59-04

HOW BAD RECEIVER COORDINATES CAN AFFECT GPS TIMING

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Abstract

Many sources of error are possible when GPS is used for time comparisons. Some of these errors have been listed by Lewandowski[1]. Because of the complexity of the system, an error source could have more than one effect. This paper will present theoretical and observational results by offsetting a receiver's coordinates. The calculations show how an error as small as 3 meters in any direction can result in a timing error of more than 10 nanoseconds. The GPS receiver must be surveyed to better than 0.2-meter accuracy for the timing error to be subnanosecond.

INTRODUCTION

GPS is a receive-only system. The user's equipment does not transmit a signal other than the intermittent frequencies used internally to the receiver. The system relies on knowing the position of the transmitter (the GPS satellite), the time of signal transmission, and the position of the receiver so the receiver can determine its time and time offset from some reference (for time transfer operations). For mobile operations, the information from at least four satellites is needed so the receiver can find its position, time, and time offset. If the satellite is at its stated location and the corrections for propagation are correct, the source of error in time transfer mode of operation must be the receiver coordinates.

THEORETICAL CALCULATIONS

A person must first understand the different coordinate systems used and put all positions in a common system. The GPS antenna used was surveyed by The Defense Mapping Agency into the World Geodetic Survey 1984 (WGS-84) coordinates[2]. The WGS-84 is based on the Earth's center of mass. The Z-axis is in the direction of the Conventional Terrestrial Pole (CTP) for polar motion. The X-axis is the intersection of the WGS-84 reference meridian plane and the plane of the CTP's equator. The reference meridian is the zero meridian as defined by the BIH for epoch 1984.0 on the basis of the coordinates adopted for the BIH stations. The Y-axis completes a right-handed, Earth-fixed orthogonal coordinate system. Programs from the Defense Mapping Agency and Mihran Miranian (USNO) were used to convert the WGS-84 coordinates to Earth-Centered, Earth-Fixed (ECEF), which is used by the GPS system.

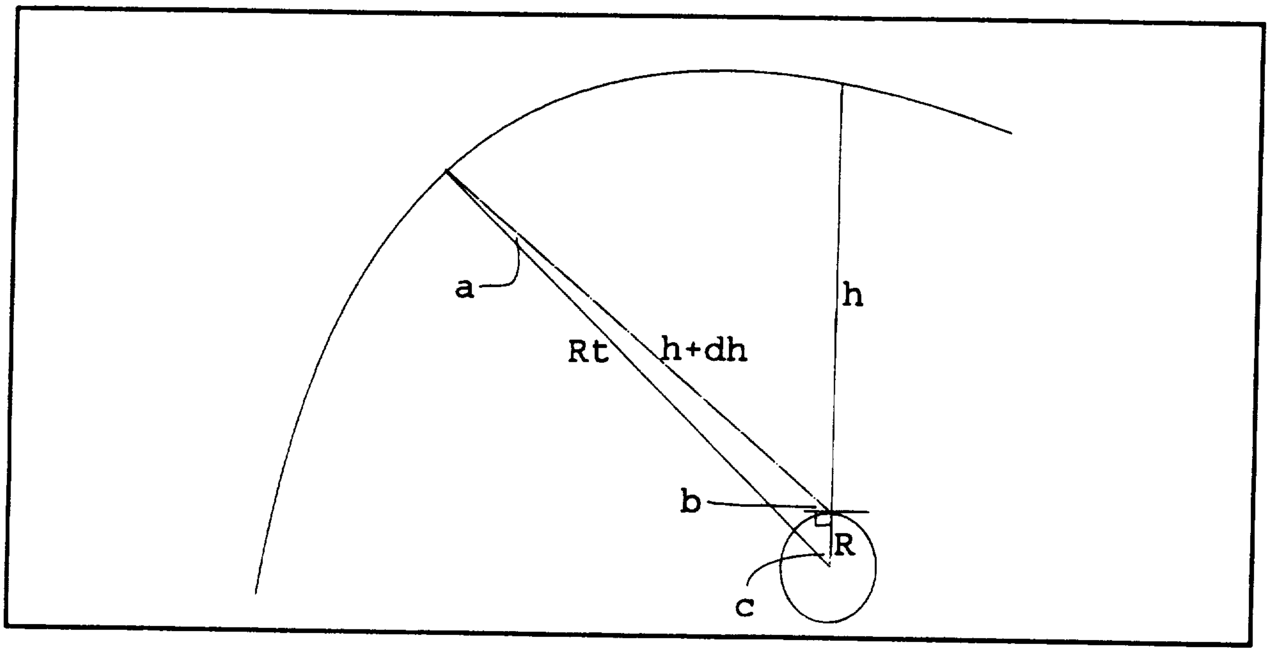
The coordinates for one GPS antenna at USNO are:

	WGS-84		ECEF		ECEF
N	38°55'13.397"	X	1112168.189m	R_e	6369795.132m
W	77°03'58.431"	Y	-4842863.286m	Θ	-77.0662308386°
A	55.5m	Z	3985479.536m	Φ	38.7324162285°
				Φ'	51.2675837715°

where R_e is the radius of the Earth (ECEF) at the receiver's location, and Φ' is measured from the Z-axis rather than from the X-Y plane.

A satellite directly at zenith is 26407545 meters from the receiver according to actual measured values. The height of the satellite above the receiver is 20037749.868 meters.

The next step is to understand how changing the position of the satellite will change the geometry of the satellite-receiver relationship and the path length.



Let:

R = radius of Earth
R_t = height of satellite above center of Earth (assumed constant)
h = height of satellite above receiver
h+dh = height of satellite above receiver plus additional distance due to change of satellite-receiver geometry
c = angle between zenith of receiver and location of the satellite
b = angle satellite is above the horizon

We have the following relations:

$$a = \arcsin\left(\frac{R \times \sin(b + 90)}{R_t}\right)$$

$$c = 180 - (b + 90) - a$$

$$\text{h+dh} = \frac{R_t \times \sin(c)}{\sin(b + 90)}$$

The angle b was varied from 90^{o} to 0^{o}. This resulted in c varying from 0^{o} to 76.375^{o} and dh varying from 0 meters to 5477587.0874 meters. These variations were then transformed to those seen by the individual receiver coordinates. These values were then converted to ECEF coordinates X, Y, Z.

This assumes that the receiver is at its proper coordinates. In order to understand of how dh changes as the satellite changes position when the receiver is NOT in its proper location, one must vary the surveyed latitude, longitude, and altitude (in WGS-84 coordinates) and determine the "new" coordinates in the ECEF coordinate system.

The altitude changes in direct proportion to the radius. However, latitude and longitude do not have such a simple transform. The latitude of the satellite is given by:

$$\text{SLAT} = \frac{S}{2\pi \times B}$$

where S is seconds per 360 degrees (1296000) and B is the Earth's polar radius (6356752.3142 meters). For the USNO receiver, SLAT = .032448″ per meter.

The longitude is given by:

$$\text{SLON} = \frac{S}{2\pi \times \cos(\text{Lat}) \times A}$$

where A is the Earth's equatorial radius (6378137.0 meters). At USNO's Latitude of $38^{o}55'13.397''$, SLON = .04156624″ per meter.

The receiver offsets, symmetric about zero, were 15m, 10m, 5m, 4m, 3m, 2m, 1m, .9m, .8m, .7m, .6m, .5m, .4m, .3m, .2m, .1m, and .05m. These offsets were transformed to altitude, latitude, and longitude offsets in the WGS-84 coordinate system. The new positions were then transformed into the ECEF coordinate system.

With the satellite and receiver in ECEF coordinates and knowing the non-offset h+dh values, a simple computer program can solve the time error equation. The time error equation is:

$$dt = \frac{\sqrt{(X_s - X_r)^2 + (Y_s - Y_r)^2 + (Z_s - Z_r)^2} - (\text{h+dh})}{c}$$

where s and r represent satellite and receiver respectively.

The results are plotted as time offset vs. offset vs. angle of satellite above the horizon in Figure 1. An error of as small as 3 meters offset in any of the three coordinates can result in a time error of more than 10 nanoseconds. For the time error to be subnanosecond, the GPS antenna must be surveyed to better than 0.2 meter accuracy.

Theoretical calculations for offsetting a receiver's coordinates, holding the other variables fixed, show some interesting results. First, a time error of 20 nanoseconds would require an antenna's coordinates to be off by more than five meters. Second, the errors are three-dimensionally symmetric.

OBSERVATIONAL RESULTS

The theoretical results are interesting, but mean nothing without some proof of observation. For this, two keyed dual-frequency receivers were used. First, both receivers were set in the time transfer mode of operation with their correct coordinates in their databases (Figures 2 and 3). After several days of observation, receiver 1 continued to operate with the correct coordinates, while receiver 2 had its coordinates offset changed daily. Receiver 2's offset were 15m, 10m, 5m, 1m, .5m, and back to 0m (for two days) to verify each offset run. The offsets were applied in altitude (Figures 4, 5, and 6), latitude (Figures 7 and 8), and longitude (Figures 9, 10, and 11). The closure checks of zero offset showed that no parameters changed during the observations. The bias of approximately 5.6 nanoseconds was between this pair of receivers. In a follow-on observational set between one of these receivers and another, the bias was 3.5 nanoseconds. All receivers were calibrated by the manufacturer.

CONCLUSION

The theoretical and observational results agree with common sense that an approximate three nanoseconds per meter error would be present because of receiver coordinates being offset. However, more important facts were found from the observational data. First, although the receivers used to collect the observational data met specification, there was an offset between them. In a follow-up observation series, using one of these two receivers and a third, this offset was found to still be present but of a different value. (The offset values differed by 2-3 nanoseconds.) Further investigation is needed to resolve these differences for higher precision time transfers. Second, although keyed dual-frequency receivers were used, evidently there are some differences between satellites. Averaging does decrease this effect. Higher accuracy time transfers will require more investigation of this effect. One needs to know if averaging is the right thing to do or if some problem must be fixed.

REFERENCES

[1] W. Lewandowski 1994, *"GPS common-view time transfer,"* Proceedings of the 25th Annual Precise Time and Time Interval (PTTI) Applications and Planning Meeting, 29 November-2 December 1993, Marina Del Rey, California, pp. 133-148.

[2] *"Department of Defense World Geodetic System 1984,"* 1992, Defense Mapping Agency Technical Report 8350.2.

Time Error vs. Receiver Offset

Figure 1

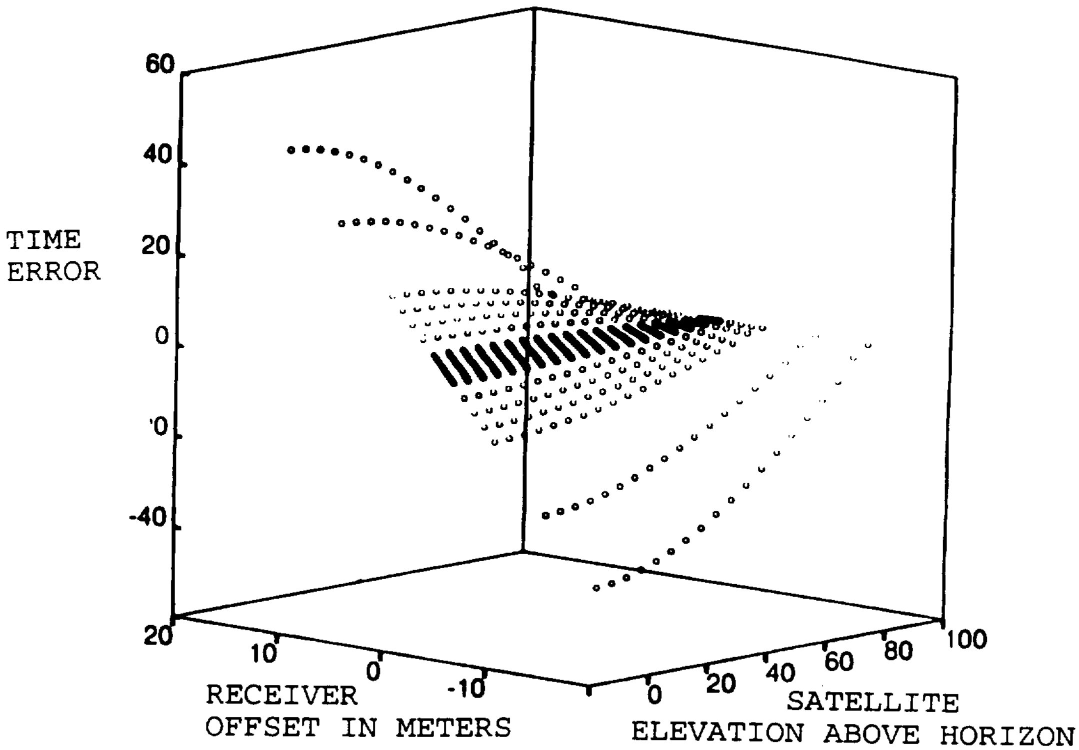


Figure 2 Theoretical minus Experimental Results each satellite observation averaged independently

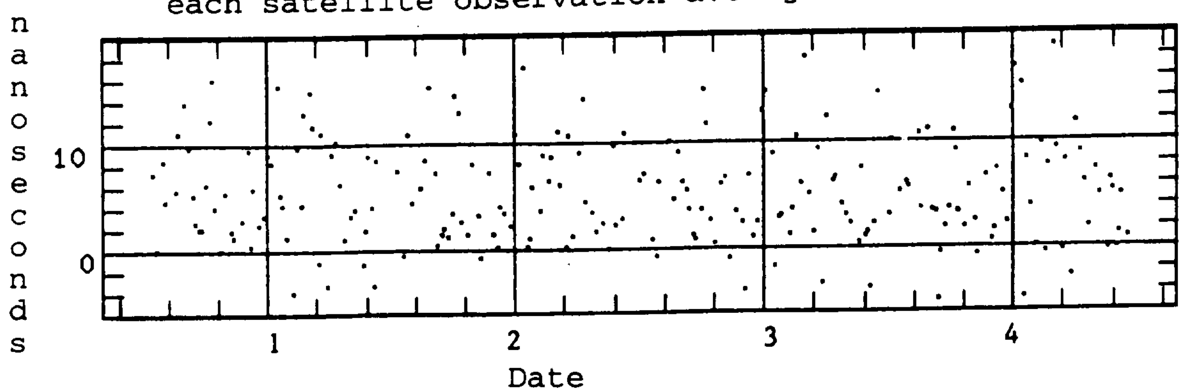


Figure 3

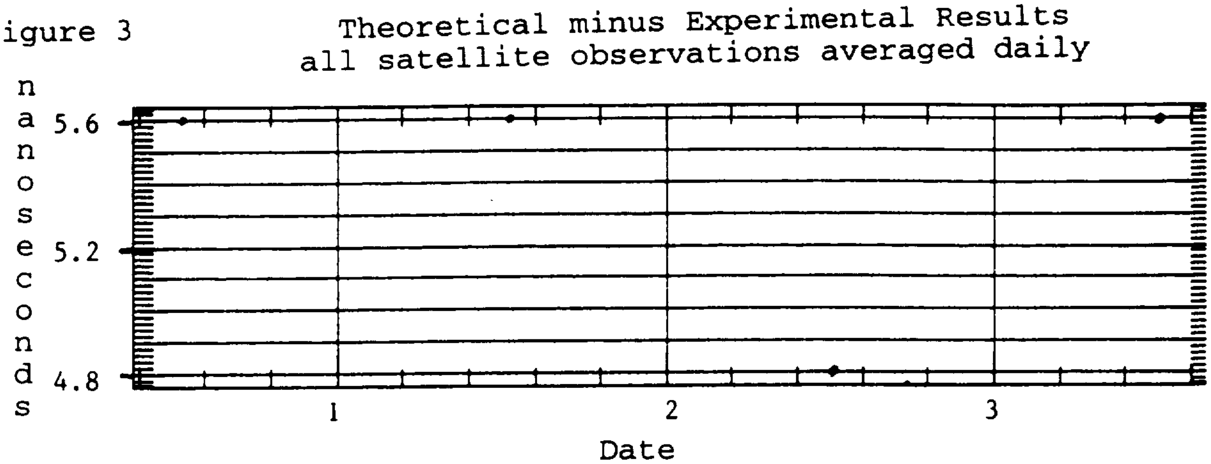


Figure 4

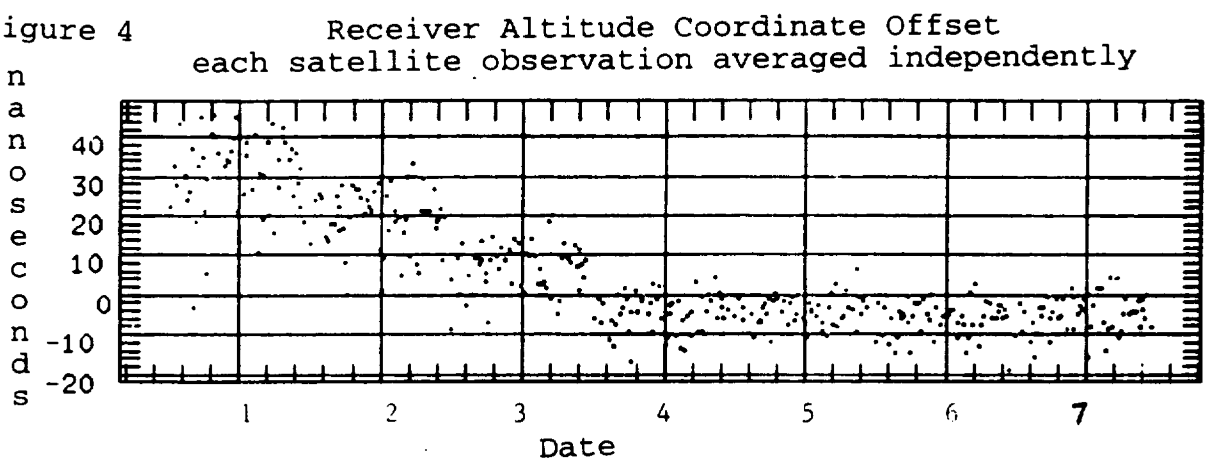


Figure 5

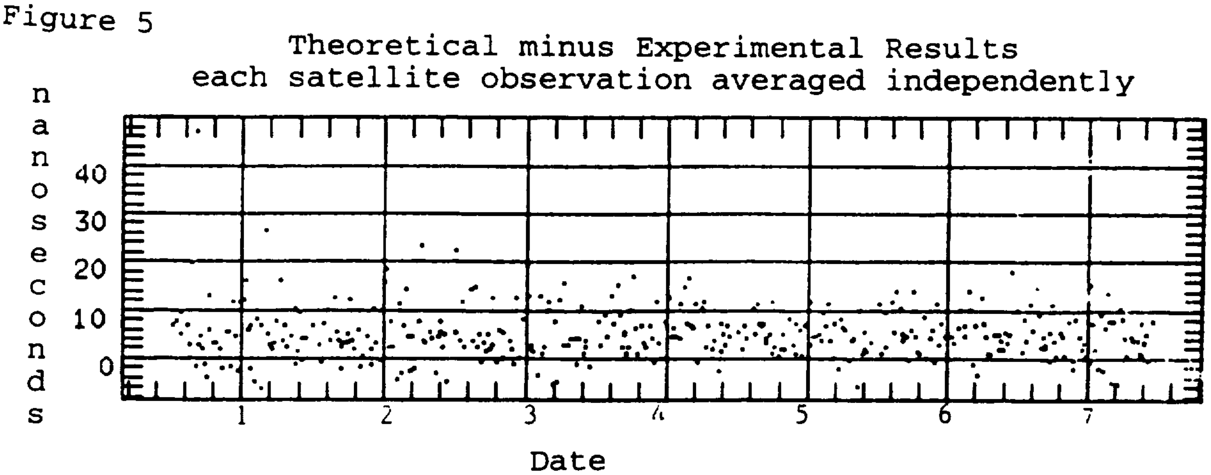


Figure 6 Theoretical minus Experimental Results all satellite observations averaged daily

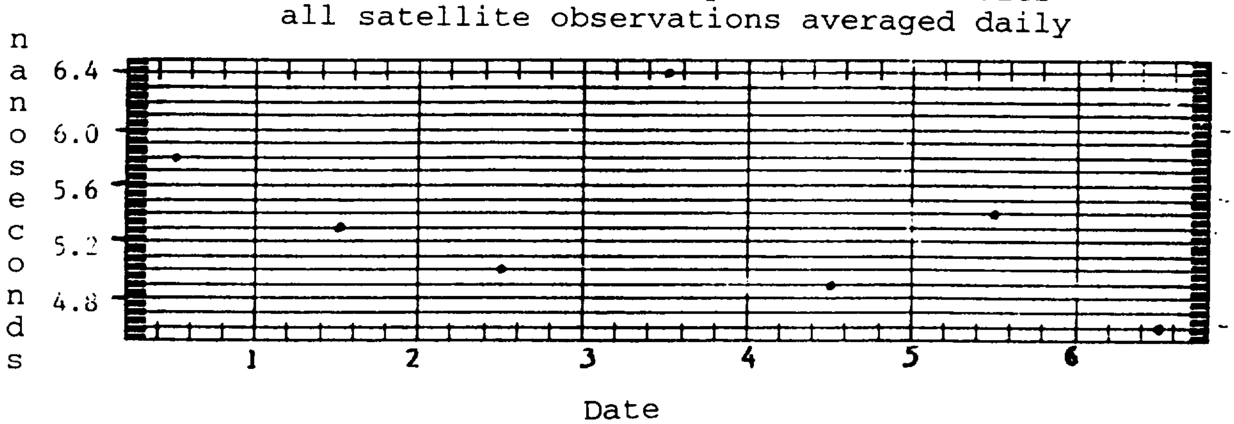


Figure 7 Theoretical minus Experimental Results each satellite observation averaged independently

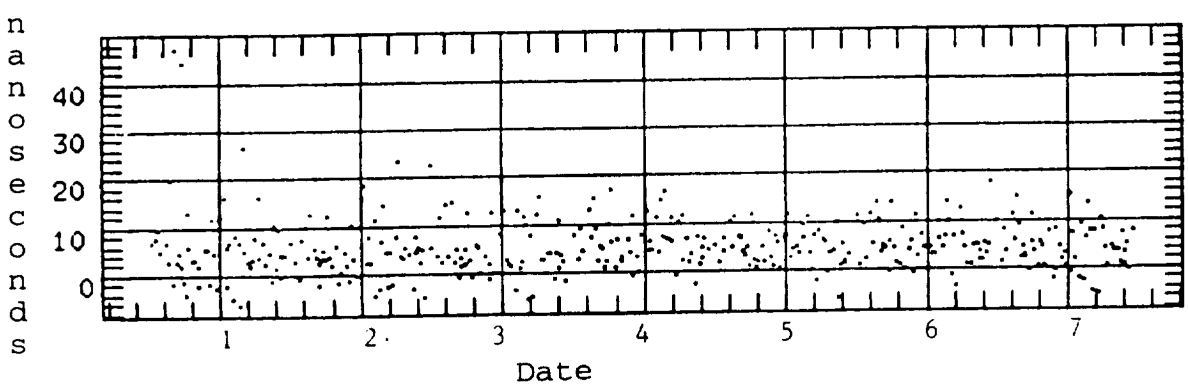


Figure 8 Theoretical minus Experimental Results all satellite observations averaged daily

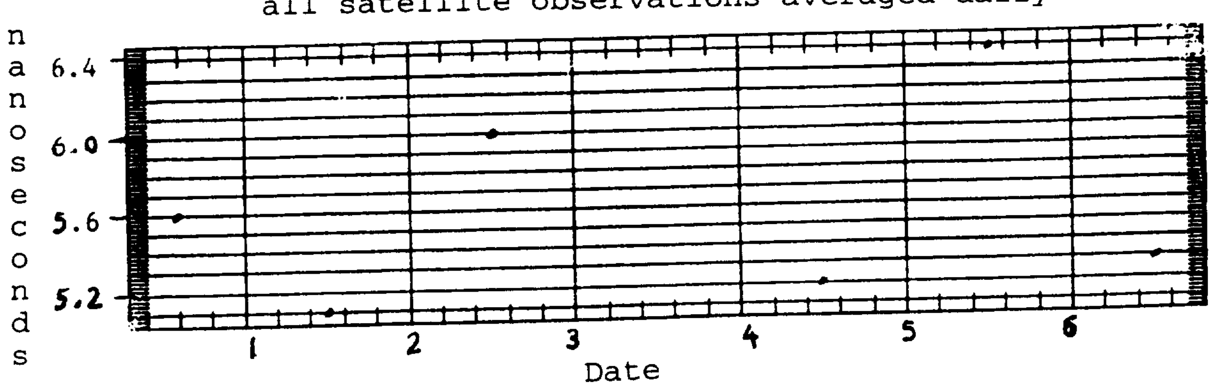


Figure 9

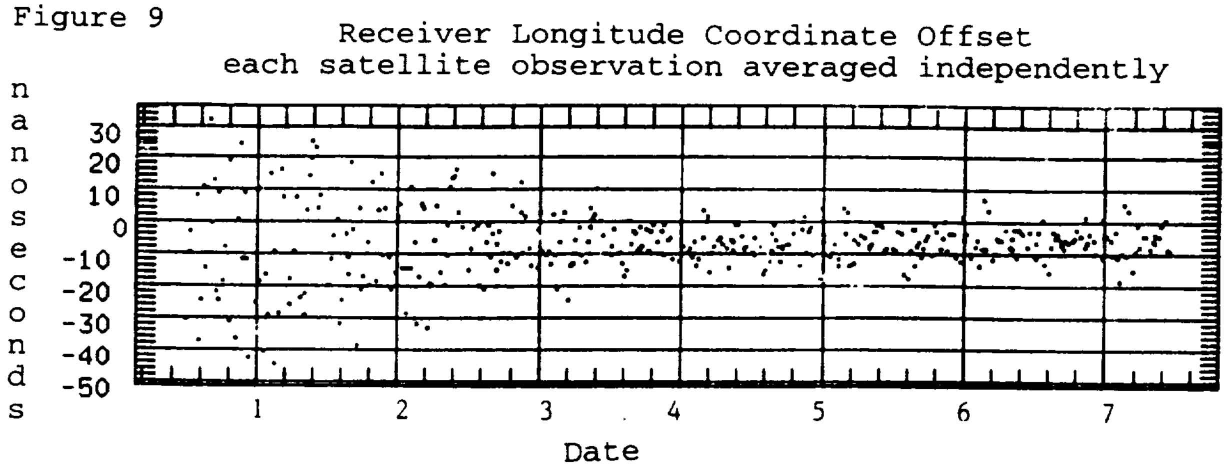


Figure 10

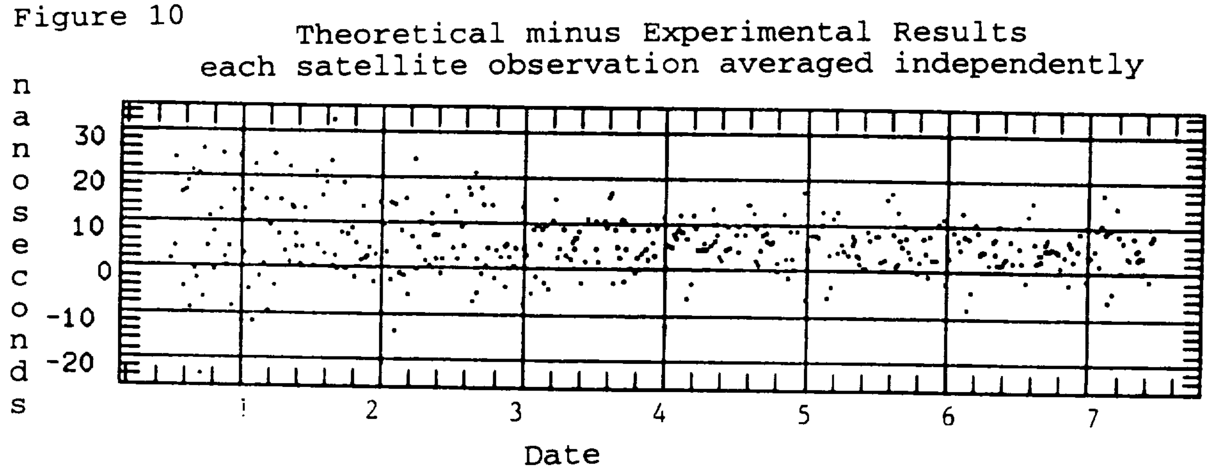
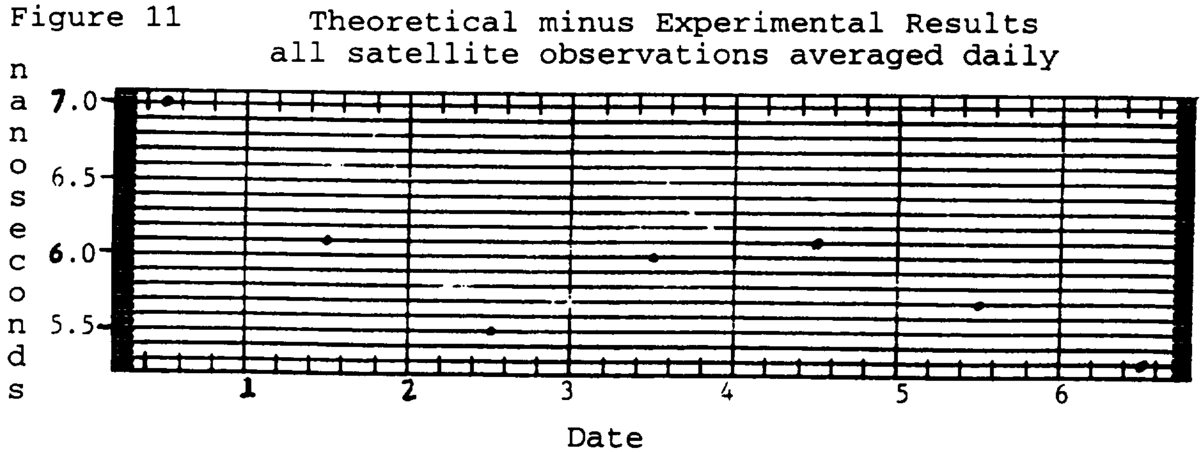


Figure 11



Questions and Answers

WLODZIMIERZ LEWANDOWSKI (BIPM): The receivers you had compared, they had exactly the same software or were they different?

HAROLD A. CHADSEY (USNO): These were two identical receivers running the same software and firmware internally.

WLODZIMIERZ LEWANDOWSKI (BIPM): The differences were not coming, for example, anomolies from the software?

HAROLD A CHADSEY (USNO): It definitely wasn't a problem of one was a TrueTime receiver and one was an S-TEL or something like that. There is a slight possibility that there may have been a small fractional difference in the software. But in talking with the manufacturer, they said that those two receivers had the same software and same firmware versions in them. And when they left the factory, they were calibrated.