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RELATING THE HADAMARD VARIANCE TO MCS KALMAN FILTER CLOCK ESTIMATION

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Abstract

The GPS Master Control Station (MCS) currently makes significant use of the Allan Variance. This two-sample variance equation has proven excellent as a handy, understandable tool, both for time domain analysis of GPS Cesium frequency standards, and for fine tuning the MCS's state estimation of these atomic clocks.

The Allan Variance does not explicitly converge for the noise types of $\alpha \leq -3$, *and can be greatly affected by frequency drift. Because GPS Rubidium frequency standards exhibit non-trivial aging and aging noise characteristics, the basic Allan Variance analysis must be augmented in order to a) compensate for a dynamic frequency drift, and b) characterize two additional noise types, specifically* $\alpha = -3$ *and* $\alpha = -4$. *As the GPS program progresses, we will utilize a larger percentage of Rubidium frequency standards than ever before. Hence, GPS Rubidium clock characterization will require more attention than ever before.*

The three-sample variance, commonly referred to as a renormalized Hadamard Variance, is unaffected by linear frequency drift, converges for $\alpha > -5$, *and thus has utility for modeling noise in GPS Rubidium frequency standards. This paper demonstrates the potential of Hadamard Variance analysis in GPS operations, and presents an equation that relates the Hadamard Variance to the MCS's Kalman Filter process noises (qs).*

INTRODUCTION

The two-sample variance, or what we commonly refer to as the Allan Variance, has been an excellent device for time domain characterization of GPS Cesium frequency standards over the past few years. Over the past year, the GPS Master Control Station (MCS) has also applied the Allan Variance towards fine tuning the MCS's state estimation of these Cesium clocks [3].

In terms of Power-Law Spectral Density exponents, the Allan Variance does not explicitly converge for noise types of $\alpha \leq -3$, and may be greatly affected by frequency drift [5]. Because GPS Rubidium frequency standards exhibit significant aging and aging noise characteristics, the Allan Variance analysis must be augmented to dynamically compensate for this frequency drift, and to characterize two additional noise types, specifically $\alpha = -3$ and $\alpha = -4$. As the GPS program progresses, we will utilize a larger percentage of *Rubidium* frequency standards than ever before. In particular, the Block IIR satellite platform will house three atomic frequency standards, and two of these three will be Rubidium. Clearly, the characterization of GPS Rubidium clocks will soon require more attention than ever before.

In contrast, the three-sample variance, commonly referred to as a renormalized Hadamard Variance, is *unaffected* by linear frequency drift, converges for $\alpha > -5$ [8], and hence has a potential utility for modeling the various noise types resident in GPS Rubidium frequency standards. This paper demonstrates this potential for Hadamard Variance analysis in GPS analysis operations, and presents the relationship between the Hadamard Variance and the MCS's Kalman Filter process noises (*q*s).

THE HADAMARD VARIANCE EQUATION

A mainstay of atomic clock characterization, the two-sample (Allan) Variance essentially examines the second difference of phase, equivalent to the first difference of the time-averaged frequencies over two successive adjacent time intervals (τ) [5]:

$$\sigma^2{}_y(\tau) = \frac{1}{2(M-1)} \sum_{i=1}^{M-1} (\bar{y}_{i+1} - \bar{y}_i)^2 \ , \ \ \bar{y}_i = \text{the time-averaged frequency over } \tau_i. \tag{1}$$

Similar in principle to the structure to the Allan Variance, the *three*-sample variance examines the *third* difference in phase, equivalent to the *second* difference of the time-averaged frequencies over *three* successive adjacent time intervals (τ). The timing community has commonly referred to this three-sample variance as the *Hadamard* Variance. Though the term *Hadamard Variance* has been used more generally in various applications of multi-sample time domain analysis, for the purposes of this paper, we shall define the Hadamard Variance as follows:

$${}_H\sigma^2{}_y(\tau) = \frac{1}{6(M-2)} \sum_{i=1}^{M-2} (\bar{y}_{i+2} - 2\bar{y}_{i+1} + \bar{y}_i)^2 \ , \ \ \bar{y}_i = \text{the time-averaged frequency over } \tau_i. \tag{2}$$

The Hadamard Deviation (the square root of the Hadamard Variance) identifies two noise types that the Allan Deviation does not *explicitly* identify [8]. For this paper, we shall name the following noise types: for $\alpha = -3$, "Flicker Walk FM"; for $\alpha = -4$, "Random Run FM" [5]. Figure 1 visually describes the noise types identified by the Hadamard Deviation [6].

In terms of phase, equation (2) converts to [8]:

$${}_H\sigma^2{}_y(\tau) = \frac{1}{6\tau^2(N-3)} \sum_{i=1}^{N-3} (x_{i+3} - 3x_{i+2} + 3x_{i+1} - x_i)^2 \ , \ \ x_i = \text{the phase measurement at } t_i. \tag{3}$$

or, equivalently:

$${}_H\sigma^2{}_y(\tau) = \frac{1}{6\tau^2} \mathrm{E}[(x_{i+3} - x_{i+2}) - (x_{i+2} - x_{i+1}) - (x_{i+2} - x_{i+1}) + (x_{i+1} - x_i)]^2 \tag{4}$$

where E[.] is the expectation operator. Each phase measurement $x_i = x(t_i)$ in equation (4) is separated from each neighboring successive phase measurement by a time interval value of τ. Meaning,

$$x(t_{i+1}) = x(t_i + \tau), \tag{5}$$

$$x(t_{i+2}) = x(t_{i+1} + \tau) = x(t_i + 2\tau), \text{ and} \tag{6}$$

$$x(t_{i+3}) = x(t_{i+2} + \tau) = x(t_{i+1} + 2\tau) = x(t_i + 3\tau) \tag{7}$$

MCS KALMAN FILTER TIME UPDATE PREDICTIONS

The propagation (time update), of Rubidium clock states in the MCS Kalman Filter, is modeled using the following polynomial expansion [7]:

$$\begin{bmatrix} x(t+\tau) \\ y(t+\tau) \\ z(t+\tau) \end{bmatrix} = \begin{bmatrix} 1 & \tau & (1/2)\tau^2 \\ 0 & 1 & \tau \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \tag{8}$$

where τ is the prediction span, and $x(t)$, $y(t)$, and $z(t)$ are the phase, frequency, and frequency drift values, respectively, of the clock in question. Note that $y(t)$ is the time derivative of $x(t)$, and $z(t)$ is the time derivative of $y(t)$. $\Delta(x)$, $\Delta(y)$, and $\Delta(z)$ are assumed to be random error increments, independent of $x(t)$, $y(t)$, and $z(t)$, having a prediction covariance P represented by a function of the Kalman Filter process noises (qs) [1,7]:

$$P = \mathrm{E}\left[\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \begin{bmatrix} \Delta x & \Delta y & \Delta z \end{bmatrix}\right] = \begin{bmatrix} q_1\tau + q_2\tau^3/3 + q_3\tau^5/20 & q_2\tau^2/2 + q_3\tau^4/8 & q_3\tau^3/6 \\ q_2\tau^2/2 + q_3\tau^4/8 & q_2\tau + q_3\tau^3/3 & q_3\tau^2/2 \\ q_3\tau^3/6 & q_3\tau^2/2 & q_3\tau \end{bmatrix} \tag{9}$$

An expansion of equation (8) produces the following equations:

$$\begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \end{bmatrix} = \begin{bmatrix} x_i + \tau y_i + (1/2)\tau^2 z_i \\ y_i + \tau z_i \\ z_i \end{bmatrix} + \begin{bmatrix} \Delta x_{i+1} \\ \Delta y_{i+1} \\ \Delta z_{i+1} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} x_{i+2} \\ y_{i+2} \\ z_{i+2} \end{bmatrix} = \begin{bmatrix} x_{i+1} + \tau y_{i+1} + (1/2)\tau^2 z_{i+1} \\ y_{i+1} + \tau z_{i+1} \\ z_{i+1} \end{bmatrix} + \begin{bmatrix} \Delta x_{i+2} \\ \Delta y_{i+2} \\ \Delta z_{i+2} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} x_{i+3} \\ y_{i+3} \\ z_{i+3} \end{bmatrix} = \begin{bmatrix} x_{i+2} + \tau y_{i+2} + (1/2)\tau^2 z_{i+2} \\ y_{i+2} + \tau z_{i+2} \\ z_{i+2} \end{bmatrix} + \begin{bmatrix} \Delta x_{i+3} \\ \Delta y_{i+3} \\ \Delta z_{i+3} \end{bmatrix} \tag{12}$$

Using equations (10), (11), and (12), and examining the differences between each successive x_i:

$$(x_{i+1} - x_i) = \tau y_i + (1/2)\tau^2 z_i + \Delta x_{i+1} \tag{13}$$

$$(x_{i+2} - x_{i+1}) = \tau y_{i+1} + (1/2)\tau^2 z_{i+1} + \Delta x_{i+2} \tag{14}$$

$$(x_{i+3} - x_{i+2}) = \tau y_{i+2} + (1/2)\tau^2 z_{i+2} + \Delta x_{i+3} \tag{15}$$

AN EXPECTATION OPERATOR EXPANSION OF $_{H}\sigma^2_y(\tau)$

Inserting equations (13), (14), and (15) into the following expression:

$$[ThirdDif] = [(x_{i+3} - x_{i+2}) - (x_{i+2} - x_{i+1}) - (x_{i+2} - x_{i+1}) + (x_{i+1} - x_i)] \tag{16}$$

obtains:

$$[ThirdDif] = \Delta x_{i+3} - 2\Delta x_{i+2} + \Delta x_{i+1} + \tau\{(y_i - y_{i+1}) - (y_{i+1} - y_{i+2})\} + (1/2)\tau^2\{(z_i - z_{i+1}) - (z_{i+1} - z_{i+2})\} \tag{17}$$

Examining the differences between each successive y_i and z_i, from equations (10), (11), and (12):

$$(y_{i+1} - y_i) = \tau z_i + \Delta y_{i+1} \tag{18}$$

$$(y_{i+2} - y_{i+1}) = \tau z_{i+1} + \Delta y_{i+2} \tag{19}$$

$$(z_{i+1} - z_i) = \Delta z_{i+1} \tag{20}$$

$$(z_{i+2} - z_{i+1}) = \Delta z_{i+2} \tag{21}$$

Equation (17) translates into:

$$[ThirdDif] = \Delta x_{i+3} - 2\Delta x_{i+2} + \Delta x_{i+1} + \tau\{(\tau z_{i+1} + \Delta y_{i+2}) - (\tau z_i + \Delta y_{i+1})\} + (1/2)\tau^2\{(\Delta z_{i+2}) - (\Delta z_{i+1})\} \tag{22}$$

With some more algebraic manipulation:

$$[ThirdDif] = \Delta x_{i+3} - 2\Delta x_{i+2} + \Delta x_{i+1} + \tau\{(\Delta y_{i+2} - \Delta y_{i+1}) + (\tau z_{i+1} - \tau z_i)\} + (1/2)\tau^2\{(\Delta z_{i+2}) - (\Delta z_{i+1})\} \tag{23}$$

$$[ThirdDif] = \Delta x_{i+3} - 2\Delta x_{i+2} + \Delta x_{i+1} + \tau\{(\Delta y_{i+2} - \Delta y_{i+1}) + (\tau\Delta z_{i+1})\} + (1/2)\tau^2\{(\Delta z_{i+2}) - (\Delta z_{i+1})\} \tag{24}$$

$$[ThirdDif] = \Delta x_{i+3} - 2\Delta x_{i+2} + \Delta x_{i+1} + \tau\{(\Delta y_{i+2} - \Delta y_{i+1})\} + (1/2)\tau^2\{(\Delta z_{i+2}) + (\Delta z_{i+1})\} \tag{25}$$

$$[ThirdDif] = \{\Delta x_{i+3}\} + \{-2\Delta x_{i+2} + \tau\Delta y_{i+2} + (1/2)\tau^2(\Delta z_{i+2})\} + \{\Delta x_{i+1} - \tau\Delta y_{i+1} + (1/2)\tau^2(\Delta z_{i+1})\} \tag{26}$$

Since we've now broken down this expansion into three independent polynomial terms:

$$\mathrm{E}[ThirdDif]^2 = \mathrm{E}[\{\Delta x_{i+3}\} + \{-2\Delta x_{i+2} + \tau\Delta y_{i+2} + (1/2)\tau^2(\Delta z_{i+2})\} + \{\Delta x_{i+1} - \tau\Delta y_{i+1} + (1/2)\tau^2(\Delta z_{i+1})\}]^2 \tag{27}$$

the independence of each term {} allows us to separate equation (27) into three individual expectation operators [4]:

$$E[ThirdDif]^2 = E[\Delta x_{i+3}]^2 + E[-2\Delta x_{i+2} + \tau\Delta y_{i+2} + (1/2)\tau^2(\Delta z_{i+2})]^2 + E[\Delta x_{i+1} - \tau\Delta y_{i+1} + (1/2)\tau^2(\Delta z_{i+1})]^2 \quad (28)$$

Expressing each term of $E[ThirdDif]^2$ as a function of the Kalman Filter prediction covariance matrix [1]:

$$E[\Delta x_{i+3}]^2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} P \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = q_1\tau + (1/3)q_2\tau^3 + (1/20)q_3\tau^5 \quad (29)$$

$$E[-2\Delta x_{i+2} + \tau\Delta y_{i+2} + (1/2)\tau^2(\Delta z_{i+2})]^2 = \begin{bmatrix} -2 & \tau & (1/2)\tau^2 \end{bmatrix} P \begin{bmatrix} -2 \\ \tau \\ (1/2)\tau^2 \end{bmatrix} \quad (30)$$

$$= 4q_1\tau + (1/3)q_2\tau^3 + (9/20)q_3\tau^5$$

$$E[\Delta x_{i+1} - \tau\Delta y_{i+1} + (1/2)\tau^2(\Delta z_{i+1})]^2 = \begin{bmatrix} 1 & -\tau & (1/2)\tau^2 \end{bmatrix} P \begin{bmatrix} 1 \\ -\tau \\ (1/2)\tau^2 \end{bmatrix} \quad (31)$$

$$= q_1\tau + (1/3)q_2\tau^3 + (1/20)q_3\tau^5$$

By adding each term:

$$E[ThirdDif]^2 = 6q_1(\tau) + q_2(\tau^3) + (11/20)q_3(\tau^5) \quad (32)$$

Hence,

$$_H\sigma^2{}_y(\tau) = \frac{1}{6\tau^2}E[ThirdDif]^2 = \frac{1}{6\tau^2}[6q_1(\tau) + q_2(\tau^3) + (11/20)q_3(\tau^5)] \quad (\text{for } \alpha = 0, -2, -4) \text{ [6]} \quad (33)$$

$$_H\sigma^2{}_y(\tau) = q_1\tau^{-1} + (1/6)q_2\tau + (11/120)q_3\tau^3 \quad (\text{for } \alpha = 0, -2, -4) \quad (34)$$

RELATING WHITE PM TO THE HADAMARD VARIANCE

Equation (34) does not, however, account for white PM noise ($\alpha = 2$) [6], sometimes also referred to as *representation error* [1]. The Hadamard Variance can be expressed, in terms of phase measurements, as follows:

$$_H\sigma^2{}_y(\tau) = \frac{1}{6\tau^2}E[x_{i+3} - 3x_{i+2} + 3x_{i+1} - x_i]^2 \quad (35)$$

When white PM is the *only* significant noise component, the individual phase values are uncorrelated with time, and may be separated [4]:

$$ {}_H\sigma^2{}_y(\tau) = \frac{1}{6\tau^2}\left\{E[x_{i+3}]^2 + E[-3x_{i+2}]^2 + E[3x_{i+1}]^2 + E[-x_i]^2\right\} \quad \text{(for } \alpha = 2\text{)} \tag{36} $$

When assuming that white PM is the primary noise source, the representation error, which we'll denote as $q_0 = E[x_i]^2$, is independent of t_i, and thus is a phase variance that is constant across time. Therefore,

$$ {}_H\sigma^2{}_y(\tau) = \frac{1}{6\tau^2}\left\{(1+9+9+1)E[x_i]^2\right\} = \frac{10}{3\tau^2}q_0 \quad \text{(for } \alpha = 2\text{)} \tag{37} $$

THE HADAMARD-Q EQUATION

In the presence of both a) white PM, and b) the three noise types modeled by P, and assuming independence between the white PM and the other noise types, equations (34) and (37) can be combined into one that models four noise types, namely $\alpha = 2$, 0, -2, and -4:

$$ {}_H\sigma^2{}_y(\tau) = (10/3)q_0\tau^{-2} + q_1\tau^{-1} + (1/6)q_2\tau + (11/120)q_3\tau^3 \tag{38} $$

Note how this equation compares to the analogous equation relating the Allan Variance to the qs [2,3]:

$$ \sigma^2{}_y(\tau) = 3q_0\tau^{-2} + q_1\tau^{-1} + (1/3)q_2\tau + (1/20)q_3\tau^3 \tag{39} $$

For white FM, the Allan and Hadamard Variances are mathematically equivalent. For white PM, the two Variances are roughly the same, and, for random walk FM, the Variances differ by a factor of two. Though an analyst may use either the Hadamard Variance or the Allan Variance for deriving MCS qs, each has its own set of advantages and disadvantages.

The primary advantage of the Hadamard Variance is the automatic removal of linear frequency drift [8]. Whereas the equation relating the Allan Variance to MCS qs assumes that the analyst must apply a continuously dynamic correction for frequency drift, the Hadamard-Q equation doesn't require this assumption. The tradeoff, however, is that the Hadamard Variance incurs an extra computational burden, simply because it examines the *third* (vice the second) difference of phase. For analyzing GPS Cesium frequency standards, the increased computational load of the Hadamard Variance proves fruitless, only because the Allan Variance gets the job done more efficiently [3].

Many timing experts, over the years, have extensively used techniques for applying a continuously dynamic correction for frequency drift, prior to using the Allan Variance for deriving q_3 values with high confidence. This paper does *not* address the issue of confidence in the q_3 value produced by the Hadamard-Q equation. This paper does, however, present an easily understood relationship between a relatively lesser known equation (the Hadamard Variance), and a set of system parameters used by the MCS (the Kalman Filter qs). On initial appearance, given the computational capability, one can see the great potential utility of an algorithm that applies a relatively simple equation onto a large measurement data base, in order to derive Kalman Filter qs, *without* the need to apply preparatory frequency drift corrections. In the future, the author hopes to further investigate a) the real-world utility of this relationship, b) the issue of estimate confidence, and c) the net gain from the increased utility balanced against the increased computational burden.

CONCLUSION

The implication of the Hadamard Variance in GPS operations is as follows: Analysts at the MCS could simply gather a large data base of clock phase measurements, apply all known step corrections, perform a number of iterations of the Hadamard Variance equation, and plot the results to visually describe the noise characteristics of GPS Rubidium clocks (including $\alpha = -4$). Consequently, the Hadamard Variance could offer GPS operators an alternate tool for characterizing GPS atomic frequency standard noise.

Perhaps more significantly, the implication of equation (38) is that GPS analysts now have an easily understood technique to relate raw clock phase measurements towards deriving important Kalman Filter clock estimation parameters (qs), that are unique to the performance of each individual clock, and that will include q_3 automatically, without any need for the preliminary removal of frequency drift.

The MCS hopes to make continued use of the Allan Variance, for both the characterization of Cesium clocks, and the derivation of their associated process noise values. This application of the Hadamard Variance widens the array of available tools for the characterization of *Rubidium* clocks, and the derivation of *their* associated process noise values. The GPS Block IIR satellite program will use a large percentage of Rubidium clocks. Certainly, the ever-important issue of refining Rubidium clock estimation may see its most important days in the years ahead.

ACKNOWLEDGMENTS

The author wishes to thank the following people and agencies for their generous assistance with both our timing improvements and this paper:

David W. Allan, Allan's TIME
James A. Barnes, Senior Member, IEEE
Kenneth R. Brown, Loral Federal Systems Division
James W. Chaffee, Associate Member, IEEE
Pamela C. Crum, B.A., Mathematics, Amherst College
Sam R. Stein, Timing Solutions Corporation
The people of the 2 SOPS
Todd E. Walter, Stanford University
Marc A. Weiss, NIST

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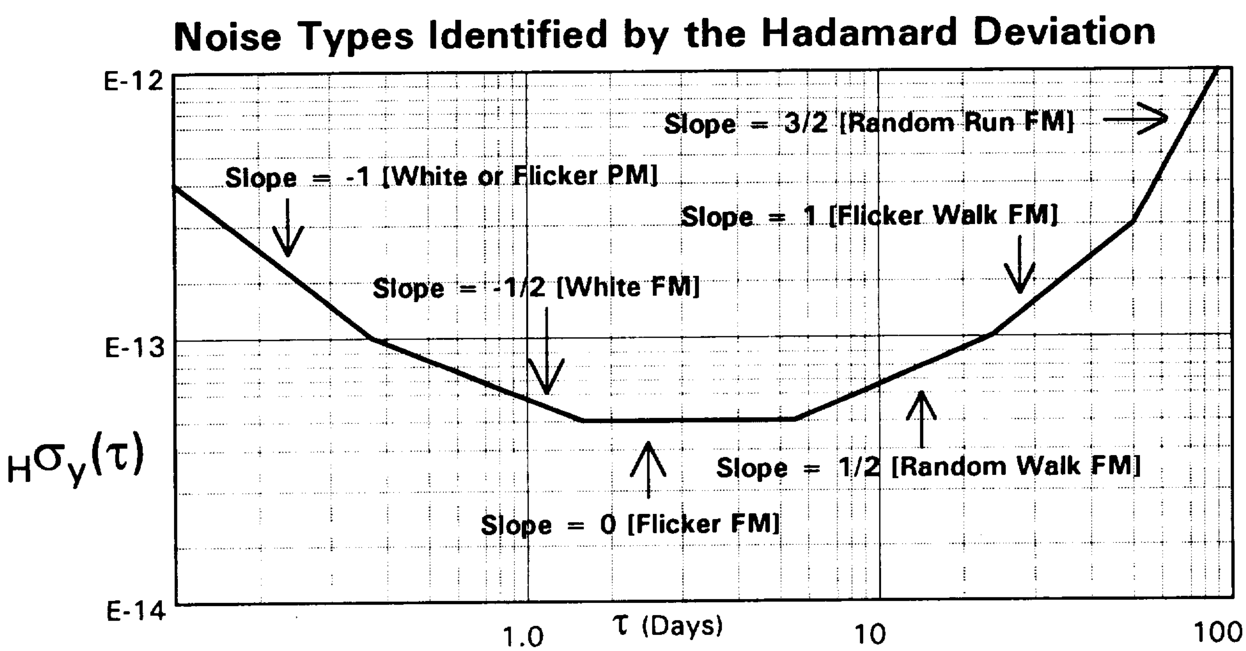

Noise Type	Power-Law Spectral Density Exponent	$_H\sigma_y(\tau)$ Log-Log Slope
White PM	$\alpha = 2$	**-1**
Flicker PM	$\alpha = 1$	**-1**
White FM	$\alpha = 0$	**-1/2**
Flicker FM	$\alpha = -1$	**0**
Random Walk FM	$\alpha = -2$	**1/2**
Flicker Walk FM	$\alpha = -3$	**1**
Random Run FM	$\alpha = -4$	**3/2**

Figure 1

Questions and Answers

SERGEY V. ERMOLIN (HEWLETT-PACKARD): The Hadamard Variance does remove linear drift, that's true; and that saves you some time on preprocessing. But, it doesn't remove any drift beyond linear. As you showed on one of your first slides, that rubidium standards on board one of the space vehicles show not only linear drift, but possibly quadratic drift to some other power.

So still, if you wanted to go ahead with longer operational time, you would still have to do some preprocessing, even if you used the Hadamard Variance.

CAPTAIN STEVEN HUTSELL (USAF): The analogy is the Allan Variance does not care about a constant frequency offset. For instance, our atomic clocks can have a 1×10^{-11} frequency offset; but if it's stable enough, the Allan Variance will be low, regardless of that frequency offset. The analogy is in the Hadamard Variance, if the satellite clock, or whatever, has an already existing common offset of, say, 3×10^{-18} seconds per second squared of frequency drift, that will not adversely affect the Hadamard Variance calculations.

However, in the Allan Variance, if there is a frequency drift, it will affect it. But with the Hadamard, it won't. In the same sense, a random walk in random walk FM will affect the Allan Variance by causing a positive one slope. But the frequency offset itself will not affect it.

DAVID ALLAN (ALLAN'S TIME): Dr. Barnes did some work three decades ago on the confidence question, actually related to it, in the third difference estimate. The bottom line was that the confidence is worse by a significant amount, especially when you have finite data lengths that come into impact you quite adversely.

The other point is that it turns out a logarithmic drift estimator, both for quartz and rubidium is much better than linear. I think the graph that you showed outside of the turn-on transient probably would fit a logarithmic curve quite well.

So, one might be better doing logarithmic modeling if we deal with a lot of rubidiums in the future. Yes, one would expect that the logarithmic function could be fit to this quite well, outside of the first point.

The last one I would like to suggest that you think about – and I want to say that I think you've done a beautiful piece of work, but you can actually remove the effect of drift, kind of in real time, from the second difference operator, because you know the exact equation for the effect of drift on it until you can subtract that from and get an estimator variance without the drift effectively in real time. So, it doesn't need to impact the value of the variance if you don't want it to; and it gives you a tighter confidence of the estimate.

CAPTAIN STEVEN HUTSELL (USAF): Yes, and the intent of this is not really to present the best way to estimate frequency drift dynamically. Really, what I wanted to do was examine the way the MCS is currently set up. And right now, it does not have the capability to do what you just described. The MCS is only set up as a dynamic Kalman filter three-state vector that needs process noise values.

I completely agree that are far more sophisticated techniques to look at frequency drift than what's set up in the Kalman filter. Sorry, your first comment?

DAVID ALLAN (ALLAN'S TIME): [Inaudible].

CAPTAIN STEVEN HUTSELL (USAF): Yes, we see that too. We see it start to converge - - -

DAVID ALLAN (ALLAN'S TIME): [Inaudible].

CAPTAIN STEVEN HUTSELL (USAF): We agree. Over time, the frequency drift goes from a negative value, around 3 or 4×10^{-18}, and gradually starts logarithmically to approach zero.

At the beginning, however, sometimes we see it start hugely negative, like -1×10^{-17}. Sometimes $+1 \times 10^{-16}$. We're talking about over the first 48 hours that we turn it on. We would need to address how we try to model that. It's also probably appropriate to ask what's causing that, what physical phenomenon is causing it to be positive at the beginning for some clocks, and negative for the others. But, I do agree.