LIMITS TO THE STABILITY OF PULSAR TIME

Gérard Petit
Bureau International des Poids et Mesures
92312 Sèvres Cedex, France

Abstract

The regularity of the rotation rate of millisecond pulsars is the underlying hypothesis for using these neutron stars as "celestial clocks." Given their remote location in our galaxy and to our lack of precise knowledge on the galactic environment, a number of phenomena affect the apparent rotation rate observed on earth. This paper reviews these phenomena and estimates the order of magnitude of their effect. It concludes that an ensemble pulsar time based on a number of selected millisecond pulsars should have a fractional frequency stability close to 2×10^{-15} for an averaging time of a few years.

INTRODUCTION

Millisecond pulsars have a very regular period of rotation, which suggests that they may be considered as flywheels, generating a pulsar time scale (PT) to which atomic time (AT) can be compared over time intervals of several years. Presently the relative frequency instability of (AT-PT) is at the level of one to two parts in 10^{14} for averaging times of several years. From the observation of many pulsars, an ensemble pulsar time can be derived which is more stable than each individual scale[1].

The apparent period of rotation observed on earth differs from the proper period by terms originating, on the one hand, in the relativistic transformation between the reference frame of the pulsar and the geocentric one and, on the other, in variations in the time of flight of the signal caused by the interstellar medium or gravitation. For this reason, the observed instability of pulsar time may be intrinsic or may arise in the effects just mentioned.

In this paper the phenomena that can affect the apparent period of rotation of a pulsar are reviewed and their magnitudes are estimated with a view to computing the relative frequency stability that can be expected from pulsar time. This stability is evaluated by means of the Allan deviation $\sigma_y(\tau)$ for an averaging duration τ. In this study we are mainly interested in the frequency stability over several years, which implies that pulsar observations must cover ten years or more. A synthesis of the known limits to the stability of pulsar time is presented in the Allan deviation curves of Figure 1.

BASIC CONSIDERATIONS

A first component limiting the stability of PT is the measurement noise: this is considered to be white phase noise with a measurement uncertainty of order one microsecond. This type of noise averages to zero as the number of measurements increases so that it is always possible to consider that its effect can be made arbitrarily small by increasing the number of the observations and the duration over which they take place. In practice, a reasonable supposition is that one observation, with an uncertainty of one microsecond, is taken every ten days, yielding a stability of 1×10^{-15} for an averaging time of three years (Figure 1).

A second component arises from the need to adjust the timing data to estimate the astrometric parameters of the pulsar: its position and proper motion are determined by a fit of periodic terms with a period of one year; its period P and period derivative $\dot{P}$ are determined by a parabolic fit. This procedure sets limits to the stability of PT that are linked to the stability of the reference atomic time scale for integration durations of six months, due to the periodic terms, and of order the total duration of the observations, due to the parabolic fit[1]. At very long averaging times, the frequency stability of atomic time is believed to be essentially constant, to within the uncertainty of the primary frequency standards $(1 \rightarrow 2 \times 10^{-14}$ over the period 1980–1995). For this reason all phenomena that have power law spectra $S(f) \sim f^{-\alpha}$, where α is 4 (random walk in frequency) or 5 and 6 ("red noise"), are affected by the parabolic fit[2]. These types of noise would have Allan deviations varying as $\sigma_y(\tau) \sim \tau^{(\alpha-3)}$, which would mean that the frequency instability they introduce increases indefinitely with the averaging time: the effect of the adjustment is to prevent this from happening (Figure 2). This situation occurs for many of the physical phenomena described in the following section.

PHYSICAL PHENOMENA AFFECTING THE APPARENT PERIOD OF ROTATION

Intrinsic Variations

Regular lengthening of the period of rotation of a pulsar (represented by a constant $\dot{P}$) results from the loss of energy by emission of electromagnetic waves and relativistic particles. This is not discussed further as it is considered an (as yet) inescapable fact that the period changes with a rate $\dot{P}$ that must be measured, since it cannot be predicted by theory. This phenomenon is a "secular" one modelled by a single parameter. The second-order period derivative $\ddot{P}$ is not expected to contribute significantly[3] for reasonable durations of observation (centuries).

Another phenomenon which may affect the period of rotation is precession of the spin axis. Several theories have been proposed that involve different modes of precession that would cause changes in the rotation rate or in the direction of the emission[4, 5]. Some of these phenomena would be periodic, and could eventually be modelled by a small number of parameters, but as no firmly established theory is available it would be necessary to determine the parameters by adjustment of the timing data. The effect of such periodic signatures is treated below, in the section dealing with orbiting companions.

Geodetic precession of the spin axis has been observed for a pulsar in a binary system[6]. It

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causes long-term variations in the timing data because the part of the emission zone which happens to be directed towards the earth changes. If the geodetic precession is small (weakly relativistic binary system), the quadratic fit (P and $\dot{P}$) should absorb most of the effect. For a strongly relativistic system that might not be the case, but in such a system the pulsar might become unobservable, its emission not being any more in the direction of the earth. So the geodetic precession is not expected to significantly affect the observed rotation rate of most binary millisecond pulsars ($\sigma_y(\tau) \sim 1 \times 10^{-15}$).

Other phenomena linked to the internal structure of pulsars are abrupt changes in the rotation rate, followed by a relaxation period, that have been observed in many pulsars, mainly young ones[7]. Such glitches have not been observed in millisecond pulsars, but it is conceivable that they occur with an amplitude so small that they cannot readily be identified. In any case, the rotation rate must experience intrinsic noise caused by events affecting the structure of the pulsar itself or the emission mechanism. For obvious reasons, these phenomena are not well known.

Intrinsic noise is more easily studied on "long period" pulsars because the population is more numerous, because it has been observed for a longer time, and because the changes in the period of rotation are relatively larger than for millisecond pulsars so that they stand out clearly in the stability analysis. Some authors have tried to relate these instabilities with the period and period derivative using ad-hoc formulas. For example, Cordes and Helfand[8], followed by Dewey and Cordes[9], defined an "activity parameter" A by comparing the RMS of the timing residuals of a pulsar with that of the Crab pulsar. They showed experimentally that A could be related to P and $\dot{P}$ by the ad-hoc formula:

$$A = a \log P + b \log \dot{P} + c$$

where a, b and c are constants. Similarly Arzoumanian et al.[10] defined an absolute parameter $\Delta(T)$ as the logarithm of the magnitude of the cubic term that can be adjusted to the timing residuals over an interval of duration T. They showed that, provided that the timing residuals are reasonably cubic, the parameters A and Δ are related. They also determined experimentally an ad-hoc formula:

$$\Delta(T = 10^8 \text{ s}) = 6.6. \times 0.6 \log \dot{P},$$

where all quantities with dimension of time are expressed in seconds. From this, it is possible to estimate the frequency variations due to the cubic-like residuals. One finds, for τ significantly lower than the total observation duration T,

$$\sigma_y(\tau) \approx \dot{P}^{0.6}(4 \times 10^6/T)(\tau/T). \tag{1}$$

This formula has been determined by analysis of standard pulsars, not millisecond pulsars. When applying it, by extrapolation, to millisecond pulsars for which the smallest values of the period derivative are of order $10^{-20} s s^{-1}$, it suggests that these pulsars have an intrinsic stability of a few parts in 10^{15} for averaging durations of a few years (Figure 1). Formula (1) may provide only a very crude estimate, but it is consistent with the fact that, of the two pulsars

observed over the longest duration, 1855+09 ($\dot{P} = 1.78 \times 10^{-20} ss^{-1}$) has a better apparent stability than 1937+21 ($\dot{P} = 1.05 \times 10^{-19} ss^{-1}$).

Gravitational Interactions at the Position of the Pulsar

The ideal pulsar is an isolated object in uniform motion, having no gravitational interaction with other bodies. Its apparent period of rotation then differs from the proper period by a constant factor (Doppler effect). In the real world, we consider three classes of gravitational interaction that affect the pulsar, and therefore the long-term stability of its apparent rotation rate.

The first is gravitational interaction with other masses not in a bound orbit with the pulsar. If a pulsar is submitted to a constant acceleration, its period derivative is biased, but the timing residuals (after the quadratic fit) are not affected. If, however, the pulsar moves in a gravitational field its period derivative, to first order, varies linearly with time: $\ddot{P}$ is then proportional to the first derivative of the gravitational acceleration, usually called the jerk. Orders of magnitude of the acceleration a, and therefore of the jerk $\dot{a}$, have been estimated for various cases: pulsar in a cluster[11, 12], encounter with a star[12], average effect of galactic stars, white dwarfs, and giant molecular clouds[13]. As expected, the largest effect is for the pulsar in a cluster, where the jerk can reach $10^{-19} ms^{-3}$, which would cause cubic-like timing residuals of amplitude tens of microseconds. This is a maximum value, so it is not possible to unambiguously identify this effect in the actual observations of pulsars in clusters (such as 1821–24 and 1620–26). Gravitational acceleration is identified, however, as the cause for a negative value of $\dot{P}$ for some pulsars in clusters. The gravitational jerk could cause a frequency instability in the 10^{-14} range for τ of a few years (Figure 1). For pulsars not in clusters, this effect is several orders of magnitude lower, except in the case of an improbable close encounter with a star.

The second gravitational interaction is with one or several companions in a bounded system. In this case the position of the pulsar moves with respect to the barycenter of the system, so causing a periodic signature in the timing data. The interaction is described by a small number of parameters (Keplerian parameters) that have to be determined from the timing data. If the duration of observation does not span several orbital periods, it is not possible to determine the orbital parameters reliably, so possible companions with orbital periods comparable with the duration of the observations have to be considered as a source of long-term instability. The order of magnitude of the effect is proportional to the mass of the companion: For a Jupiter-like planet orbiting the pulsar in a few tens of years, the amplitude of the periodic effect would be several seconds and, after quadratic fit, the effect on the timing residuals would still be to cause variations in the observed rotation rate of order 10^{-10}. Even a Pluto-like planet in a similar orbit would cause variations close to 10^{-14} (Figure 1). It is, therefore, to be hoped that millisecond pulsars do not have sizeable planets, with long periods, around them.

The third interaction is with gravitational waves. Gravity waves sweeping over an area change the space-time metric and, therefore, affect the proper time of clocks in this area relative to clocks elsewhere. Pulsar Time and Atomic Time are affected differently and this shows up in the timing residuals. Due to the short-term noise of pulsar timing and to the adjustment

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of yearly terms, measurements are sensitive only to waves with periods of several years for which the only source is probably stochastic background from the early universe (primordial gravitational waves or waves originating in vibrations of cosmic strings). Their power spectrum is expected to vary as f^{-5}, but their expected amplitude is uncertain by at least fifteen orders of magnitude[14]. It is not possible, therefore, to specify an order of magnitude for this effect in Figure 1. Rather, millisecond pulsar timing results make it possible to constrain the energy density of the gravitational wave background[15], and so to discriminate among cosmological models of the universe.

Signal Propagation

Two effects can be held responsible for variations in the time of propagation from the pulsar to the earth which can cause long-term variations in the apparent rotation rate: one is the dispersion by the interstellar medium, and the other is the gravitational effect of intervening masses.

The interstellar medium is dispersive, like the earth's ionosphere, and this causes a propagation delay along with variations in the angle of arrival. These variations cause scintillation phenomena in which can be distinguished the diffractive regime (short-term variations, $\sim$ 100s) and the refractive regime (at longer averaging times). These phenomena change continuously due to the relative motion of the pulsar, the solar system, and the medium itself. Their effect on pulsar timing has been studied by many authors[16, 17, 18]. As the effect is essentially proportional to the inverse square of the frequency, it can be determined by dual-frequency observations. Uncertainties in this determination originate in non-dispersive effects (that can be approximated by higher order terms of the frequency) and possible changes in the pulse shape with frequency. It is expected that, after taking into account the dispersive effect by dual-frequency observations, the remaining effect has the power spectrum of white or flicker phase noise and induces a frequency instability of not more than $2 \times 10^{-14}/\tau$ with τ in years (Figure 1) if the signal from the pulsar crosses a large part of the galaxy[18]. For a pulsar closer to us, this value can be one order of magnitude smaller.

Masses close to the path from the pulsar to Earth cause a gravitational delay and a "bending" of the path. The bending causes a change in the angle of arrival (similar to refraction by the interstellar medium), but this is lower than 10^{-8} rad for an average star crossing the line of sight so has limited impact on the long-term stability. On the other hand, the gravitational delay can be significant[19]: A star of one solar mass with a closest approach of 10^{-8} rad to the pulsar-earth path could perturb the frequency stability by a few parts in 10^{14} (τ = a few years, see Figure 1). This is very unlikely: typically, the star closest to the path might have an angular separation of order 10^{-5} rad (a few seconds of arc), but it is unlikely that this star and the pulsar would also have a relative proper motion large enough for this angle to change by several times its value during the period of observation. The average value of the gravitational delay effect can be estimated to be no larger than about 2×10^{-15} (Figure 1).

Gravitational Interactions at the Position of Earth

All effects noted that can modify the position of the pulsar can also affect the position of the barycenter of the solar system (SSB) and the position of the earth relative to the SSB. Gravitational effects on the SSB are of similar magnitude to those on the pulsar itself, or are smaller. The solar system is not inside a cluster of stars and the surrounding star population is quite well known, so direct gravitational effects on the position of the SSB are expected to have a negligible influence on the long-term stability of pulsar time. The effect of gravitational waves is similar to that on the pulsar.

Presently it seems that the most important source of uncertainty in the SSB position is that due to uncertainty in the masses of outer planets, or to the omission of some masses. As an example, an error in the mass of one of the large asteroids by the amount of its present uncertainty would correspond to a periodic change in the SSB position of about 300m (i.e. one microsecond in timing residuals) with a period of a few years. Uncertainties in the masses of the outer planets[20] may be the cause of a larger shift of the SSB, but with periods from tens of years to a few hundred years so that the effect on the timing residuals, after quadratic fit, is not larger. Overall it is thought that the uncertainty in the SSB position and in the relativistic transformation from the topocentric frame of observation to the barycentric reference frame does not cause frequency variations greater than a few parts in 10^{15} (τ = a few years, see Figure1). In addition, any error originating in the orbit of the earth would, to first order, be absorbed by the parameters representing the position and proper motion of the pulsar, as these are determined from the timing data. Such errors would not directly affect the long-term stability of pulsar time.

CONCLUSIONS

From the above presentation, it seems that the long-term (τ = a few years) stability of pulsar time should be a few parts in 10^{15} for a number of millisecond pulsars. In selecting pulsars for computing an ensemble average, one should avoid, or give a low weight to, the following:

- Pulsars inside clusters because of the direct gravitational effect: the inclusion of a second period derivative might be sufficient to account for this effect, but it would require a longer observation period to decorrelate this term from other effects.

- Pulsars for which the dispersion measure is very large, which are more likely to show residual effects of the interstellar medium (after removal of the dispersive effect with dual-frequency observations).

- Pulsars with high values of the period derivative, which might be intrinsically more noisy.

Periodic terms with periods of years to tens of years (e.g. due to pulsar companion, precession, motion of the SSB) and other systematic effects (gravitational delay) may be detectable only after a very long period of observation. In the mean time, their effect should be treated as noise. In general, all the effects noted above are uncorrelated among pulsars and can be reduced by averaging. Gravitational effects on the SSB would show a definite signature, which

could ultimately make it possible to discriminate among them. Provided the distribution of pulsars is isotropic, however, the magnitude of these effects would also be reduced by averaging. In conclusion, an ensemble pulsar time should be stable to a few parts in 10^{15} for averaging durations of several years. This value is comparable to the stability of current atomic time scales for averaging durations of 10 days to a few months[21]. It is also close to the uncertainty of recent atomic frequency standards[22] which will provide the accuracy and, therefore, the long-term stability, of atomic time scales in the future. Comparisons of atomic time and pulsar time are, therefore, useful, although their main function is to examine the physical phenomena affecting pulsars themselves, the propagation of signals, or the ephemerides of the solar system.

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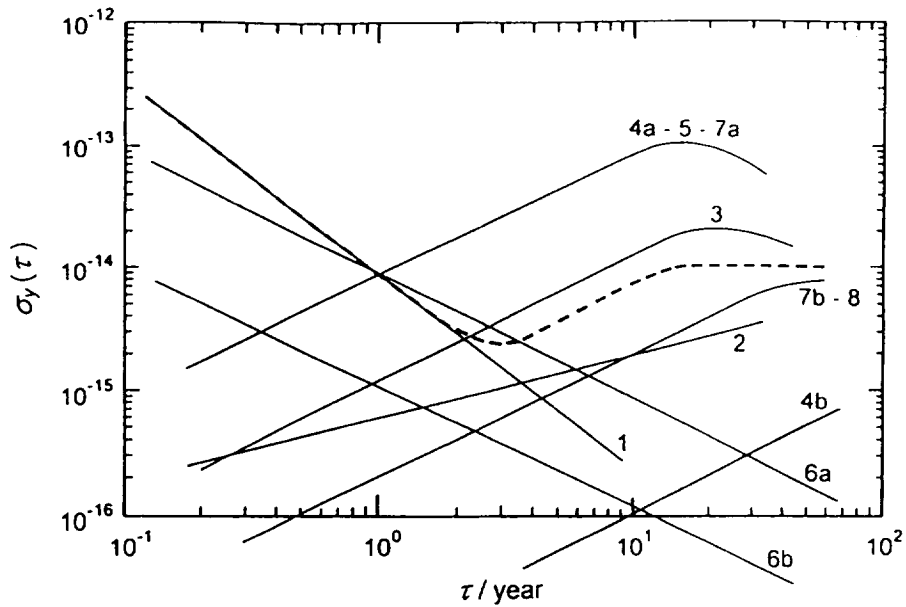
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Figure 1. Fractional frequency stability of the effect on pulsar timing of several phenomena: 1 = measurement noise; 2 = geodetic precession; 3 = intrinsic noise; 4 = gravitational jerk (a = maximum, b = typical); 5 = gravitational pull of long period companion; 6 = interstellar medium (a = maximum, b = typical); 7 = gravitational delay of intervening mass (a = maximum, b = typical); 8 = error in solar system barycentre. The dashed curve represents the stability of pulsar time, assuming average conditions for the effects above.
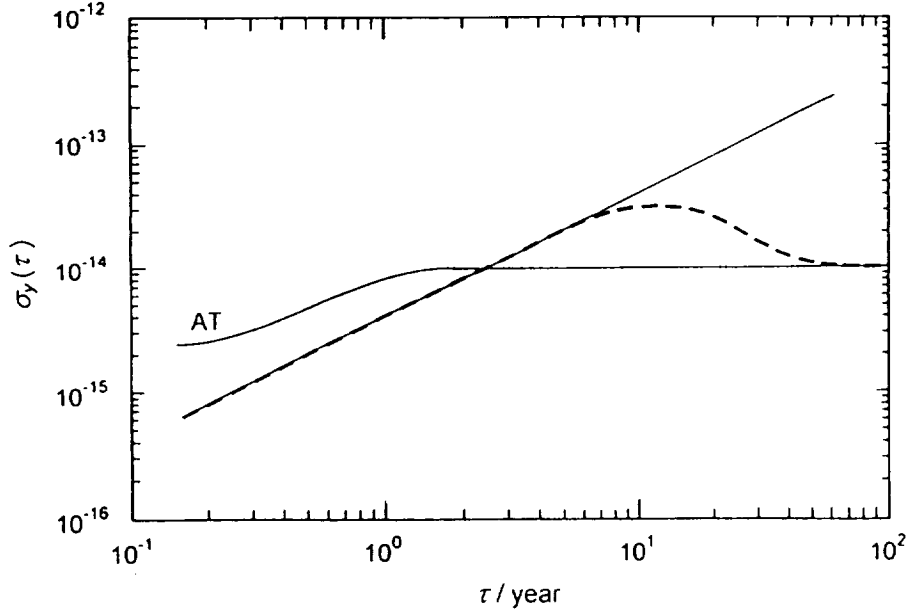

Figure 2. Fractional frequency stability of the current atomic time (AT) and of the effect on pulsar timing of a phenomenon with "red noise" spectrum before (solid line) and after (dashed line) the quadratic fit that determines the period P and its derivative $\dot{P}$.

Questions and Answers

DAVID ALLAN (ALLAN'S TIME): According to Professor Backer at UC Berkeley, and I think Professor Taylor also confirms this, that they have intrinsically within the model, that there is a P double dot as well as a P dot. And they don't know what it is and how big it is. And so, this gives you other low frequency problems which probably don't impact things at 2×10^{-15}.

GERARD PETIT (BIPM): Yes, the intrinsic P double dot is at a level which is several magnitudes lower at the time of infusion. Of course, if you have 1000, you have a better length to stop the range.

ROGER FOSTER (NRL): Can you make a comment about what do you think it's going to take to establish pulsar time? At least, what are your thoughts at BIPM?

GERARD PETIT (BIPM): Do you mean the time it takes?

ROGER FOSTER (NRL): What kind of effort does it take on the part of the community to establish that?

GERARD PETIT (BIPM): Well, more than 50 millisecond pulsars which are known, the first thing is to observe them regularly. That's not really the case because it would require a continuous report from the observatory.

The second thing is that those who observe the pulsar data are available, which is not always the case.

The third thing is to discover new pulsars so as to be able to choose among a larger set to try to find the right pulsar, which has very good stability. I would say that if we had, say, a half a dozen pulsars in the 10^{-15} class observed for 10, 15 years, we can learn something about atomic time.

ROGER FOSTER (NRL): I agree. I'll speak a little bit about this this afternoon.