenna location, the rotation rate, and the aspect angles relative to the uplink and downlink trackers. For the purpose of deriving this functional dependence, effects of nutation will be ignored, hence, the spin axis is assumed to be coincident with the angular momentum vector, which is fixed in inertial coordinates. The inertial coordinate system will have the spin axis as the z-axis, and the x- and y-axes orthogonal. The following parameters will be used,

$\hat{\mathbf{s}}$ - -unit vector along spin axis

$\hat{\mathbf{u}}$ - -unit vector along LOS from uplink tracker to spacecraft in inertial coordinates

$$\hat{\mathbf{u}} = \mathbf{i}u_1 + \mathbf{j}u_2 + \mathbf{k}u_3$$

$\hat{\mathbf{d}}$ - -unit vector along LOS from downlink tracker to spacecraft in inertial coordinates

$$\hat{\mathbf{d}} = \mathbf{i}d_1 + \mathbf{j}d_2 + \mathbf{k}d_3$$

r_A - -distance from spin axis to antenna

$\hat{\mathbf{s}}$ - -unit vector along spin axis

$\vec{\mathbf{r}}(t)$ - -position vector of antenna relative to spacecraft center of mass in inertial coordinates

$$\vec{\mathbf{r}}(t) = \mathbf{i}r_1(t) + \mathbf{j}r_2(t) + \mathbf{k}r_3$$

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$\vec{v}(t)$ --velocity vector of antenna relative to spacecraft center of mass in inertial coordinates

ω_s --spin rate in $\frac{\text{radians}}{\text{sec}}$

The position and velocity of the antenna as functions of time can be written,

$$\vec{r}(t) = \mathbf{i} r_A \cos(\omega_s t) + \mathbf{j} r_A \sin(\omega_s t) + \mathbf{k} r_3$$

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = r_A \omega_s \left(-\mathbf{i}\sin(\omega_s t) + \mathbf{j}\cos(\omega_s t)\right)$$

The projections of the LOS vectors on the antenna velocity are,

$$\hat{\mathbf{u}} \bullet \vec{v}(t) = \left(\hat{\mathbf{u}} - (\hat{\mathbf{u}} \bullet \hat{\mathbf{s}})\hat{\mathbf{s}}\right) \bullet \vec{v}(t) = r_A \omega_s \left(-u_1 \sin\omega_s t + u_2 \cos\omega_s t\right)$$

$$\hat{\mathbf{d}} \bullet \vec{v}(t) = \left(\hat{\mathbf{d}} - \left(\hat{\mathbf{d}} \bullet \hat{\mathbf{s}}\right)\hat{\mathbf{s}}\right) \bullet \vec{v}(t) = r_A \omega_s \left(-d_1 \sin\omega_s t + d_2 \cos\omega_s t\right)$$

Where c is the speed of light in the same units as range rate and f_T is the transmit frequency in Hz , define

$$\Delta_f = \frac{K f_T}{c}$$

K is the turnaround ratio of the spacecraft transponder: K = 240/221 for NASA S-band trackers. The range rate oscillation and associated frequency deviation observed at the downlink tracker are,

$$\dot{R}_\Delta(t) = r_A \omega_s \left(-(u_1 + d_1)\sin\omega_t + (u_2 + d_2)\cos\omega_s t\right)$$

$$f_\Delta(t) = \dot{R}_\Delta(t)\Delta_f$$

The expression for frequency deviation can be converted to amplitude-phase form,

$$f_\Delta(t) = A\cos(\omega_s t + \phi)$$

The amplitude of the frequency oscillation is,

$$\begin{aligned} A &= r_A \omega_s \Delta_f \sqrt{(u_1 + d_1)^2 + (u_2 + d_2)^2} \\ &= r_A \omega_s \Delta_f \left\| \hat{\mathbf{u}} + \hat{\mathbf{d}} - \left((\hat{\mathbf{u}} + \hat{\mathbf{d}}) \bullet \hat{\mathbf{s}}\right)\hat{\mathbf{s}} \right\| \\ &= r_A \omega_s \Delta_f \sqrt{\|\hat{\mathbf{u}}\|^2 + \|\hat{\mathbf{d}}\|^2 + 2(\hat{\mathbf{u}} \bullet \hat{\mathbf{d}}) - \left((\hat{\mathbf{u}} + \hat{\mathbf{d}}) \bullet \hat{\mathbf{s}}\right)^2} \\ &= r_A \omega_s \Delta_f \sqrt{2 + 2(\hat{\mathbf{u}} \bullet \hat{\mathbf{d}}) - \left((\hat{\mathbf{u}} + \hat{\mathbf{d}}) \bullet \hat{\mathbf{s}}\right)^2} \end{aligned}$$

For 2-way tracking, where the transmit and receive trackers are the same, the amplitude reduces to

$$A = 2 r_A \omega_s \Delta_f \sqrt{1 - (\hat{\mathbf{u}} \bullet \hat{\mathbf{s}})^2}$$

The 1-way amplitude would be half the 2-way. Note that there is a 180 deg ambiguity; if the spin vector is replaced by it's negative, i.e., the direction of spin is reversed or the spacecraft is flipped over, the apparent Doppler oscillation amplitude will be the same.

Estimating attitude. Given that the spin rate is known, all the parameters in the above equations are known except the spin axis unit vector. The absolute value of the projection of the spin axis unit vector onto a known vector can be solved for as follows:

$$(2\text{-way})\quad |\hat{\mathbf{u}} \bullet \hat{\mathbf{s}}| = \sqrt{1 - \frac{A^2}{4 r_A^2 \omega_s^2 \Delta_f^2}}$$

$$(1\text{-way})\quad |\hat{\mathbf{u}} \bullet \hat{\mathbf{s}}| = \sqrt{1 - \frac{A^2}{r_A^2 \omega_s^2 \Delta_f^2}}$$

$$(3\text{-way})\quad \left|\left(\hat{\mathbf{u}} + \hat{\mathbf{d}}\right) \bullet \hat{\mathbf{s}}\right| = \sqrt{2 + 2\left(\hat{\mathbf{u}} \bullet \hat{\mathbf{d}}\right) - \frac{A^2}{r_A^2 \omega_s^2 \Delta_f^2}}$$

The last equation can be rewritten in terms of unit vectors by normalizing,

$$(3\text{-way})\quad \left|\frac{\left(\hat{\mathbf{u}} + \hat{\mathbf{d}}\right)}{\left\|\hat{\mathbf{u}} + \hat{\mathbf{d}}\right\|} \bullet \hat{\mathbf{s}}\right| = \sqrt{1 - \frac{A^2}{r_A^2 \omega_s^2 \Delta_f^2 \left(2 + 2\left(\hat{\mathbf{u}} \bullet \hat{\mathbf{d}}\right)\right)}}$$

From this point on, it will be assumed that the Doppler tracking observations have been reduced to a set of projections and unit vectors,

$$a_k = |\hat{\mathbf{x}}_k \bullet \hat{\mathbf{s}}|; \quad 0 \le a_k \le 1$$

Each projection is equivalent to an angle,

$$\xi_k = \cos^{-1}(a_k); \quad 0 \le \xi_k \le \frac{\pi}{2}$$

When many observations (projections and unit vectors) are available, a least squares solution can be computed. Depending on geometry, several solutions are possible, so initiating the least squares algorithm properly is important. If the attitude is the only unknown, the right ascension (α) and declination (δ) of the spin axis can be solved for directly by a non-linear least squares routine,

$$\text{Minimize } e = \sum_{k=1}^{K} \left(a_k - \left| \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix} \bullet \hat{\mathbf{x}}_k \right| \right)^2$$

If another parameter such as antenna radius from spin axis is uncert[illegible] may also be solved for. The MATLAB™ Optimization Toolbox installed on FDF LANs contains built[illegible] functions for minimization.

Analysis of GOES-8 Doppler Tracking. GOES-8 was launched 13 April 1994. Doppler tracking [illegible] was received at FDF from DS46 (Canberra) 3-way with the Indian Ocean Station (IOS or SEYS) telemetry system as the transmit site. For data taken from approximately 0700Z to 0740Z, the spacecraft was spin stabilized. Figure 1 shows the Doppler oscillations from valid data obtained during this time. Shown by a dotted line is the estimate of amplitude derived from these oscillations. The measured rotation rate is 0.0182 Hz, while the distance of the antenna from the spin axis is assumed to be 3.3 meters. Figure 2 depicts contours of the surface generated by varying right ascension from 25 to 75 deg and declination from -50 to 0 deg and plotting the reciprocal of the resultant RMS error between the measured projection and the vector dot product. Further analysis showed that the fit could be improved if r_A was varied as well. The MATLAB routine LEASTSQ in the Optimization Toolbox was used to solve for right ascension, declination and antenna radius from the spin axis. The routine was run twice, initialized with each of the angle pairs found above and with r_A initialized at 3.3 meters. The solutions were,

RMS error	0.042	0.044
Right ascension (deg)	62.2	59.7
Declination (deg)	-24.7	-16.7
Antenna radius from spin axis (m)	2.79	2.76

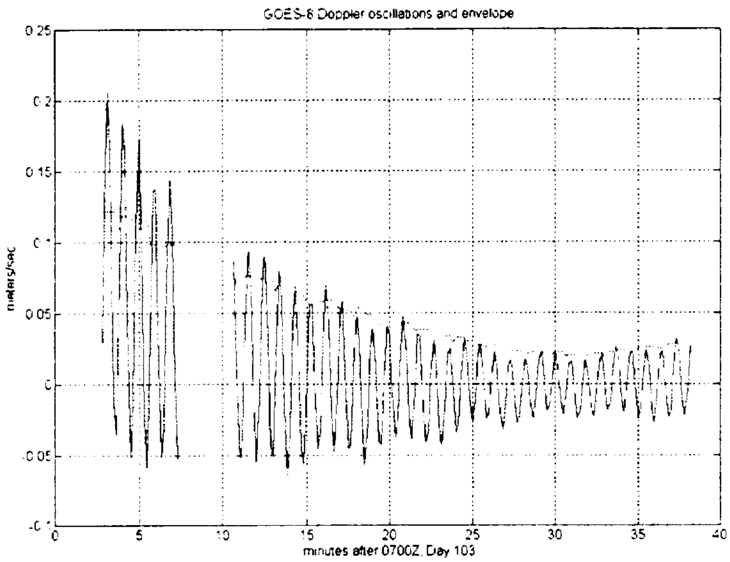

Figure 1. GOES-8 Doppler Tracking.

Since the observations were not exactly co-planar, the solution with the smaller RMS error would be the more likely candidate for the true attitude. The planned attitude after separation was $\alpha = 61.7$ deg and $\delta = -25.0$ deg, according to reference 1. Figure 3 shows the fit obtained by comparing the projection of the solved for spin attitude on the unit vectors with the projections measured from tracking data.

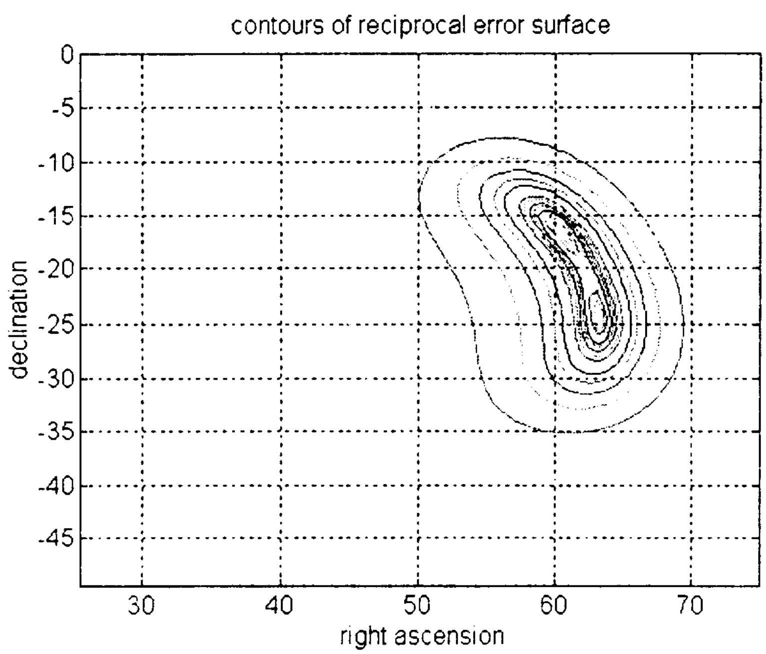

Figure 2. Error Surface.

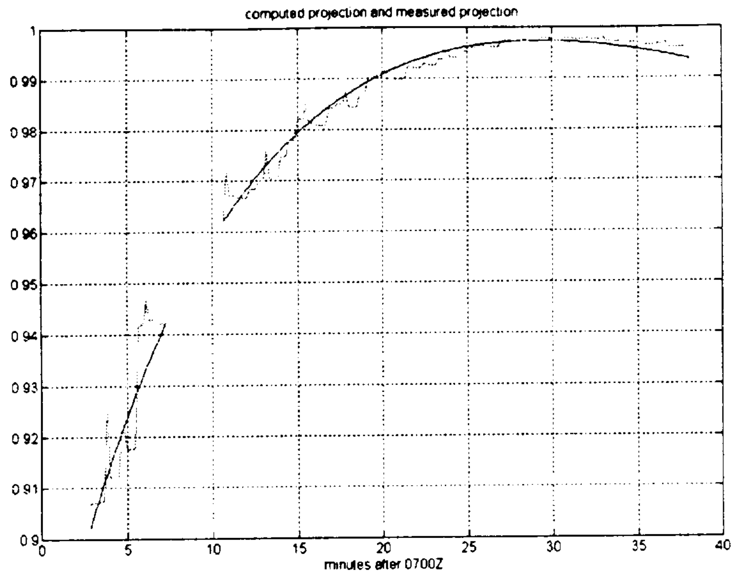

Figure 3. Measured versus computed projections.

Spinning spacecraft attitude dynamics (nutation). Analysis of the dynamics of a rigid spinning spacecraft in order to predict the effects of such motion on Doppler tracking data uses the following parameters:

1. The spacecraft major moments of inertia, in Kg-m^2. The nominal axis of rotation is assumed to be the z-axis, so the values are represented as a vector [Ix Iy Iz], where the labels x, y and z are assigned so that the entries are in monotonic (lowest to highest or highest to lowest) order. The choice of the axis with the intermediate moment of inertia as the spin axis is unstable and will not be considered.

2. The location of the spacecraft antenna in meters, as a vector $[x_A \; y_A \; z_A]$ in body coordinates. The assignment of x_A, y_A and z_A should be consistent with the moments of inertia.

3. The angle β of the LOS vector with the angular momentum vector.

4. The angular momentum magnitude L.

5. The initial nutation angle θ_0, which is assumed to be small.

Reference 2 is the source of the basic attitude dynamics relationships in the following. The convenient inertial reference frame has the angular momentum vector along the z-axis, with the x and y orthogonal. The time reference is chosen so that at t=0, the angular velocity vector is in the body x-z plane, so the angular velocity vector in body coordinates is,

$$\vec{\omega}_{b0} = \mathbf{i}_b\omega_{10} + \mathbf{j}_b 0 + \mathbf{k}_b\omega_{30} = \mathbf{i}_b \frac{L\sin\theta_0}{Ix} + \mathbf{k}_b \frac{L\cos\theta_0}{Iz}$$

$$\omega_{10} \approx \frac{L\theta_0}{Ix}, \quad \omega_{30} \approx \frac{L}{Iz}$$

The kinetic energy E is,

$$E = \frac{Ix\omega_{10}^2 + Iz\omega_{30}^2}{2} = \frac{1}{2}\left(\frac{L^2 \sin^2\theta_0}{Ix} + \frac{L^2\cos^2\theta_0}{Iz}\right)$$

The "parameter" m is given by,

$$m = \frac{(Ix-Iy)(L^2-2IzE)}{(Iz-Iy)(L^2-2IxE)} = \frac{(Ix-Iy)L^2\left(1-Iz\left(\frac{\sin^2\theta_0}{Ix}+\frac{\cos^2\theta_0}{Iz}\right)\right)}{(Iz-Iy)L^2\left(1-Ix\left(\frac{\sin^2\theta_0}{Ix}+\frac{\cos^2\theta_0}{Iz}\right)\right)}$$

$$= \frac{(Ix-Iy)\left(\frac{(Ix-Iz)\sin^2\theta_0}{Ix}\right)}{(Iz-Iy)\left(\frac{(Iz-Ix)\cos^2\theta_0}{Iz}\right)} \approx \frac{Iz(Iy-Ix)}{Ix(Iz-Iy)}\theta_0^2$$

Define the "normalized" parameter,

$$m_0 = \frac{Iz(Iy-Ix)}{Ix(Iz-Iy)} \approx m/\theta_0^2.$$

The body nutation rate is,

$$\omega_p = \pm\sqrt{\frac{(Iz-Iy)(L^2-2IxE)}{IxIyIz}} = \pm\sqrt{\frac{(Iz-Iy)L^2\left((Iz-Ix)\cos^2\theta_0\right)}{IxIyIz^2}}$$

$$\approx \pm\frac{L}{Iz}\sqrt{\frac{(Iz-Iy)(Iz-Ix)}{IxIy}} \approx \pm\omega_{30}\sqrt{\frac{(Iz-Iy)(Iz-Ix)}{IxIy}}$$

where the "+" is used if Ix > Iy > Iz and "-" if Ix < Iy < Iz.

Finally, the amplitude of the y body component of angular velocity is,

$$\omega_{20} = \sqrt{\frac{L^2-2IzE}{Iy(Iy-Iz)}} = L\sqrt{\frac{(Ix-Iz)\sin^2\theta_0}{IxIy(Iy-Iz)}} \approx L\theta_0\sqrt{\frac{(Ix-Iz)}{IxIy(Iy-Iz)}}$$

$$= \frac{L\theta_0}{Iy}\sqrt{\frac{IxIy-IyIz}{Ix(Iy-Iz)}} = \frac{L\theta_0}{Iy}\sqrt{\frac{IxIy-IyIz-IxIz+IxIz}{Ix(Iy-Iz)}} = \frac{L\theta_0}{Iy}\sqrt{1+\frac{Iz(Ix-Iy)}{Ix(Iy-Iz)}}$$

$$= \frac{L}{Iy}\sqrt{\theta_0^2+m} = \frac{L}{Iy}\theta_0\sqrt{1+m_0}$$

The angular velocity vector as a function of t (sec) is given by,

$$\omega_1 = \omega_{10}\mathrm{cn}(\omega_p t|m)$$
$$\omega_2 = -\omega_{20}\mathrm{sn}(\omega_p t|m)$$
$$\omega_3 = \omega_{30}\mathrm{dn}(\omega_p t|m)$$

where sn, cn and dn are the Jacobian elliptic functions. When m << 1, approximations for the Jacobian elliptic functions are,

$$sn(x|m) \approx \sin(x)$$
$$cn(x|m) \approx \cos(x)$$
$$dn(x|m) = 1 - \frac{m}{2}\sin^2(x) = 1 - \frac{m}{4} + \frac{m}{4}\cos(2x)$$

So the angular velocity vector in body coordinates can be written,

$$\omega_1 = \omega_{10}\text{cn}(\omega_p t|m) \approx \omega_{10}\cos(\omega_p t)$$
$$\omega_2 = -\omega_{20}\text{sn}(\omega_p t|m) \approx -\omega_{20}\sin(\omega_p t)$$
$$\omega_3 = \omega_{30}\text{dn}(\omega_p t|m) \approx \omega_{30}\left[1 - \frac{m}{4} + \frac{m}{4}\cos\left(2\omega_p t\right)\right]$$

The instantaneous velocity of the antenna in body coordinates is found by crossing the angular velocity vector with the antenna position vector. The antenna velocity is then rotated to inertial coordinates and projected on the LOS vector to give the range-rate of the antenna as a function of spinning.

The Euler rotation angles for angles θ and ψ can be found by solving,

$$\begin{bmatrix} L_{b1} \\ L_{b2} \\ L_{b3} \end{bmatrix} = A_{313}(\phi,\theta,\psi)\begin{bmatrix} 0 \\ 0 \\ L \end{bmatrix} = L\begin{bmatrix} \sin\theta\sin\psi \\ \sin\theta\cos\psi \\ \cos\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} \dfrac{L\sin\theta\sin\psi}{Ix} \\ \dfrac{L\sin\theta\cos\psi}{Iy} \\ \dfrac{L\cos\theta}{Iz} \end{bmatrix}$$

So the minimum and maximum values of nutation angle are,

$$\theta_{min} = \theta_0$$
$$\theta_{max} = \sqrt{\theta_0^2 + m} \approx \theta_0\sqrt{1 + m_0}$$

This implies,

$$\frac{\theta_{max}^2}{\theta_0^2} \approx \frac{\theta_0^2 + m}{\theta_0^2} = 1 + \frac{m}{\theta_0^2} = \frac{Iy(Iz - Ix)}{Ix(Iz - Iy)}$$

which agrees with equation 16-114 in reference 2.

The angular velocity vector in body coordinates can now be written as,

$$\omega_1 \approx \frac{L}{Ix}\theta_0 cn(\omega_p t|m)$$

$$\omega_2 \approx \frac{-L}{Iy}\theta_{\max} sn(\omega_p t|m)$$

$$\omega_3 \approx \frac{L}{Iz}$$

Sine and cosine of the Eulerian angle ψ can be found as functions of time by solving,

$$\tan\psi = \frac{Ix\omega_1}{Iy\omega_2} \Rightarrow \sin\psi = \frac{Ix\omega_1}{\sqrt{Ix^2\omega_1^2 + Iy^2\omega_2^2}};\cos\psi = \frac{Iy\omega_2}{\sqrt{Ix^2\omega_1^2 + Iy^2\omega_2^2}}$$

$$\sqrt{Ix^2\omega_1^2 + Iy^2\omega_2^2} \approx L\theta_0\sqrt{cn(\omega_p t|m)^2 + (1+m_0)sn(\omega_p t|m)^2}$$
$$= L\theta_0\sqrt{1+m_0 sn(\omega_p t|m)^2} = L\theta$$

$$\sin\psi = \frac{Ix\omega_1}{\sqrt{Ix^2\omega_1^2 + Iy^2\omega_2^2}} \approx \frac{Ix\left(\frac{L\theta_0}{Ix}\right)cn(\omega_p t|m)}{L\theta} = \frac{\theta_0}{\theta}cn(\omega_p t|m)$$

$$\cos\psi = \frac{Iy\omega_2}{\sqrt{Ix^2\omega_1^2 + Iy^2\omega_2^2}} \approx \frac{Iy\left(\frac{-L\theta_{\max}}{Iy}\right)sn(\omega_p t|m)}{L\theta} = \frac{-\theta_{\max}}{\theta}sn(\omega_p t|m)$$

A differential equation relating ϕ to θ and ψ is,

$$\dot{\phi} = (\omega_1\sin\psi + \omega_2\cos\psi)/\sin\theta = L\left(\frac{\sin^2\psi}{Ix} + \frac{\cos^2\psi}{Iy}\right) = \frac{L}{Ix} + \frac{L(Ix-Iy)\cos^2\psi}{IxIy}$$
$$\approx L\left(\frac{1}{Ix} + \frac{\theta_0^2(Ix-Iz)(Ix-Iy)sn(\omega_p t|m)^2}{\theta^2 Ix^2(Iy-Iz)}\right)$$
$$\approx L\left(\frac{1}{Ix} + \frac{\theta_0^2(Ix-Iz)(Ix-Iy)sn(\omega_p t|m)^2}{\left(\theta_0^2 + m\,sn(\omega_p t|m)^2\right)Ix^2(Iy-Iz)}\right)$$

To get the frequency of oscillation of terms involving $\cos\phi$ and $\sin\phi$, the average is needed.

$$<\dot{\phi}> \approx L\left(\frac{1}{Ix} + \left(\frac{\theta_0^2(Ix-Iz)(Ix-Iy)}{Ix^2(Iy-Iz)}\right)\frac{1}{2K}\int_{-K}^{K}\frac{sn(\omega_p t|m)^2\,dt}{\left(\theta_0^2 + m\,sn(\omega_p t|m)^2\right)}\right)$$

where K is the half period of the Jacobian elliptic functions (ref. 3, section 16). The integral may be approximated by substituting sin for sn since m is small,

$$\frac{1}{2K}\int_{-K}^{K}\frac{sn(\omega_p t|m)^2\,dt}{\left(\theta_0^2+m\,sn(\omega_p t|m)^2\right)}\approx\frac{\omega_p}{\pi}\int_{\frac{-\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}}\frac{\sin^2(\omega_p t)dt}{\left(\theta_0^2+m-m\cos^2(\omega_p t)\right)}$$

$$=\frac{1}{\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2(x)dx}{\left(\theta_0^2+m-m\cos^2(x)\right)}=\frac{1}{m}\left(1-\sqrt{\frac{\theta_0^2}{\theta_0^2+m}}\right)$$

(integration formula 262 in ref. 4)

Substituting for the integral,

$$<\dot{\phi}>\approx L\left(\frac{1}{Ix}+\left(\frac{\theta_0^2(Ix-Iz)(Ix-Iy)}{Ix^2(Iy-Iz)}\right)\frac{1}{m}\left(1-\sqrt{\frac{\theta_0^2}{\theta_0^2+m}}\right)\right)$$

$$=L\left(\frac{1}{Ix}+\left(\frac{(Ix-Iz)(Ix-Iy)}{Ix^2(Iy-Iz)}\right)\frac{\theta_0^2}{m}\left(1-\sqrt{\frac{1}{1+m/\theta_0^2}}\right)\right)$$

$$=L\left(\frac{1}{Ix}+\left(\frac{(Ix-Iz)(Ix-Iy)}{Ix^2(Iy-Iz)}\right)\frac{Ix(Iz-Iy)}{Iz(Iy-Ix)}\left(1-\sqrt{\frac{Ix(Iz-Iy)}{Iy(Iz-Ix)}}\right)\right)$$

$$=L\left(\frac{1}{Ix}+\left(\frac{(Ix-Iz)}{IxIz}\right)\left(1-\sqrt{\frac{Ix(Iz-Iy)}{Iy(Iz-Ix)}}\right)\right)$$

$$=\frac{L}{Iz}+\left\{\begin{array}{ll}+\frac{L}{Iz}\sqrt{\frac{(Iz-Ix)(Iz-Iy)}{IxIy}} & ;Iz>Ix\\ -\frac{L}{Iz}\sqrt{\frac{(Iz-Ix)(Iz-Iy)}{IxIy}} & ;Ix>Iz\end{array}\right\}\approx\omega_{30}-\omega_p$$

Thus, the signals cosϕ and sinϕ oscillate at an average frequency approximately equal to the difference of the spin rate and the nutation frequency.

Estimation. With the preceding derivation, it is now possible to write the component of measured range-rate caused by antenna motion as a function of known parameters, signals with known frequencies and amplitudes and parameters to be estimated. The parameters are:

parameter	Name	known or to be estimated
L	angular momentum magnitude	either
θ	nutation angle	to be estimated
β	LOS angle	either
Ix, Iy, Iz	moments of inertia	known
m_0	normalized parameter of elliptic function	known
x_A, y_A, z_A	body coordinates of antenna	known

$$\bar{v}(t) = \frac{d}{dt}\left\{ A_{313}^{-1}(\phi,\theta,\psi)\begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix}\right\}$$

$$\approx \frac{d}{dt}\begin{bmatrix} (\cos\psi\cos\phi - \sin\psi\sin\phi)x_A + (-\sin\psi\cos\phi - \cos\psi\sin\phi)y_A + \theta z_A\cos\phi \\ x \\ \theta\sin\psi x_A + \theta\cos\psi y_A + z_A \end{bmatrix}$$

$$= \frac{d}{dt}\begin{bmatrix} \cos(\phi+\psi)x_A + -\sin(\phi+\psi)y_A + \theta\cos\phi z_A \\ x \\ \theta\sin\psi x_A + \theta\cos\psi y_A + z_A \end{bmatrix}$$

The second component will not be used and so is not calculated, since the LOS vector in inertial coordinates is assumed to be,

$$\mathbf{LOS} = \begin{bmatrix} \sin\beta \\ 0 \\ \cos\beta \end{bmatrix}$$

The measured range-rate will be the dot product of the LOS vector and the inertial velocity, hence the incremental range-rate due to antenna motion that will be measured by the tracker is the sum of two components. The two components will be evaluated separately.

If the LOS direction were parallel to the angular momentum vector, the range-rate due to antenna motion is, where Φ is an unknown phase angle,

$$\begin{aligned} \dot{R}_V &\approx \frac{d}{dt}(\theta\sin\psi x_A + \theta\cos\psi y_A + z_A) \\ &\approx \frac{d}{dt}\left(x_A\theta_0 cn(\omega_p t|m) - y_A\theta_{max} sn(\omega_p t|m) + z_A\right) \\ &= \left(-x_A\theta_0 sn(\omega_p t|m) - y_A\theta_{max} cn(\omega_p t|m)\right)\omega_p dn(\omega_p t|m) \end{aligned}$$

This will be called the "vertical component" of the antenna motion. It can be rewritten as,

$$\begin{aligned} \dot{R}_V &\approx \left(-x_A\theta_0 sn(\omega_p t|m) - y_A\sqrt{1+m_0}\,\theta_0 cn(\omega_p t|m)\right)\omega_p dn(\omega_p t|m) \\ &\approx \theta_0\omega_p\sqrt{x_A^2 + (1+m_0)y_A^2}\,\cos(\omega_p t + \Phi) \end{aligned}$$

So the vertical component is a signal oscillating at the nutation rate and with amplitude proportional to the minimum nutation angle.

The horizontal component is a bit more complicated.

$$R_H \approx \frac{d}{dt}[(\cos\psi\cos\phi - \sin\psi\sin\phi)x_A + (-\sin\psi\cos\phi - \cos\psi\sin\phi)y_A + z_A\theta\cos\phi]$$
$$\approx \frac{d}{dt}\left[x_A\left(\frac{-\theta_{max}}{\theta}sn(\omega_p t|m)\cos\phi - \frac{\theta_0}{\theta}cn(\omega_p t|m)\sin\phi\right) + y_A\left(\frac{\theta_{max}}{\theta}sn(\omega_p t|m)\sin\phi - \frac{\theta_0}{\theta}cn(\omega_p t|m)\cos\phi\right) + z_A\theta\cos\phi\right]$$
$$\approx \frac{d}{dt}\left[\frac{\theta_0}{\theta}\left(x_A\left(-\sqrt{1+m_0}\,sn(\omega_p t|m)\cos\phi - cn(\omega_p t|m)\sin\phi\right) + y_A\left(\sqrt{1+m_0}\,sn(\omega_p t|m)\sin\phi - cn(\omega_p t|m)\cos\phi\right)\right) + z_A\theta\cos\phi\right]$$

The ratio θ_0 / θ has essentially no dependence on θ, so the nutation component of antenna motion is, where Φ' and Φ'' are unknown phase angles,

$$\dot{R}_{HN} = \frac{d}{dt}(z_A\theta\cos\phi)$$
$$= z_A\left(\dot{\theta}\cos\phi - \theta\dot{\phi}\sin\phi\right) \approx z_A\left(\dot{\theta}\cos\phi - \theta(\omega_{30} - \omega_p)\sin\phi\right)$$
$$\approx -z_A\theta_0\sqrt{1+\frac{m_0}{2}}(\omega_{30} - \omega_p)\sin((\omega_{30} - \omega_p)t + \Phi') +$$
$$\frac{z_A\theta_0 m_0}{2\sqrt{4+2m_0}}\left[(\omega_{30} + \omega_p)\sin((\omega_{30} + \omega_p)t + \Phi') - (\omega_{30} - 3\omega_p)\sin((\omega_{30} - 3\omega_p)t + \Phi')\right]$$

So the horizontal nutation component is a signal oscillating at a frequency equal to the difference of the spin rate and the nutation rate, together with sidebands at intervals of $\pm 2\omega_p$, all with amplitudes proportional to the minimum nutation angle.

WIND Nutation Estimation. During initial phases of FDF support of the WIND mission, Doppler tracking data was used to monitor the spin rate and nutation angle of the spacecraft. The purpose was to provide a back-up to on-board attitude sensor telemetry, with the emphasis on measurement of nutation angle immediately prior to the Lanyard Boom deployment. The WIND spacecraft was launched on 1 November 1994. WIND is spin stabilized with a nominal spin rate of 20 RPM. WIND is equipped with a nutation damper with a nominal time constant of 1400 seconds. The spacecraft is cylindrical, with the z-(spin) axis being the axis of the cylinder. The Medium Gain Antenna (MGA) is extended from the negative z face of the cylinder. Figure 4 shows the WIND spacecraft configuration. The WIND attitude is maintained with the spin axis parallel to the south ecliptic pole, effectively perpendicular to the LOS.

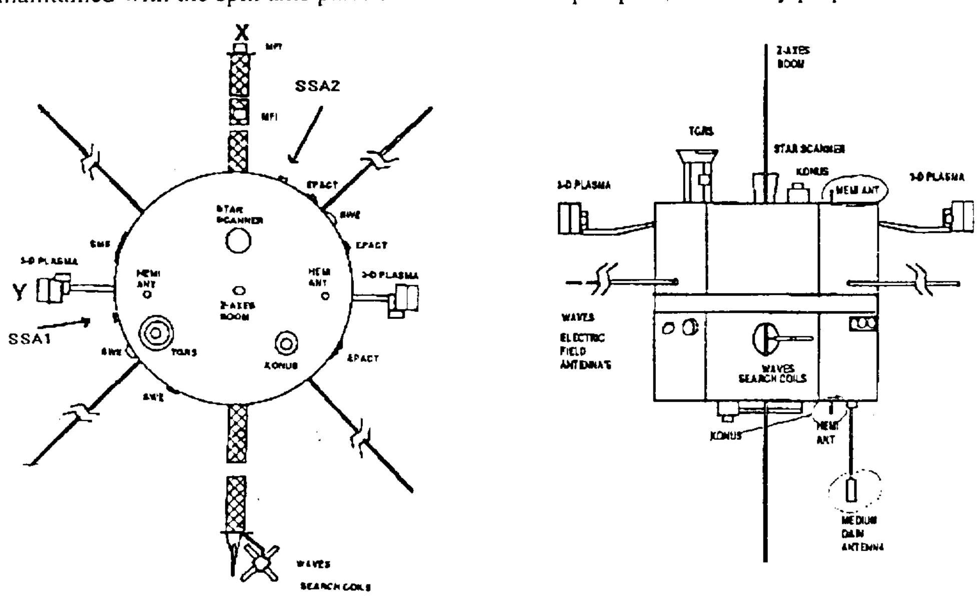

Figure 4. WIND Configuration.

The nutation estimation results for the first (of three) thruster flush burns will be presented. During the maneuver, the WIND moments of inertia were 741, 678 and 912 Kg-m^2. The MGA body coordinates are (0.14,-1.15,-2.68) meters. For WIND's attitude, according to the analysis above, the Doppler signal will contain a component at a frequency approximately equal to 1.3 times the spin rate, with an amplitude proportional to the minimum nutation angle (the nutation angle oscillates between minimum and maximum values since WIND is not axially symmetric). For the assumed WIND configuration, the minimum nutation angle in degrees can be estimated by finding the amplitude of this component in meters/sec, and dividing it by (0.007SR), where SR is the spin rate in RPM.

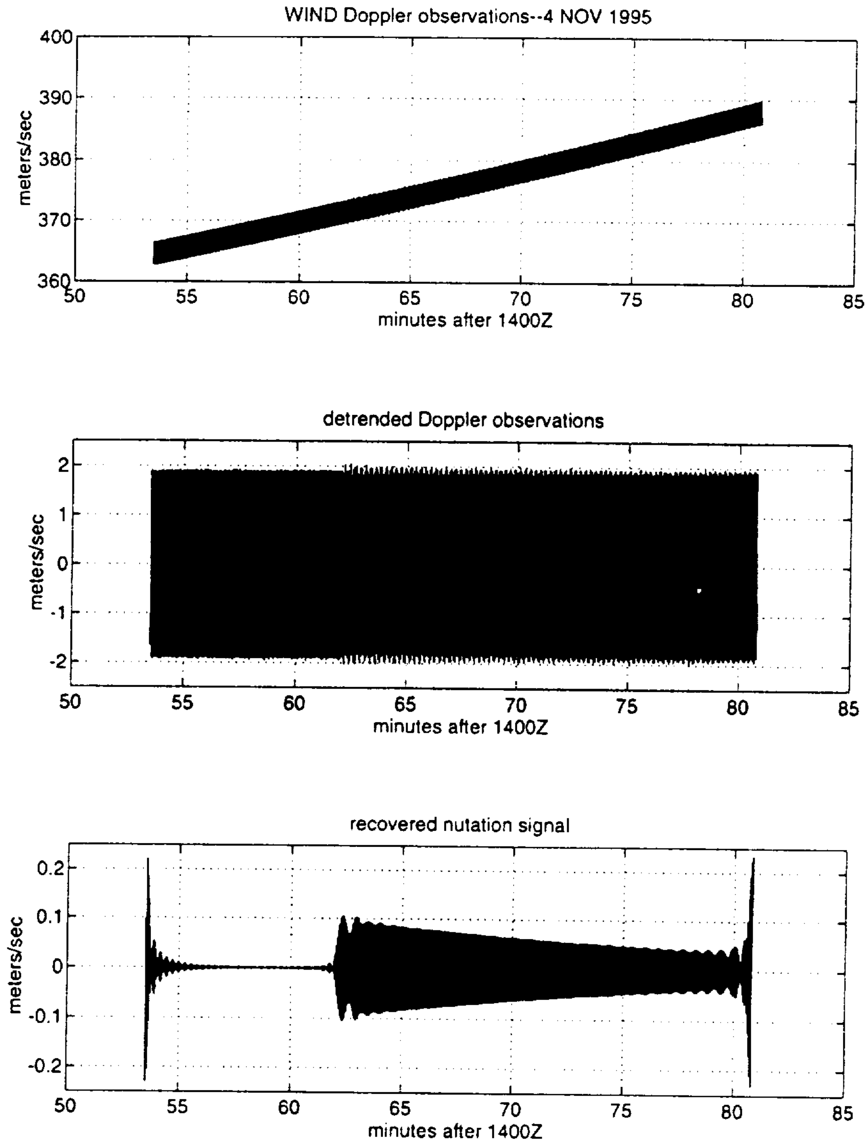

Figure 5. WIND Doppler Signal Processing

The topmost plot in figure 5 shows WIND range-rate for the period of the thruster flush maneuver. The middle plot shows the detrended data, used as the input to a Discrete Fourier Transform (DFT) algorithm. The lowest plot is the recovered nutation signal obtained by performing the inverse DFT on only those frequency components near the expected location of the nutation signal. Since some frequency components of the desired signal are lost, and some of other signals are included, there are large oscillations at the beginning and end of the recovered signal; these are known as the Gibb's Phenomenon (ref. 5, pp. 73-75). The nutation signal was "envelope detected" and scaled by 1/(.007SR) to obtain the minimum nutation angle (θ_0) as a function of time. The comparison is with Sun Sensor Assembly #2 on WIND (listed as SSA2 in figure 4.). SSA2 is 30 degrees away from the X-axis, so the largest nutation angle it sees is between θ_0 and θ_{MAX}. The Doppler nutation signal was further scaled by ,

$$\sqrt{1+\frac{m_0}{2}-\frac{m_0}{2}\cos\left[2\left(\frac{30\pi}{180}\right)\right]};m_0\approx 0.49$$

to put the SSA2 and Doppler signals on the same reference. Figure 6 shows the comparison of the detrended SSA2 angles and the scaled Doppler nutation signal amplitude.

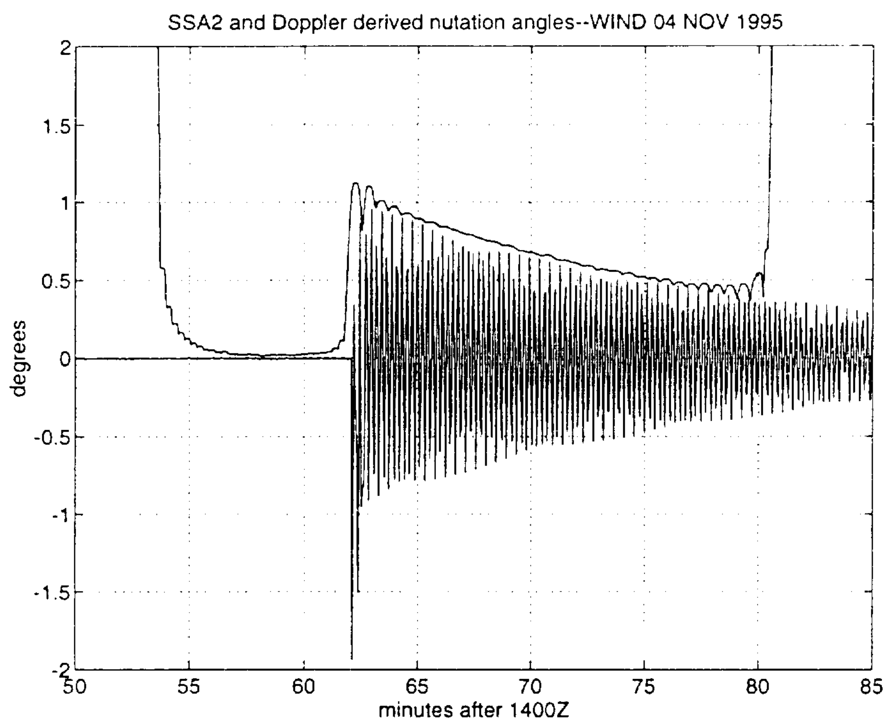

Figure 6. Comparison of Sun Sensor 2 and Doppler Nutation.

Conclusions. For spinning spacecraft, processing of Doppler tracking data for attitude parameters can be a useful adjunct to on-board attitude sensors. For "smaller, cheaper" missions it could possibly replace on-board sensors. In order to be useful for real time attitude estimation, further algorithm development should be done; for example, a digital phase-locked loop (DPLL) implementation for tracking spin rate and demodulating the nutation signal.

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