JOSEPHSON FREQUENCY METER FOR MILLIMETER AND SUBMILLIMETER WAVELENGTHS.

S.E.Anischenko, S.Y.Larkin, V.I.Chaikovsky, P.V.Khabayev, V.V.Kamyshin
State Research Center "Fonon", 37, Pobedy Ave., KPI, Kiev, 252056 Ukraine.

1. INTRODUCTION

Frequency measurements of electromagnetic oscillations of millimeter and submillimeter wavebands with frequency growth due to a number of reasons become more and more difficult. First, these frequencies are considered to be cutoff for semiconductor converting devices and one has to use optical measurement methods instead of traditional ones with frequency transfer. Second, resonance measurement methods are characterized by using relatively narrow bands and optical ones are limited in frequency and time resolution due to the limited range and velocity of movement of their mechanical elements as well as the efficiency of these optical techniques decreases with the increase of wavelength due to diffraction losses. That requires the apriori information on the radiation frequency band of the source involved. Method of measuring frequency of harmonic microwave signals in millimeter and submillimeter wavebands based on the ac Josephson effect in superconducting contacts is devoid of all the above drawbacks. This approach offers a number of major advantages over the more traditional measurement methods, that is the one based on frequency conversion, resonance and interferometric techniques. It can be characterized by high potential accuracy, wide range of frequencies measured, prompt measurement and the opportunity to obtain panoramic display of the results as well as full automation of the measuring process.

2. THEORY

All known frequency measurements methods using the ac Josephson effect are mainly based on the major relationship binding the frequency of the external electromagnetic radiation Ω with the voltage V_Ω across the Josephson junction:

$$\Omega = 2e\, V_\Omega / \hbar \qquad (1)$$

This specific feature has the form of a Shapiro step on the voltage-current curve (V-I curve), i.e. the voltage across the junction is remaining constant with the biasing current that is varying in the sync zone. From this point on let us call this part of the V-I curve a "Special feature existence" zone. Thus, functionally, the frequency measurement procedure of the monochromatic signal that is disturbing the Josephson junction reduces to the coordinate estimation of a "special feature existence" zone on the V-I curve.

In practice the pattern of measuring the frequency of monoharmonic signals based on the ac Josephson effect described above can be carried out using different techniques. Let the external disturbance be described by the expression for the current amplitude:

$$I = A \cos \Omega t \qquad (2)$$

Let us introduce for convenience the current peak value of the disturbing external source normalized in current:

$$A_\Omega = A / 2I \qquad (3)$$

Then, the V-I curve of the Josephson junction being irradiated by the described signal can be presented as follows [1]:

$$V = V_\Omega + \begin{cases} 0, & \text{for } \left|I - I_\Omega\right| < A_\Omega \\ \operatorname{sign}(I - I_\Omega) R_d \left[(I - I_\Omega)^2 - A_\Omega^2\right]^{1/2}, & \text{for } \left|I - I_\Omega\right| \geq A_\Omega \end{cases} \qquad (4)$$

where V_Ω is a bias voltage across the junction which produces the coincidence of the Josephson oscillation frequency ω_J with the frequency of the external signal Ω, I_Ω is a hypothetical current of the junction corresponding to the voltage V_Ω without incident radiation; R_d is a dynamic resistance. At this, beyond the "special feature existence" zone the V-I curve of irradiated junction is monotonically approaching the curve of the non-irradiated one which is splitting the discontinuous section into two equal parts at the point with the coordinates (I_Ω, V_Ω) . Let us call this point a "central point" of the "special feature existence" zone on the V-I curve. It should be pointed out that the expression (4) does not take into account the effect of the junction fluctuations as the peak value of the induced current is assumed to be far in excess of the current amplitude of the natural fluctuations. This inclusion of fluctuations when analyzing the V-I curve leads to the quadratic smoothing of the hyperbolic sections.

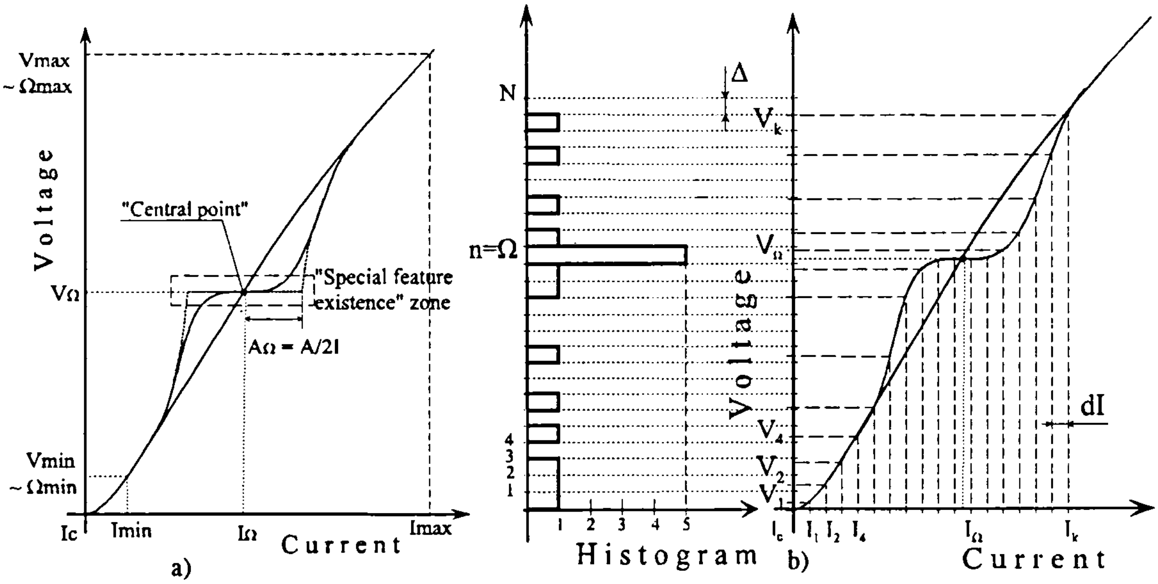


Fig.1. a) Schematical illustration of both V-I curves with and without incident radiation in the coordinate system displaced to the point $(I_c,0)$;

b) Procedure of forming the histogram P_n.

Fig. 1a schematically displays the diagrams of both V-I curves with and without incident radiation in the coordinate system displaced to the point (I_c, 0). In this figure the V-I curve of the non-irradiated junction is linearly approximated. Analyzing a shape of the V-I curve displayed in Fig.1 and described by expression (4) the frequency measuring procedure can be roughly presented as the procedure of identifying the "special feature existence" zone with the subsequent determination of the central point coordinates. The advantages of the frequency measurement technique, based on the ac Josephson effect, are the best seen during digital registration and functional processing of the disturbed V-I curve with a certain rational algorithm. For this purpose it is necessary to carry out the quantization of the biasing current and to represent the V-I curve as the latticed function that is in shape of the finite sequence of discrete counts in the prescribed system of nodes and with the prescribed quantization increment of their instantaneous values. The counts for the V-I curve are represented on the set of nodes (see Fig.1b): $v = f(i) \rightarrow \{v_k\}$, $k \in K$.

In our previous paper [2] we offered the frequency measurement algorithm based on the analysis of the first differences of the V-I curve of irradiated Josephson junction. The present paper suggests for consideration the algorithm forming the histogram of the count's sequence of this curve. In this case the rationality criterion is marked by several features:

- information productivity or possibility of insuring measurement accuracy approaching limiting values;

- functional simplic.ty allowing one-increment processing procedure;
- minimization of thւ computation time and expenses;
- possibility of panoramic displaying the results in the real time scale.

Finalizing the theoretical analysis it should be pointed out the important restriction for the amplitude of the external effect that defines the application scope of the above approaches. The signal measured is considered to be a low-power signal on carrying out the following inequality:

$$A << I_c*\Omega^2/\Omega^2_c \tag{5}$$

where I_c is a critical current of the junction; Ω_c is a characteristic frequency. This restriction breaks down when the frequency band is higher than Ω_c but the junction sensitivity sharply falls off.

3. TECHNIQUE

To carry out digital processing technique for the V-I curve it is necessary to have the discrete image of it by setting the discrete current counts, dI (see Fig.1b). The value of the elementary current count influence upon the performance of the algorithm and evidently should be chosen as minimum as it could be. Evidently to identify a Shapiro step it is necessary that the minimum size of discrete count dI_{min} be related to minimum-discernible disturbance of $A_{\Omega min}$ as: $A_{\Omega min} = 3dI_{min}$. Furthermore, the amplitude of the external effect should at least three times exceeded the intrinsic noise of the Josephson junction (when no external noise exist). It follows that:

$$dI_{min} \approx 10\, I_f \approx 10\sqrt{\frac{4k_B T}{R}\frac{R^2_{d\,max}}{R^2}} \tag{6}$$

where I_f is a fluctuating current of the junction; R is a normal resistance; R_{dmax} is a maximum dynamic resistance of the non-radiated junction for the frequency band being analyzed.

For processing the discrete image of the V-I curve of the irradiated Josephson junction and terminating the coordinates of the central point of the "special feature" we use the algorithm that forms the histogram of the V-I curve. To identify the "special feature existence" zone on the V-I curve the algorithm mentioned provides the criterion of equation of all count ordinates of the V-I curve which are attributed to the Shapiro step. The V-I curve of non-irradiated junction is a monotonically increasing function. Its discrete representation is equivalent to carrying out the inequality $V_{k+1} > V_k$ on the hole set of nodes $\{V_k\}$. There exists the final window Δ (see Fig.1b) of the uniform splitting of the interval for the possible values of the V-I curve. As this takes place, not more than one count of the V-I curve should correspond to each splitting window Δ_n (or the window limited by levels of $n*\Delta$ и $(n+1)*\Delta$ out of $\{V_k\}$). At this, some number P, that is equal to the quantity of counts thrown into the specified splitting interval is taken the value to: 1 or 0. It is sufficient for the minimum value out of the set of first differences $d_k^{(A)}$ for the V-I curve of the non-irradiated junction:

$$d_k^{(A)} = V_{k+1}^{(A)} - V_k^{(A)}\ ,\ \inf\{d_k^{(A)}\} > \Delta \tag{7}$$

The V-I curve of the irradiated junction is characterized by inherence of the "special feature existence" zone including more than 2 counts that are equal to each other:

$$V_k = V_{k+1}\ ,\ \text{for } K_{min} \le K \le K_{max} \tag{8}$$

where the limits of zone are determined by the following relationships:

$$K_{min} = \text{fix}\,(K_\Omega - 0.5P)$$
$$K_{max} = \text{fix}\,(K_\Omega + 0.5P) \tag{9}$$

where K_Ω is the abscissa dimensionless index of the "special feature" central point. Then more than 2 counts ($P > 2$) of the V-I curve get into the window Δ_n with index $n = \Omega$ covering the "special feature existence" zone. Having completed the registration procedure for the quantity of counts P_n captured by each window out of the continuous set, a certain latticed function $\{P_n\}$ is formed (see Fig.1b). The hole interval of possible values of ordinates for the V-I curve is spanned in the process. This function or histogram takes the value of 0 or 1 in all the nodes except $n = \Omega$ node. The window of it spans the "special feature existence" zone. There are 2 counts or more ($P > 2$) within this node and in this manner it is easy to identify the "special feature existence" zone on this indicator. The dimensionless integer index $n = \Omega$ (i.e. the dimensionless abscissa of the histogram overshoot) is easily converted into ordinate V_Ω of the "special feature" in accordance with the following rule:

$$n*\Delta \leq V_\Omega \leq (n+1)*\Delta, \qquad (10)$$

where $\Delta \sim dI_{min}$ is a window (i.e. the established size of the interval or the splitting increment for the position of existence of the V-I curve registered counts $\{V_k\}$. Value of voltage V_Ω determined in a such way can be easily recalculated into the desired frequency Ω of the incident radiation according to Eq. (1).

Besides measurement of the frequency Ω, the result of histogram construction enables comparing value P_n with peak $2A_\Omega$ of the "special feature existence" zone in terms of the current interval dI. By this means value P_n at $n = N$ is a measure of intensity of the incident power for amplitudes A_Ω limited by inequality (5)

$$2A_\Omega = P_n*dI \qquad (11)$$

There exists a limit for the reasonable size reduction of window Δ for the histogram P_n. It approximately corresponds to the equation of the window size Δ and the interval dV of the amplitude quantization of the registered counts. The choice of size Δ less than interval dV does not lead to the reduction in error of the measurement of the ordinate of the "special feature" central point V_Ω. The reason is that the registration error of the estimated value V_Ω as a result of the finite length of its digital representation makes itself evident and comes on to dominate. As the interval dV is proportional to the discrete count dI the window size Δ is bound with frequency of the current quantization. Finally, the window size Δ can be presented as follows:

$$\inf\{d_k^{(A)}\} \leq \Delta \leq dV \qquad (12)$$

So, the algorithm of measuring frequency of incident radiation with the use of histogram analysis incorporates a sequence of the following steps:

1. Registration of sequence of V-I counts of the Josephson junction without incident radiation $\{V_k^{(A)}\} \leftrightarrow \{I_k^{(A)}\}$. At this, frequency of current quantization dI is being determined in accordance with Eq. (6). The number of registered counts K of discrete V-I curve is defined by a current peak and quantization interval for overlapping the working interval of voltages $V_{min}...V_{max}$: $K = (I_{max} - I_{min}) / dI$.
2. Obtaining a file of the first differences $\{d_k^{(A)}\}$ according to Eq. (7)
3. Determination of the minimum value out of the file $\inf\{d_k^{(A)}\}$ as well as the window size of histogram Δ according to Eq. (12).
4. Registration of the counts sequence of V-I curve in case of incident radiation.
5. Selection of the V-I curve section bearing information by means of recalculation of the boundary conditions of the measured frequency band Ω_{min} ... Ω_{max} using Eq. (1) into lower and upper boundaries of the interval $\inf\{V_k\}$ and $\sup\{V_k\}$. Determination of the number of intervals (i.e. windows) N of the histogram formed:

$$N = (\mathrm{fix}[\sup\{V_k\} - \inf\{V_k\}]) / \Delta \qquad (13)$$

6. Formation of the histogram $\{P_n\}$ for the selected counts of the V-I curve.
7. Conversion of node arguments of the histogram into the frequency Ω_n.
8. Registration of arguments of the histogram overshoots as the frequency estimations of the harmonics making up an incident radiation.

4. EXPERIMENTAL RESULTS

In the course of the experiment the task was directed to the evaluation of the simulation model of the method and its comparison with the real results obtained with the experimental set-up. Fig.2 presents a block diagram of the experimental set-up described in [3]. As the Josephson junction the low temperature niobium edge contact with the following parameters was utilized: the normal resistance is about 1 Ohm; the critical current is 0.4 ma; the junction area is 0.5 μm^2.

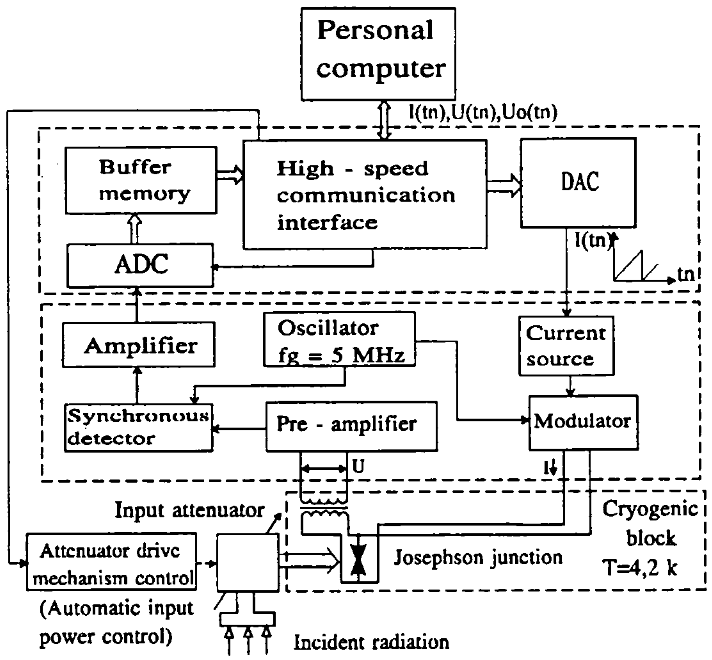


Fig.2. Block diagram of the experimental set-up.

Fig.3 shows the V-I curve of the Josephson junction simulated on a computer. The radiation frequency was chosen to be 60 GHz and the fragment of the junction V-I curve corresponds to the classic theoretical behavior in the neighborhood of the "special feature existence" zone. The software allows to get the idealized histogram that is consistent with such representation. Fig.4 also gives the simulation results without taking into account the fluctuations influence. They refer to the minimum amplitude signal allowed. The response size perfectly corresponds to three points along the sweep tone of the V-I curve. Fig.5 illustrates the behaviour of the model under the condition of strong external induction. The histogram can confirm that the method is stable to external parasitic sources. Fig.6 displays the fragment of the real V-I curve of the Josephson junction irradiated by 140 GHz oscillator. The line width of the intrinsic Josephson oscillation constitutes 190 MHz. The line width of the oscillation source was certified with a filter and runs to 1,3 GHz that corresponds to the histogram obtained on an experimental set-up. Thus, the experimental set-up and software developed allow to get 300 MHz frequency resolution, at this, the line width of the intrinsic oscillation being 200 MHz. Fig.7 presents the results of a more complex experiment. The Josephson junction was exposed to the influence of three oscillators with different frequencies: f_1 = 135 GHz with generation line width of

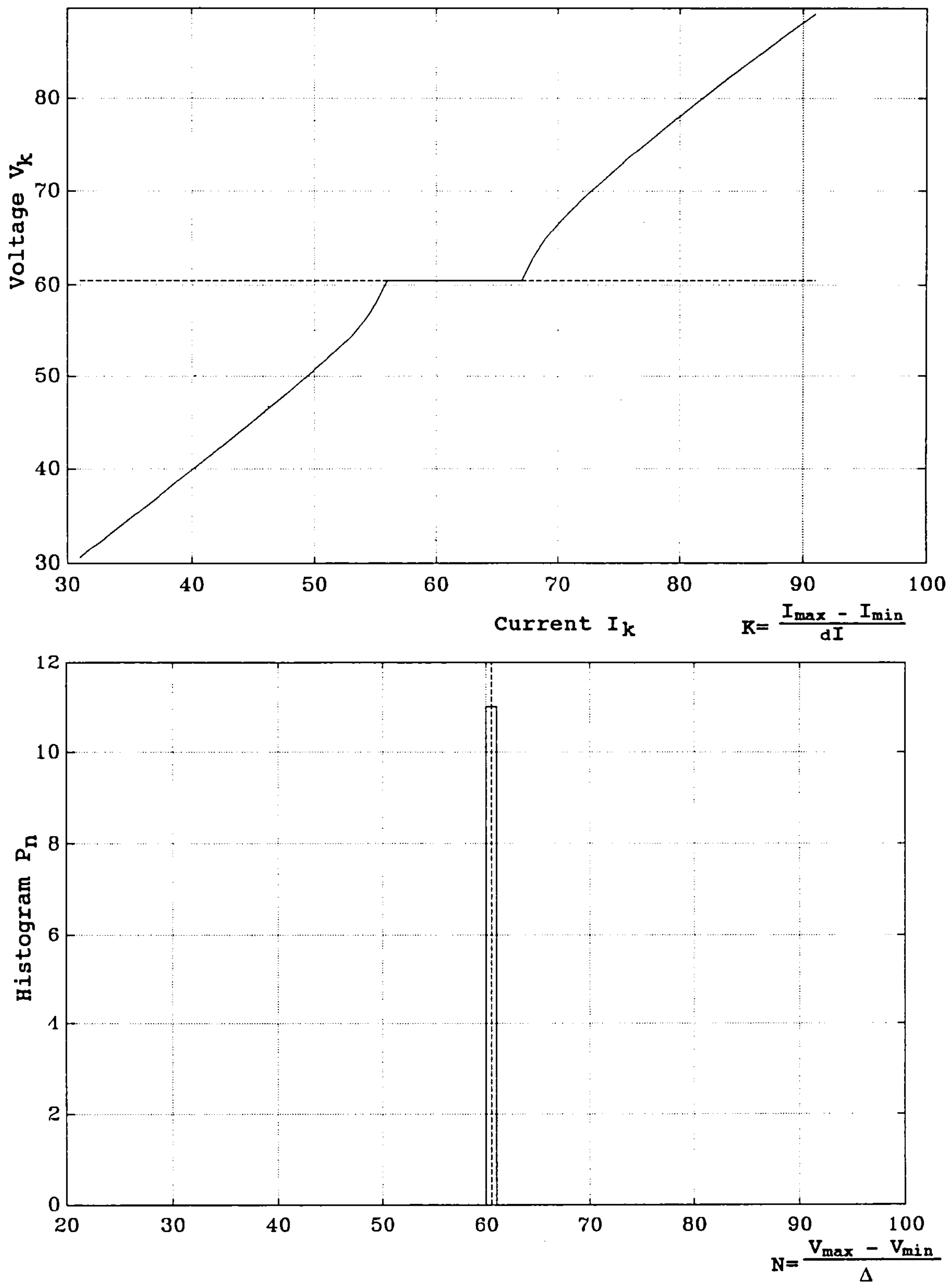
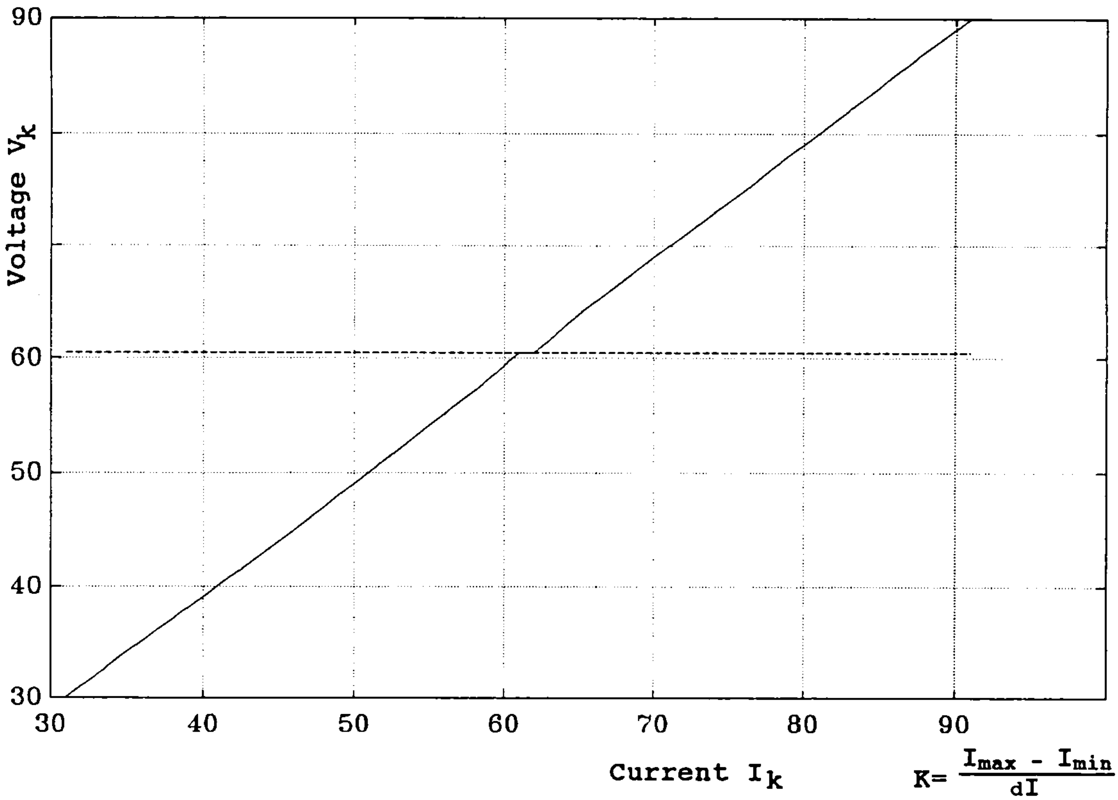


Fig.3 The fragment of the V-I curve of the Josephson junction simulated on a computer and the results of the experiments in the form of a histogram for the radiation frequency chosen to be 60 GHz.



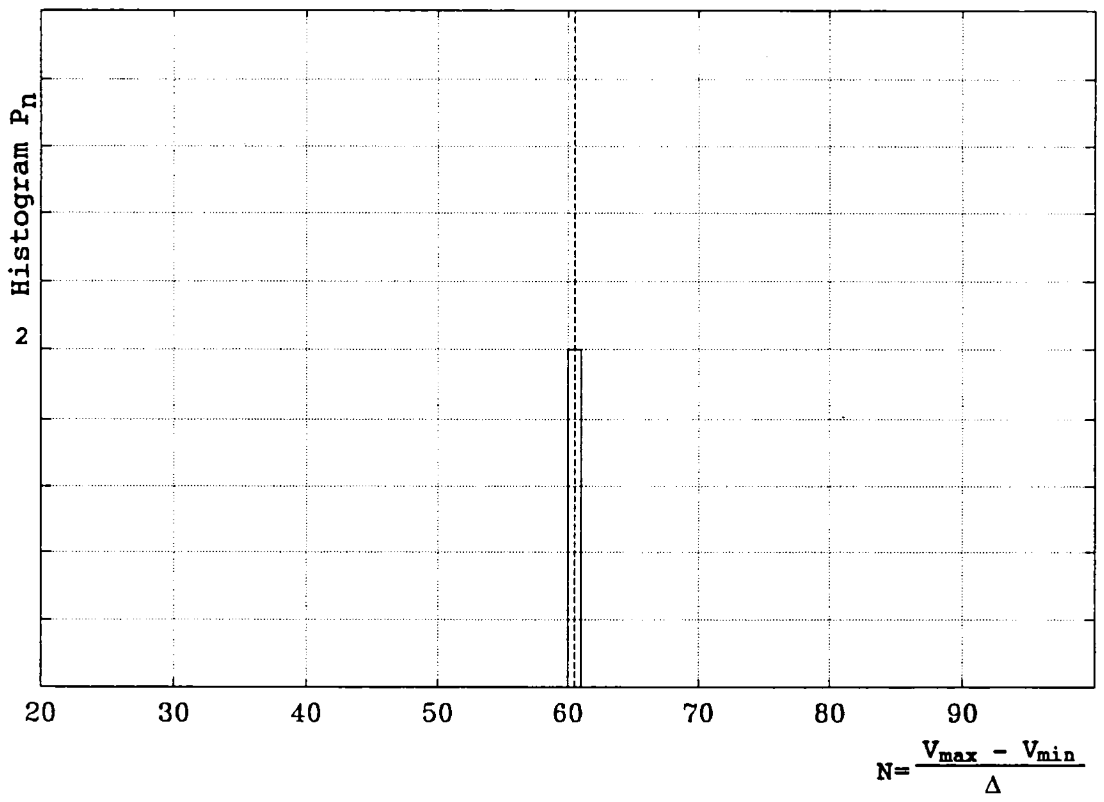


Fig.4. The results simulated on a computer for the experiments of obtaining a histogram for the signal of 60 GHz with minimum amplitude allowed and without fluctuations influence.

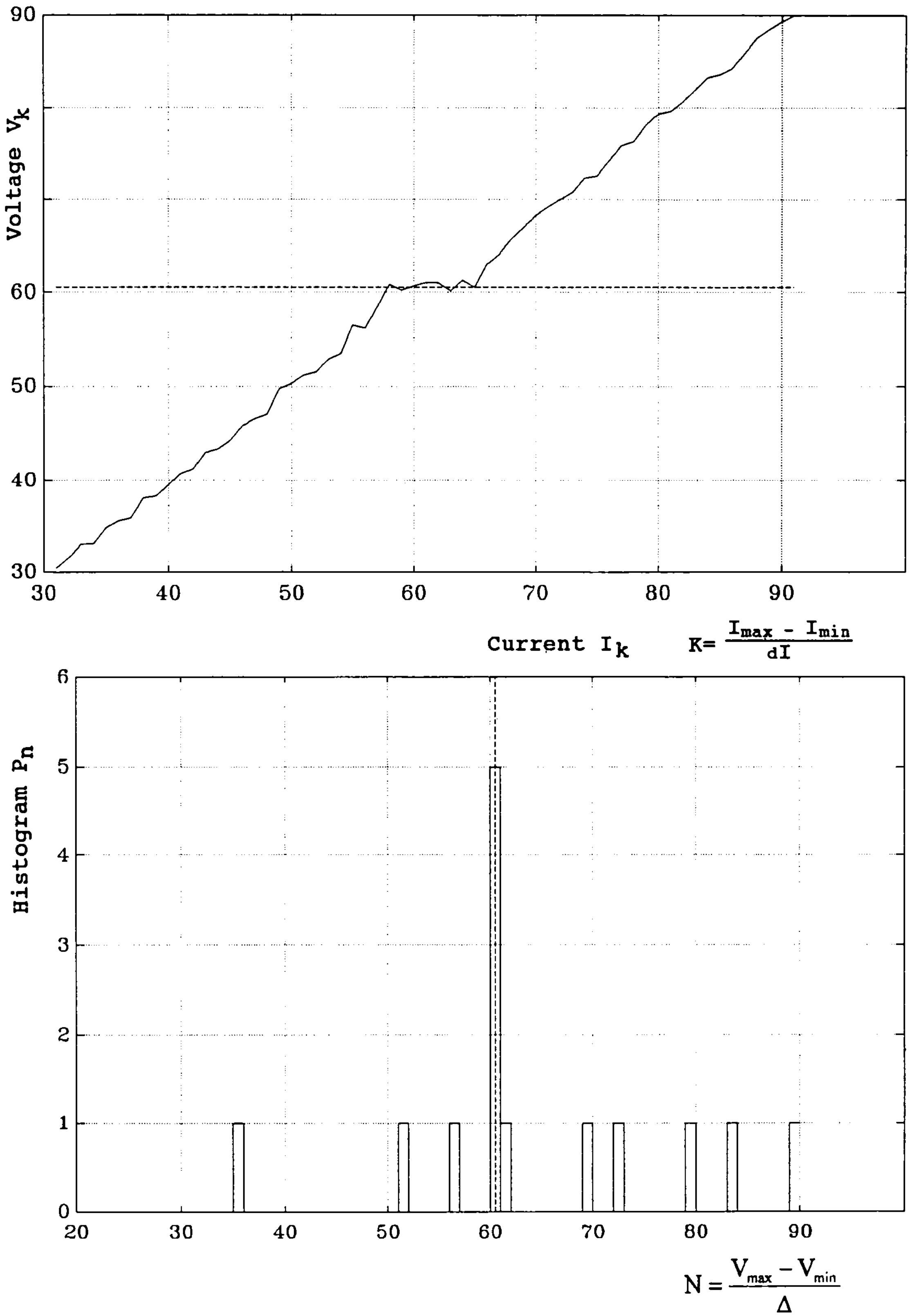
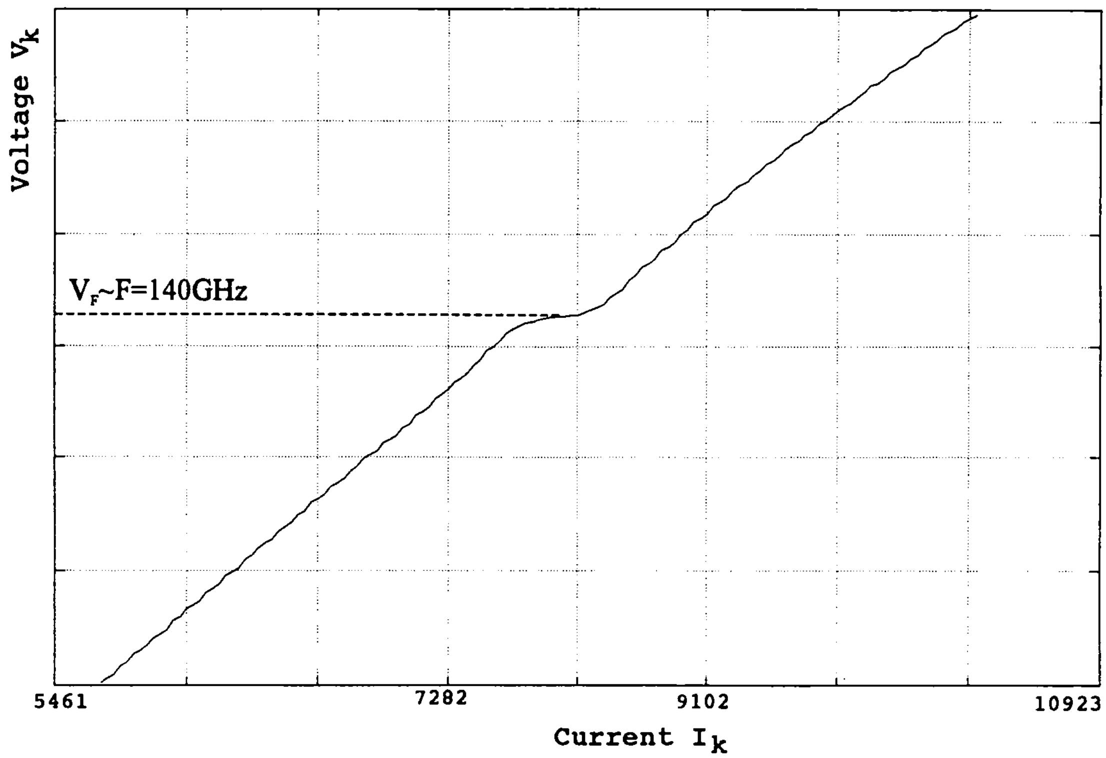


Fig.5. The behaviour of the computer model for the signal of 60 GHz under the condition of strong external induction. The histogram confirms that the method is stable to external parasitic sources.



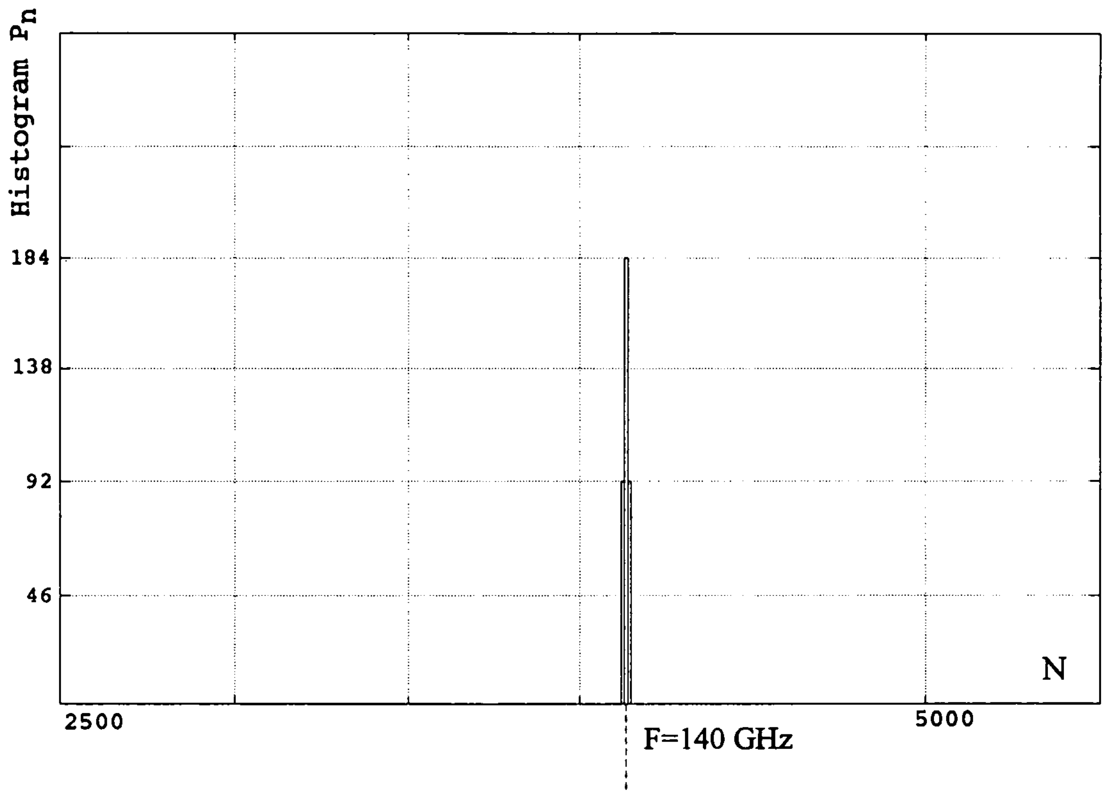


Fig.6. A fragment of the real V-I curve of the Josephson junction irradiated by 140 GHz oscillator and the corresponding histogram as a result of frequency measurement.

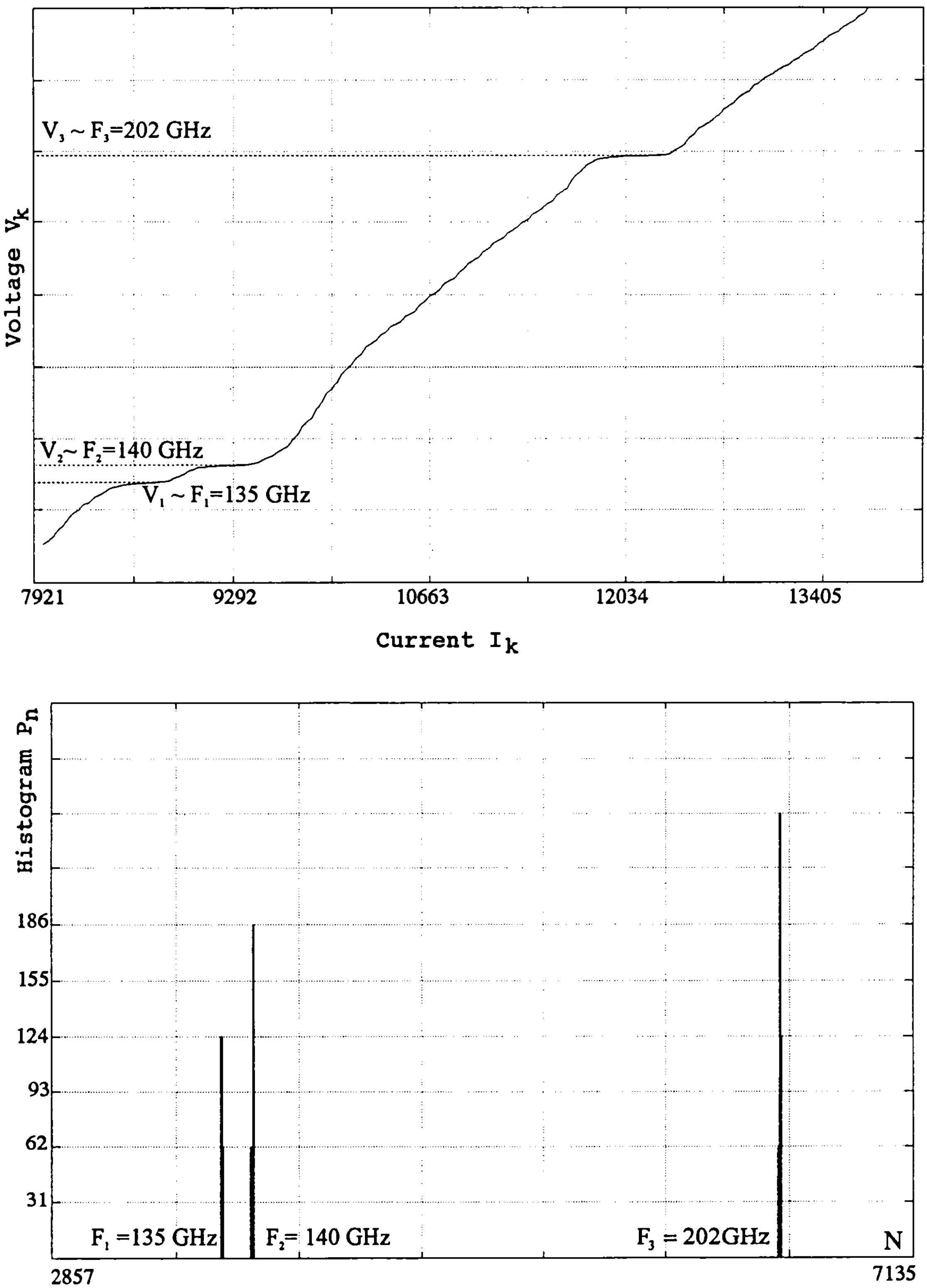


Fig.7. A fragment of the real V-I curve of the Josephson junction irradiated by the complex polyharmonic signal and the corresponding histogram as a result of frequency measurement.

Δf_1 = 500 MHz, f_2 = 140 GHz with Δf_2 = 1,3 GHz and f_3 =200 GHz with Δf_3 = 2 GHz. The histogram depicts the conditions of the experiment with high accuracy (not worse than 10^{-4} of relative units to the accuracy of the evaluation of the average source frequency). The amplitude of the lab sources was measured with the use of standard power meters and controlled by way of introducing attenuation into the microwave transmission line in accordance with block-diagram given in the technical specifications of the experimental set-up. The analysis of the determination of the microwave signal amplitude according to the histogram shows that in conformity with theoretical background and on carrying out the inequality (5) the response value of the Josephson junction is practically proportional to the square root of the external radiation power. At this, the less is the amplitude of the external source influence and its frequency related to the typical frequency of the Josephson junction, the closer this dependence is to the linear form. It takes 1 sec to make analysis and display the results. This time can be minimized at least one order higher without any loss for the accuracy of measurements with the use of high-speed DAC/ADC of the corresponding number of bits.

We would like to finalize that the proposed technique related to the panoramic analysis of the frequency composition of microwave signals and the equipment involved can be used:
- in radio engineering research of various microwave radiation sources;
- radioastronomy and navigation;
- for specific task on radio provision.

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