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DESIGN OPTIMIZATION FOR A MAGLEV SYSTEM EMPLOYING FLUX ELIMINATING COILS

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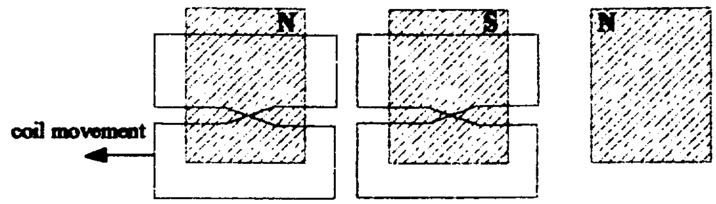
ABSTRACT

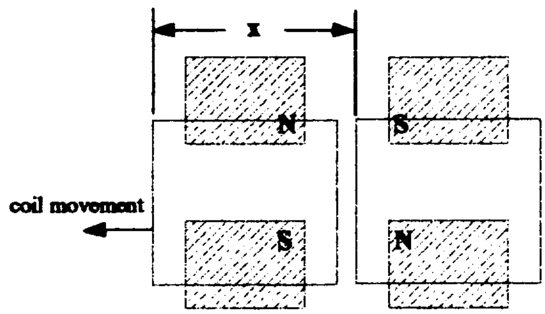
Flux eliminating coils have received no little attention over the past thirty years as an alternative for realizing lift in a MAGLEV system. When the magnets on board the vehicle are displaced from the equilibrium or null flux point of these coils, they induce current in those coils which act to restore the coil to its null flux or centerline position. The question being addressed in this paper is that of how to choose the best coil for a given system. What appears at first glance to be an innocent question is in fact one that is actually quite involved, encompassing both the global economics and physics of the system. The real key in analyzing that question is to derive an optimization index or functional which represents the cost of the system subject to constraints, the primary constraint being that the vehicle lift itself at a certain threshold speed. Outlined in this paper is one scenario for realizing a total system design which uses sequential quadratic programming techniques.

INTRODUCTION

Figure 1 shows a simple magnet and coil layout involving null flux and flux eliminating coils. In inset (a), a pair of null flux coils is being moved past a set of magnets which direct flux in a single direction through the coil. When the coil is displaced vertically downward with respect to the magnet, the upper window of the null flux coil begins to link more flux than the lower window. Because that flux is also changing with time, an induced voltage causes a current to flow which acts to restore the coil to its centerline position, yielding a force in the upward direction of the coil. A similar process is involved in the lower inset (b) of that figure. Here the magnets are stacked, unlike poles above each other, unlike those in inset (a). The coils now form single loops. When the single loop coil is offset from its null flux position, it begins to link flux in a similar fashion to the null flux coil. As drawn, it is clear that the left most coil has more south pole shadowing it than north pole. It will therefore have a net flux linking it which induces a current to again restore it to its null flux position. The one advantage that the stacked magnet design has over the null flux design is that the closure path for the magnetic field is shorter and therefore more efficient.



(a) Null Flux Coil



(b) Flux Eliminating Coils

Figure 1 Magnet and coil geometry used for getting lift for null flux and flux eliminating coils.

Figure 2 shows a cross-section of the second embodiment, the flux eliminating coil. In this cross-section is clear that there are two loops or "O" rings that are arranged side by side, one of the "O"

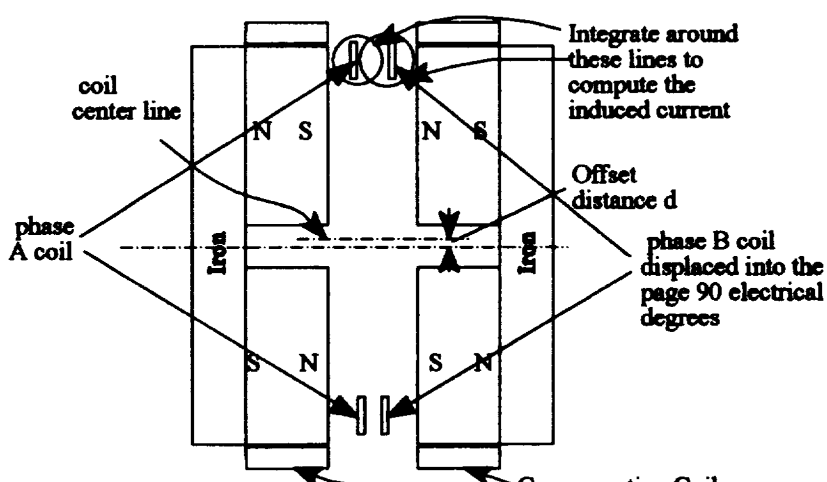


Figure 2 Overlapped Composite coils analyzed with a compensation winding.

rings is in fact displaced into the page with respect to the other, so that the two form a phase shifted pair. The lift associated with the coil pattern shown in Figure 2 is a function of both the offset displacement d of the centerline of the coils with respect to the centerline of the magnets and the excitation frequency. The excitation frequency is in fact specified by the velocity of the coils past the magnets, i.e., $f = \frac{v}{\lambda}$, *where* $\lambda \equiv$ *wavelength into the page*. Note that the coils that are displaced electrically axially into the paper 90^0 with respect to the first set, link no flux at the instant in time when the phase A coil links maximum flux. The objective is to suggest the best track design based on the information realized through a computational analysis, delivering force as a function of displacement and frequency. Specifically the objective would be to define the following:

1. The number of magnet c-sets on the vehicle.
2. The spacing of the magnets and the coils in the track.
3. Displacement distance d at lift off.
4. The commensurate properties associated with these parameters including the system cost per mile, the vehicle weight, the drag forces, and the lift to weight ratio.

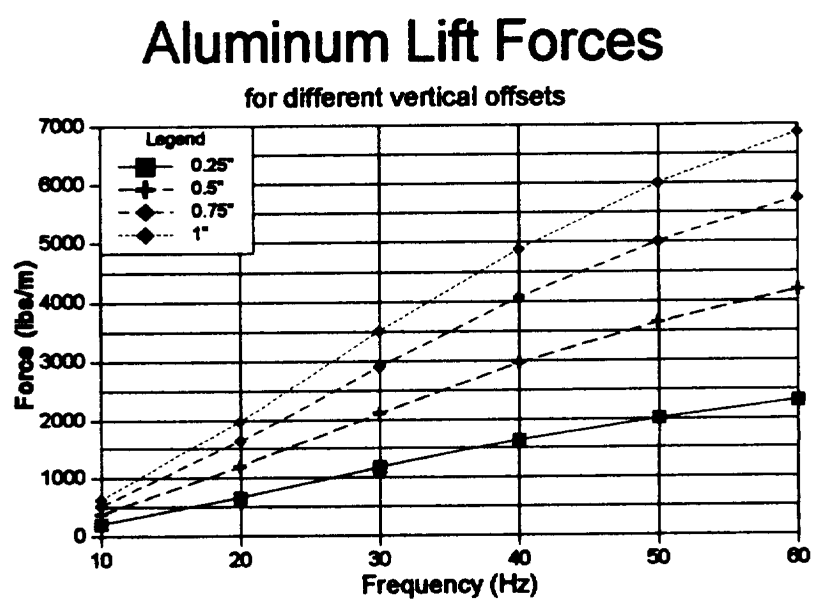


Figure 3 *Lift force on overlapped aluminum composite coils.*

The forces on the coils in Figure 2 are analyzed for a range of displacements and frequencies for both aluminum and copper using a Boundary Element eddy current package (Oersted from Integrated Engineering Software in Winnipeg, Canada). Shown in Figure 3 are the lift forces on aluminum overlapped composite coils. The reader should recognize the familiar induction motor torque/speed profile within these shapes. Because the forces were analyzed in 2D, all forces are reported in lbs/m of depth. Connectivity of the coils is specified by constraining the vector potential within each of the coils, and demanding that the $\oint \vec{H} \cdot d\vec{l}$ around a closed-loop surrounding the coils be constrained so that the induced current within the top two coils is opposite in sign to that in the lower two coils. In addition, the induced current was constrained to be the same for each conductor cross-section.

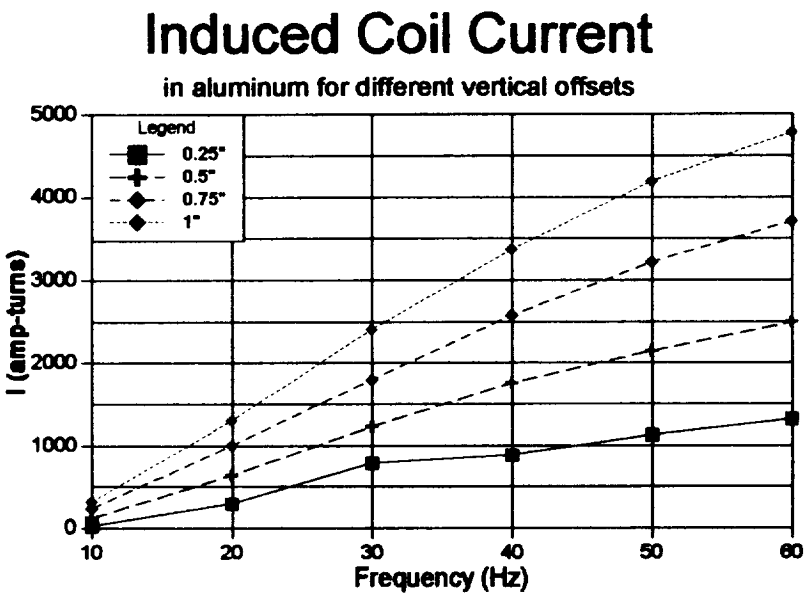


Figure 4 Current induced in the aluminum overlapped composite.

After the field is found everywhere, the induced current within the coils is determined also by integrating $\oint \vec{H} \cdot d\vec{l}$ around each of the coils. Shown in Figure 4 is the current induced in the coils as a function of the same parameters. Note that this current is independent of depth since both the inductance/resistance and flux linkage scale the same with depth extension.

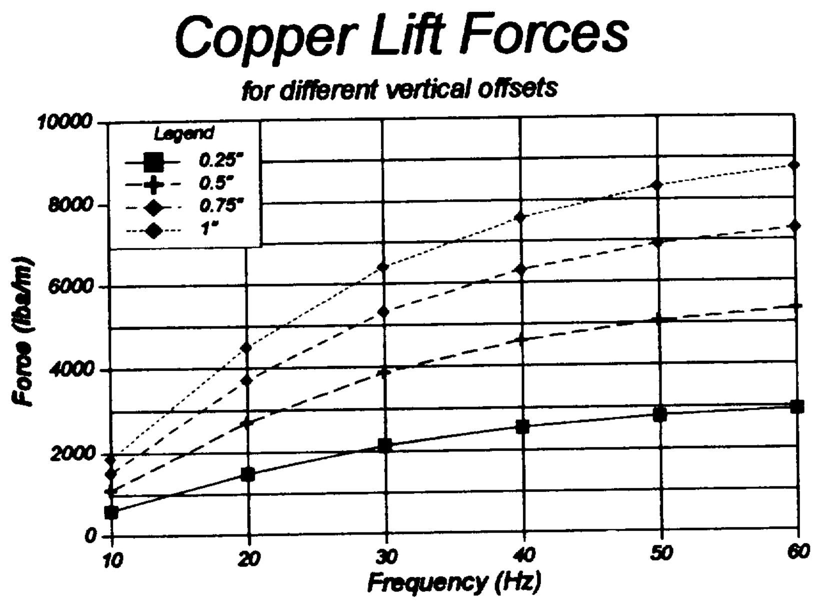


Figure 5 Lift forces on copper overlapped composite coils.

The primary reason why the force using aluminum coils is low is due to timing. The current is not peaking at the right time. If the coil were resistance dominated, the current would be 90 ° out of phase with the inducing current in the magnet, and no net current would result. Changing the coils to copper roughly doubles the L/R time constant and greatly helps the force as witnessed by Figure 5.

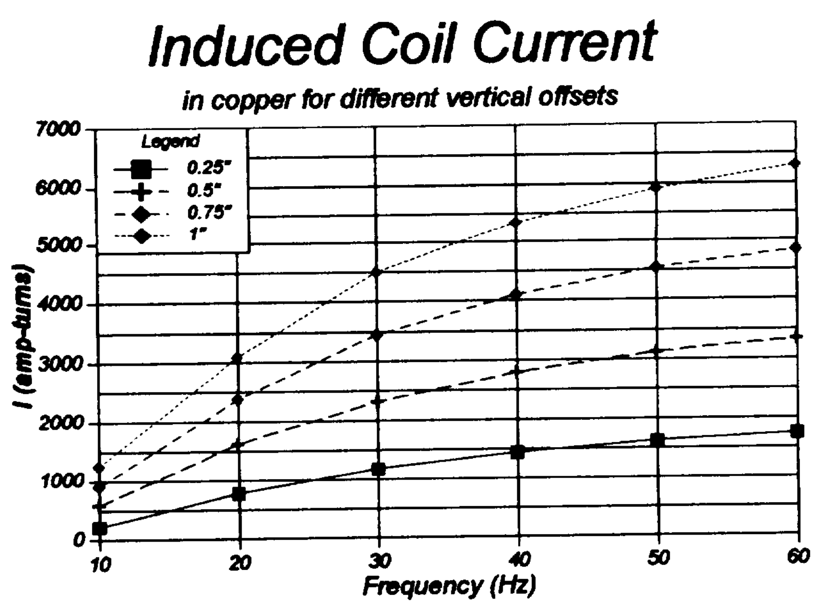


Figure 6 Current induced in copper composite coils.

The commensurate current induced in these coils is displayed in Figure 6. As expected, the force and current follow the same pattern.

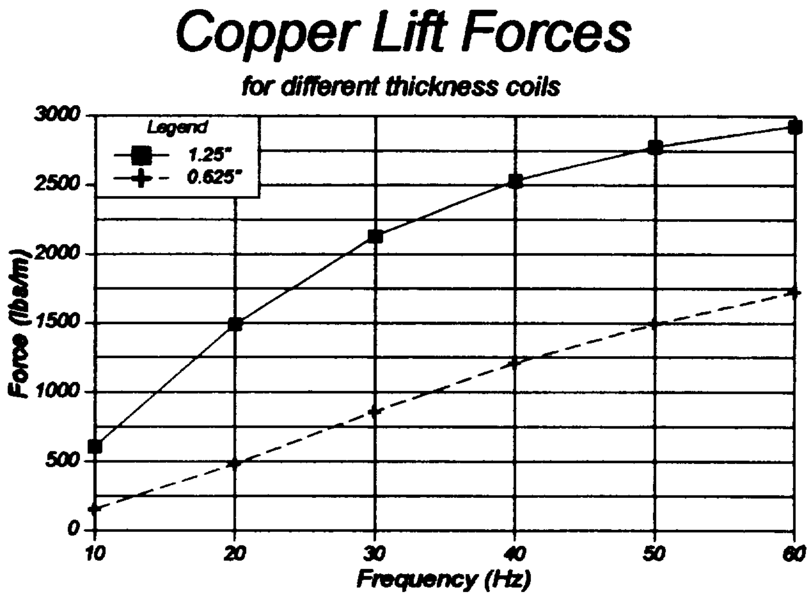


Figure 7 Lift force on two copper coils each with a different thickness.

The ultimate objective is to minimize the cost of the long member, the track. Two coil thicknesses were examined, one having a cross-section of 0.625" by 0.25" (10 turns of #9 wire) and a second having a cross-section of 1.25" by 0.25" (20 turns of #9 wire). By way of underscoring the importance of the larger 1.25" coils over the previous 0.625" coils, Figure 7 displays the different forces expected when 1.25" copper coils are condensed to 0.625" in height. The force reduction results from two issues. First, the timing due to the L/R ratio is such that the currents do not come on opposite in phase to their source. Second, the currents are physically positioned closer to the outer periphery of the field, where the fields are reduced in magnitude.

OPTIMIZATION SETUP

The optimization objective will be to minimize the cost of the magnets and wire in the track as well as the drag at lift off,

$$\mathscr{F} = (\$wire + \$magnets) * drag \tag{1}$$

subject to the constraint that lift $\geq$ weight at lift off. **The system design is dictated by the desired liftoff speed**. Once the desired liftoff is defined, the four objective parameters listed on page 3 are known. The approach adopted is as follows:

1. Predict the forces and induced currents as a function of both offset distance d and frequency f.
2. Fit a complex polynomial involving d and f to the force and induced current.
3. Use a sequential quadratic program to determine the best design topology.

The equivalent frequency seen by the coils is dictated by the product of wave number k and velocity v as

$$kv=\omega \Rightarrow \frac{2\pi}{2x}v = 2\pi f. \tag{2}$$

Where x is the axial distance between coils (see Figure 1). Thus the equivalent frequency is related to the spacing between coils x as

$$f = \frac{v}{2x}. \tag{3}$$

The process begins by fitting the force F per C set and induced current I to a vector of unknowns c_f and c_I such that

$$A\vec{c}_f = \vec{F}/C \tag{4}$$

$$A\vec{c}_I = \vec{I}. \tag{5}$$

The actual current for each is fitted as

$$F=c_1+c_2x+c_3x^2+c_4x^3+c_5d+c_6d^2+c_7xd+c_8x^2d+c_9xd^2+c_{10}(xd)^2+c_{11}x^4. \tag{6}$$

with a similar fit for the induced current I. A total of m=24 trials were investigated, allowing the construction of a matrix equation to determine the coefficients based on the m trials,

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^4 \\ 1 & x_2 & x_2^2 & \cdots & x_2^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_m & x_m^2 & \cdots & x_m^4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{11} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_m \end{bmatrix} \tag{7}$$

Equation (5) was found to accurately track both force and induced current.

THE DESIGN SETUP

The objective function (1) involves 3 parameters which serve as the unknowns - the axial winding displacement x, the offset distance d of the coils with respect to the magnets, and the number of magnet C sets N. The magnet costs, wire costs, and drag at liftoff must be represented in terms of these three parameters. The distance x represents the outside axial distance of the coils. In terms of the average height of the coils (h=7.325") and the average width (x-1.25"), the wire density ρ, the price per pound P, and the length L of the track, the cost of the wire is

$$\$wire = 2_{sides/coil}(h+(x-1.25))*4_{coils/Cset}*0.22*1.25*(2L/x)\rho P*4_{rails/system}. \tag{8}$$

The factor 2 multiplying L accounts for the half pole pitch placed coil. Each of the 12" magnets employed cost $10,000. To allow room for the placement of the compensation winding, the magnets must be 0.75" shorter than the pole pitch distance x. Thus the cost of N magnets for vehicles is

$$\$magnets = N*\$10,000*Veh*(x-0.75)/12. \tag{9}$$

From the induced current in each coil, it is possible to compute the drag force in terms of the average B field in the air gap. This drag of course depends on the relative offset distance d of the coils with respect to the magnets. For the 4 sets of coils (2 sets being displaced a half pole pitch axially), the drag force in pounds is

$$Drag = N*8*d*0.0254_{m/in}*I*B/4.48_{N/lb}. \tag{10}$$

Each brush set for a 12" section of magnet is estimated to weigh 100# with an additional 50# being needed to account for the weight of the air cylinder controlling the actuators, yielding a brush weight

$$brush = 100\,x/12+50. \tag{11}$$

Each magnet C set weighs approximately 400 lbs with an additional 150 lbs required to account for the support struts. In terms of the burden, the vehicle weight is

$$wt = burden+N*[650*(x-0.75)/12+50]. \tag{12}$$

The burden for the test sled is only 640 lbs, whereas the burden for a typical people mover is 27,000 lbs. The expected burden for the fully deployed 90' long high speed cruiser is 43,000 lbs. The lift to drag ratio l/d is

$$l/D = wt/drag. \tag{13}$$

The cost per mile for a length L of track is

$$cost/mile = (\$magnets+\$wire)/L*5280_{ft/mile}*12_{in/ft}. \tag{14}$$

RESULTS

Table I System Design with a 700lb load/C set

L	Mat.	spd	# veh	x	d	N	drag /1000	l/D	wt /1000	$wire $*10^6$	$mag $*10^6$	cost/ mile $*10^6$	I kA	F/C /1000
4000'	Cu	30	6	12	1	51	4.35	14	60.7	1.13	2.87	5.28	3.42	1.19
4000'	Al	35	6	12.3	1	38.2	3.71	14.3	52.86	1.12	2.21	4.4	3.87	1.38
4000'	Al	40	2	12	1	145	8.83	13.9	123.1	0.331	2.73	4.04	2.43	0.846
10 miles	Al	40	2	"	"	"	"	"	"	4.36	"	0.709	"	"
4000"	Cu	40	2	14.8	0.906	35.6	2.74	20.4	55.9	1.06	0.834	2.50	3.4	1.57
10 miles	Cu	40	2	17.12	0.82	45.9	2.55	27.4	70.0	13.42	1.25	1.47	2.70	1.53
4000'	Al	40	6	12	1	145	8.83	13.9	123.1	0.33	8.21	11.26	2.43	0.846
10 miles	Al	40	6	12	1	145	8.83	13.9	123.1	4.36	8.21	1.25	2.43	0.846
4000'	Cu	40	6	13.77	.969	32	2.99	17.1	51.2	1.08	2.09	4.18	3.86	1.60
4000'	Al	80	2	14.7	0.929	29.3	2.55	19.8	50.6	0.31	0.683	1.31	3.76	1.73
10 miles	Al	80	2	16.98	0.795	38.4	2.18	29	62.7	3.93	1.04	0.497	2.85	1.63
4000'	Cu	80	2	26.2	0.77	17.1	1.07	48	51.4	0.924	0.725	2.18	3.26	3.07
10 miles	Cu	80	2	29.3	0.66	23.2	977	65.6	64.1	11.95	1.11	1.31	2.55	2.76

Table I shows the results for a variety of liftoff speeds, coil materials, lengths of track, and number of vehicles for a 4 rail people mover system based 27,000 lb burden system. The design variables are chosen to minimize (1) subject to the constraint that the vehicle lift itself at the specified liftoff speed. Because the winding end turns become comparable to the coil length for x<12", the additional constraint that x ≥ 12" was also enforced. As expected, when the design is asked to lift at low speeds, x is forced to the smallest axial extension in an attempt to drive the effective frequency up. In addition, the vehicle is forced to ride "low in the water" at a large "d"; this of course translates into a low lift/drag ratio, but the system has no choice at these low speeds. The superior L/R time constant of the copper allows it to deliver a solution when none is available for aluminum at low speeds. As the lift threshold rises, the optimization algorithm attempts to pick the number of magnets N to decrease both d and the drag as desired. The following trends are also registered as expected

1. As the length of the track L increases, the algorithm drives the cost of the wire down by extending x.
2. As the number of vehicles increases, the algorithm drives N down to reduce the total expenditure required for the magnets.
3. As x is increased to reduce wire cost, the vehicle will lift with a greater d; the lift/drag ratio is compromised at the expense of track cost reduction.

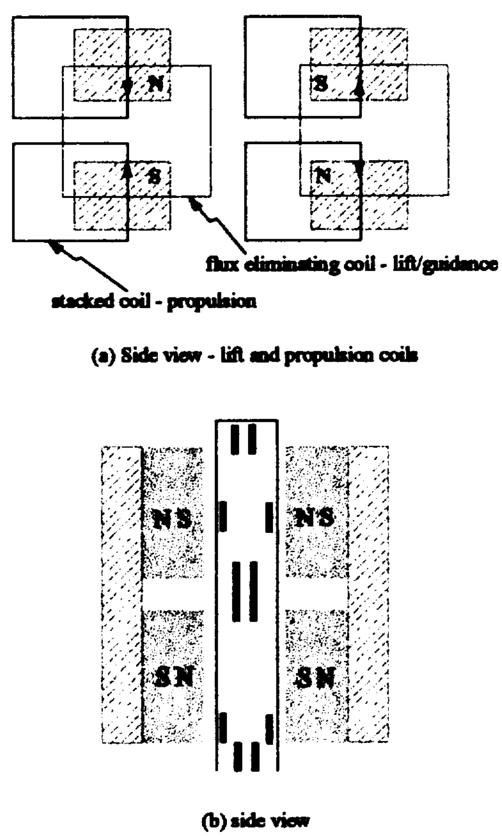


Figure 8 Combination of flux eliminating and stacked coils for lift and propulsion.

SYSTEM STAGING-A 3 STAGE RAIL BUILDUP

It should be apparent that the flux eliminating coil presented in Figure 8 (b) is not useful for realizing propulsion forces. A separate stacked coil is recommended for propulsion with this design. Figure 8 shows the incorporation of stacked coils for propulsion as well. American MAGLEV is however testing a proprietary coil at present which performs all three functions - lift, guidance, and propulsion. The first section of track up to 40 MPH where lift is unnecessary should use only stacked coils centered on the magnets. The centered stacked coil makes the best use of the field for propulsion. The lift coils in the track section between 40 and 80 MPH should be copper, while above 80 MPH they should be aluminum. The stacked coils remain aluminum throughout. With the proprietary single composite coil system being tested at present, the same sequence should be followed - aluminum stacked 0 - 40 MPH, copper composite (40 MPH - 80 MPH), aluminum composite (> 80 MPH).

Table II Distance staging @ 1.5m/s^2 acceleration

Stage	velocity range (m/s)	δ time (s)	distance (m)	distance ft	Accumulate Distance ft
acceleration 0 - 40 MPH	0 - 18	12	108	354	354
St. lift-off copper 40 - 80MPH	18 - 36	12	324	1063	1417
lift-off Al 150MPH	36 - 66.96	20.6	1063	3487	4,904

Table II shows the appropriate distance for each of these stages assuming the vehicle accelerates at a speed of 1.5m/s^2. In this scenario, the stacked coils would be used for the first 108m, then copper composites for the next 324m, and overlapped aluminum composites for the remainder of the track.

Table III Distance staging @ 1 m/s^2 acceleration

Stage	velocity range (m/s)	δ time (s)	distance m	distance ft	Accumulated Distance ft
acceleration 0 - 40MPH	0 - 18	18	162	531	531
1st lift off copper 40 - 80MPH	18 - 36	18	486	1,594	2126
lift off Al 150MPH	36 - 66.96	30.96	1594	5,229	7355 (1.4 miles)

Table III suggests the corresponding lengths if the vehicle accelerates at only $1 m/s^2$.

In such a hybrid system, a different optimization must be enforced, one in which a single coil span distance x is selected which reflects the use of both copper and aluminum overlapped coils. The modified merit function corresponding to (1) is

$$\mathcal{F} = \$wire_{Cu} * drag_{Cu} + \$wire_{Al} * drag_{Al} + \$magnets\sqrt{drag_{Cu} * drag_{Al}} \quad (15)$$

Note that the modified merit function must separately account for the drag during the copper stage $drag_{Cu}$ and the aluminum stage $drag_{Al}$. The dual constraint is that the vehicle lift itself at 40MPH with the copper coils and also at 80MPH using the aluminum coils.

The results of the hybrid optimization are summarized in Table IV

Table IV Hybrid System 10 miles 486m copper @ each end, 4 vertical rails
Cu lift @ 40MPH, Al lift @ 80MPH

$$Minimize[\$wire_{Cu} * drag_{Cu} + \$wire_{Al} * drag_{Al} + mags * \sqrt{drag_{Cu} * drag_{Al}}]$$

Row	Burden	L	veh	x	d_{Cu}	d_{Al}	N	$\frac{Drag_{Cu}}{1000}$	$\frac{Drag_{Al}}{1000}$	ℓ/D_{Cu}
1	27000	10 mile	6	15.84	0.94	0.82	35.1	2.72	2.25	21.1
2	43000	10 mile	6	15.35	0.96	0.84	53.1	4.46	3.7	24.3
3	*640	1.4miles	1	13.99	0.46	0.42	4	0.087	0.077	43

Row	wt/ 1000	ℓ/D_{Al}	wire Cu	wire Al	I_{Cu}	I_{Al}	cost/ mile	F/C set 1000
1	57.4	25.4	0.827	3.76	3.3	3.14	0.724	1.64
	87.7	23.7	0.835	3.8	3.48	3.31	0.851	1.65
3	3.77	48.4	0.429	.165	1.88	1.84	0.456	0.927

* 2 rails only

The 3 rows of the lower table inset correspond to those in the upper one. The weight burden for the third row corresponds to that for the test sled given the weight associated with each magnet as dictated by (12). This would suggest that the ideal spacing x of the coils for the test track is 13.99", but is closer to 16" for a 10 mile people mover having a burden of 27,000 lbs.

SYSTEM SENSITIVITY

To what extent are the results sensitive to the weight load associated with each magnet C set? As suggested by (12), the weight associated with each 12" C set is 386-lbs self weight, 164-lbs support structure, and 150-lbs of brush hardware for a total load of 700 lbs. By more efficient support of both the brush mechanisms and perhaps carbon composite struts, this weight could be substantially reduced. If 150-lbs could be shed from this figure, the modified vehicle weight would become

$$wt = burden + N^*[550^*(x-0.75)/12]. \tag{16}$$

Table V System design with a 550lb load/Cset

L	Mat.	speed	#veh	x	d	N	drag/1000	l/D	wt/1000	$wire *10^6	$mag *10^6	cost/mile *10^6	I kA	F/C 1000
4000ft	Cu	30	6	12.0	.943	42.7	3.26	15.04	49.1	1.13	2.41	4.67	3.24	1.15
4000ft	Al	35	6	12.0	1	153	7.6	13.9	105.9	0.33	8.61	11.8	1.99	0.69
4000ft	Cu	35	6	12.8	0.87	36.7	2.65	17.8	47.23	1.11	2.21	4.37	3.31	1.29
4000ft	Al	40	2	12.0	1	82	4973	14	69.3	0.33	154	2.47	2.43	0.85
10 mile	Al	40	2	12.0	0.995	83	4963	14.1	69,6	4.36	1.55	0.59	2.42	0.84
4000ft	Cu	40	2	15.4	0.778	36	1.964	26	50.9	1.05	0.867	2.53	2.84	1.43
10 mile	Cu	40	2	26	0.932	45	1,793	45	81.1	12.1	1.97	1.41	1.71	1.80
4000ft	Al	40	6	12.0	1	82.0	4973	13.9	69.3	0.33	4.62	6.53	2.43	0.846
10 mile	Al	40	6	12.0	1	82.0	4973	13.9	69.3	4.36	4.62	0.90	2.93	0.846
4000ft	Cu	40	6	14.2	0.844	31.4	2180	21.3	46.7	1.07	2.116	4.21	3.3	1.48
4000ft	Al	80	2	15.2	0.815	28.5	1,880	24.2	45.9	0.31	0.688	1.31	324	1.61
10 mile	Al	80	2	17.5	0.674	39.3	1,562	36.6	57.2	3.91	1.1	0.50	2.36	1.45
4000ft	Cu	80	2	27	0.67	17.5	0.82	58.7	48.1	0.92	0.767	2.22	2.79	2.79
10 mile	Cu	80	2	30.5	0.56	25	0.73	83	60.8	11.9	1.23	1.31	2.10	2.45

Table V shows how the design parameters change if such a reduction could be realized. The lift to drag ratios are greatly enhanced and the overall weight of the vehicle is reduced by 10,000-lbs. The advantages underscore the importance of expending every effort to keep the support and brush weight burden per magnet C set to a minimum. The column marked I indicates the current induced in any one (1 turn) coil of the overlapped coil.

CONCLUSIONS

The design of a complete MAGLEV system is indeed somewhat complicated, involving most critically the desired threshold speed as an input parameter. The directive to minimize cost is of course integral to the design. Towards this end a constrained optimization using sequential quadratic programming is employed to great benefit for minimizing an energy functional. A high order polynomial fit easily obtained using QR decomposition, proves to have a smoothly differentiable function to be operated on by the optimization program. The end result is a process that allows for the characterization of a pole pitch of the winding in the track and the cost for the entire system.