DESIGN AND ANALYSIS OF AN ELECTROMAGNETIC THRUST BEARING

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SUMMARY

A double-acting electromagnetic thrust bearing is normally used to counter the axial loads in many rotating machines that employ magnetic bearings. It essentially consists of an actuator and drive electronics. Existing thrust bearing design programs are based on several assumptions. These assumptions, however, are often violated in practice. For example, no distinction is made between maximum external loads and maximum bearing forces, which are assumed to be identical. Furthermore, it is assumed that the maximum flux density in the air gap occurs at the nominal gap position of the thrust runner. The purpose of this paper is to present a clear theoretical basis for the design of the electromagnetic thrust bearing which obviates such assumptions.

INTRODUCTION

The basic design analysis of an electromagnetic thrust bearing is well known (1, 2, 3, 4). In these analyses, the maximum bearing force is generally assumed to equal the external load. However, this assumption ignores the inertia forces due to the vibrating shaft. These inertial forces may be larger than the external loads under certain situations, thus invalidating the conventional design analysis.

Another assumption normally used is that the maximum force requirement occurs at the nominal or equal gap position, when the thrust runner is centered between the two stators. This assumption holds only if the maximum movement of the runner is negligibly small compared to the air gap. An adequate design should take the motion of the runner into account when computing the maximum ampere turns required.

A_p single pole face area [m^2]

B_1, B_2 flux density in left and right electromagnet respectively [T]

c_{brg} bearing damping coefficient [N-s/m]

k_{brg} bearing stiffness coefficient [N/m]

m total moving mass (shaft and runner) [kg]

f_c coil copper factor

f_p coil packing factor

F_1, F_2 force on runner from left and right electromagnet respectively [N]

F_{brg} net bearing force on runner [N]

F_{dyn} amplitude of dynamic component of external force on shaft [N]

F_{ext} external force on shaft [N]

F_{stat} static component of external force on shaft [N]

g_0 mean air gap [m]

g_1, g_2 air gap for left and right electromagnet respectively [m]

h_s radial height of slot [m]

I_1, I_2 current in the left and right electromagnet coil respectively [A]

I_b bias current in coil [A]

I_{dyn} amplitude of the dynamic component of coil current [A]

I_{max} maximum coil current [A]

I_{stat} static component of coil current [A]

J maximum coil current density [A/m^2]

l_s axial length of slot [m]

N number of turns in coil

R resistance of coil [Ω]

t time [s]

t_0 instant when bearing force is maximum [s]

V power supply voltage [V]

x axial displacement of runner [m]

x_{dyn} amplitude of axial displacement of runner [m]

α correction factor accounting for flux leakage and fringing

ϕ phase lag of runner displacement with respect to dynamic component of external force [rad]

Φ Flux in magnetic circuit [Weber]

μ_0 permeability of free space [$4\pi \times 10^{-7}$ N/A^2]

ω frequency of dynamic external force [rad/s]

ψ phase lag of coil current with respect to dynamic component of external force [rad]

THEORY

A double-acting electromagnetic thrust bearing, Fig. 1, is made up of two electromagnets (stators), one on each side of the thrust runner (rotor) and separated by an air gap. Such a system is inherently unstable. It is stabilized by sensing axial position and using this information to control the current in the stators via an electronic controller. The controller is typically of the proportional-integral-derivative (PID) type, and may be implemented in analog or digital form.

Force Capability

Each of the stators applies an attractive force on the runner. The free-body diagram of the thrust runner is shown in Fig. 2. For dynamic equilibrium, the equation of motion is

$$m\ddot{x} = F_{ext} - F_{brg} \tag{1}$$

where the external force and the resulting motion are

$$F_{ext} = F_{stat} + F_{dyn} \sin \omega t \tag{2}$$

$$x = x_{dyn} \sin(\omega t - \phi) \tag{3}$$

On substitution, we get

$$F_{brg} = F_{ext} - m\ddot{x} = F_{stat} + F_{dyn} \sin \omega t + m\omega^2 x_{dyn} \sin(\omega t - \phi) \tag{4}$$

The bearing has to be designed to accommodate the vector sum of the external force *and* the inertia force. The phase lag determines whether the inertia force adds to or subtracts from the external dynamic force, Fig. 3. For subcritical operation ($\phi < \pi/2$), neglecting the inertia force can result in an undersized thrust bearing that "bottoms out" during operation. For supercritical operation ($\phi > \pi/2$), the inertia force fights the external dynamic force, and ignoring this can result in an overdesigned bearing. If the phase lag is small

$$\phi << \pi/2 \tag{5}$$

the bearing must be designed for a load capacity of

$$F_{brg,max} = F_{stat} + F_{dyn} + m\omega^2 x_{dyn} \tag{6}$$

This force requirement occurs at $\omega t = \pi/2$. Since the two electromagnets act in pull-pull mode, F_1 and

F_2 are always positive. The net bearing force is the difference of the two:

$$F_{brg} = F_1 - F_2 \tag{7}$$

The bearing force is at a maximum when one electromagnet is applying its maximum pull force and the other is turned off. Thus, for the double-acting arrangement, we must have

$$F_{1,max} = F_{2,max} = \left| F_{brg,max} \right| \tag{8}$$

Each electromagnet must be designed for this load capability. For the special case where $F_{ext} = 0$, the bearing must still be capable of withstanding the inertia force at the natural frequency of axial vibration of the system. System design is then dictated by desired transient response, characterized by overshoot and settling time (5).

Ampere Turns

The thrust bearing consists of two electromagnetically biased and excited magnetic circuits with two air gaps per circuit. Each circuit has an outer and an inner pole in the stator, and back iron to complete the flux path in the stator, Fig. 1. The flux lines traverse the air gaps and complete their path in the runner. Both the runner and the stator are made of magnetically permeable material. The pole face areas are typically made equal in order to ensure uniform flux density in the stator.

The bearing force is related to the flux densities in the two electromagnets by

$$F_{brg} = \frac{A_p}{\mu_0} \left(B_1^2 - B_2^2 \right) \tag{9}$$

while the two air gaps are given by

$$\begin{aligned} g_1 &= g_0 + x \\ g_2 &= g_0 - x \end{aligned} \tag{10}$$

The corresponding currents in the two electromagnets are

$$\begin{aligned} I_1 &= I_b + I_{stat} + I_{dyn} \sin(\omega t - \psi) \\ I_2 &= I_b - I_{stat} - I_{dyn} \sin(\omega t - \psi) \end{aligned} \tag{11}$$

The maximum external force, $F_{ext,max}$ occurs when $\omega t = \pi/2$. This is given by

$$F_{ext,max} = F_{ext} \big|_{\omega t = \pi/2} = F_{stat} + F_{dyn} \tag{12}$$

However, F_{brg} may not reach its maximum at the same instant due to the effect of rotor inertia. Let the bearing force reach a maximum at some time t_0. At this time t_0, then, we must also have, from (9)

$$\left(B_1^2 - B_2^2\right)\Big|_{t=T} = \left(B_1^2 - B_2^2\right)\Big|_{max} \tag{13}$$

Since

$$B_1^2 \geq 0, \qquad B_2^2 \geq 0 \tag{14}$$

the difference reaches a maximum only when $B_2 = 0$ and B_1 is at a maximum, or $B_1 = 0$ and B_2 is at a maximum. Let's consider the former case:

$$F_{brg,max} = \frac{A_p}{\mu_0}\left(B_{1,max}^2 - 0\right) \tag{15}$$

$$B_2 = 0 \quad \Rightarrow \quad I_2 = 0 \quad \Rightarrow \quad I_{stat} + I_{dyn}\sin(\omega T - \psi) = I_b = I_{max}/2 \tag{16}$$

$$I_1 = I_b + I_{stat} + I_{dyn}\sin(\omega T - \psi) = I_b + I_b = I_{max} \tag{17}$$

$$B_{1,max} = \left(\frac{\mu_0 N}{2\alpha}\right)\frac{I_1}{g_1}\Bigg|_{t=T} = \left(\frac{\mu_0 N}{2\alpha}\right)\frac{I_{max}}{g_0 + x_{dyn}\sin(\omega T - \phi)} \tag{18}$$

We can rearrange (18) to determine the $(NI)_{max}$ required to produce the magnetic flux density, $B_{1,max}$:

$$(NI)_{max} = \alpha\frac{2B_{max}}{\mu_0}\left[g_0 + x_{dyn}\sin(\omega T - \phi)\right] \tag{19}$$

When the material operates up to saturation, $B_{max} = B_{sat}$, and $(NI)_{max}$ can be calculated, provided $t0$ is known. The correction factor α may be determined using the techniques discussed in (2). The pole face area required may be determined using Eqn. (15) above.

The time t_0 when the bearing force developed is maximum can be determined analytically, using Eqn. (4). It is given by

$$T = (\pi - \delta)/\omega \tag{20}$$

Here, the angle δ can be calculated from

$$\delta = \tan^{-1}\left(\frac{F_{dyn} + m\omega^2 x_{dyn}\cos\phi}{m\omega^2 x_{dyn}\sin\phi}\right) \tag{21}$$

When the small phase lag condition of Eqn. (5) holds, the maximum force occurs at $x \approx x_{dyn}$. This

implies that the designer must ensure sufficient ampere turns to saturate the magnetic material of the left electromagnet for its maximum gap position of the runner. In general, however, the phase angle ϕ is not known beforehand, since the bearing design affects the system dynamics. Thus, an iterative process must be employed or bearing stiffness and damping parameters assumed in order to determine this phase:

$$F_{brg} = c_{brg}\dot{x} + k_{brg}x \tag{22}$$

For a given dynamic to static load ratio and a required vibration to gap ratio, the required stiffness may be calculated using the methodology outlined in [6].

Number of Turns and Maximum Current

Assuming a supply voltage v driving the coil of the left electromagnet, we must have

$$v = N\frac{d\Phi}{dt} + I_1 R \tag{23}$$

The resistive load is typically small compared to the inductive load, and may be neglected. Then,

$$\Phi = \frac{1}{N}\int v\,dt = \frac{1}{\omega N}V_{max}\sin\omega t \tag{24}$$

assuming a sinusoidal supply voltage of the form $v = V_{max}\cos\omega t$ [7]. The maximum flux level is therefore

$$\Phi_{max} = B_{max}A_p = \frac{V_{max}}{\omega N} \tag{25}$$

The power supply voltage required is often determined by force slew rate requirements [8]. However, V_{max} is limited by available power supplies that operate at the frequency of interest. The number of turns is thus fixed as

$$N = \frac{V_{max}}{\omega A_p B_{max}} \tag{26}$$

This, in conjunction with Eqn. (19), fixes the maximum coil current:

$$I_{max} = \frac{(NI)_{max}}{N} \tag{27}$$

The maximum coil current density is typically fixed at around $J = 5 \times 10^6$ A/m^2, using which the coil wire gage may be obtained

$$A_w = \frac{I_{max}}{f_c J} \qquad (28)$$

The coil copper factor f_c — the ratio of bare copper cross-section to wire cross-section — is typically less than 0.7. The coil occupies a slot of axial length l_s and radial height h_s, so that the slot cross-section is

$$A_c = l_s h_s = \frac{(NI)_{max}}{f_c f_p J} \qquad (29)$$

EXAMPLE — HIGH SPEED FLYWHEEL

Let us consider a flywheel of mass 1 kg (2.2 lb) rotating at 6283 rad/s (60,000 RPM) and supported on magnetic thrust bearings. Let the magnitude of the dynamic external force be 445 N (100 lbf), so that

$$F_{ext} = 445 \sin 6283t$$

It is desired that the maximum excursion of the flywheel under this loading be limited to 2.5×10^{-5} m (1 mils). So, the runner motion is

$$x = 5 \times 10^{-5} \sin(6283t - \phi)$$

If ϕ satisfies Eqn. (5), we must design for

$$\left| F_{1,max} \right| = \left| F_{2,max} \right| = \left| F_{brg,max} \right| = 445 \, \text{N} + 1974 \, \text{N} = 2419 \, \text{N}$$

as required by Eqn. (6). It is evident that a bearing designed to withstand only the 445 N of dynamic external force would be inadequate for this system.

If the mean air gap is $g_0 = 2.5 \times 10^{-4}$ m (10 mils), $B_{max} = 1.0$ T and Eqn. (19) is applied now, the maximum ampere turns required is

$$(NI)_{max} = 438 \, \text{A - turns}$$

assuming that the correction factor is $\alpha = 1$. This is about 40 A-turns more than for conventional designs, which assume saturation at the equal gap position. Also, using Eqn. (15), the pole-face area required is

$$A_p = 5.59 \times 10^{-4} \text{ m}^2 \quad (0.87 \text{in}^2)$$

We can now determine the number of turns N by applying Eqn. (26) and assuming a supply voltage of, say, 200 V:

$$N = 57 \text{ turns}$$

The maximum current in the coil is

$$I_{max} = 7.7 \text{ A}$$

CONCLUSION

We have shown that two assumptions often employed in designing electromagnetic thrust bearings can lead to inadequate products. The inertia force must be taken into account when sizing the thrust bearing for load capacity. For subcritical operation, this inertia force increases the bearing load capacity requirement. However, if one designs for supercritical operation, the inertia force fights the external force and reduces the load capacity requirement. The bearing size may be *reduced* by taking advantage of this fact.

Moreover, during the vibration of the thrust runner in response to an excitation force, the maximum bearing load capacity is required at a position that is different from the nominal gap position. The implication is that the ampere turns required is more than that for the maximum-force-at-the-nominal-gap assumption. The maximum available voltage supply to energize the electromagnets then determines the maximum coil current and the number of turns.

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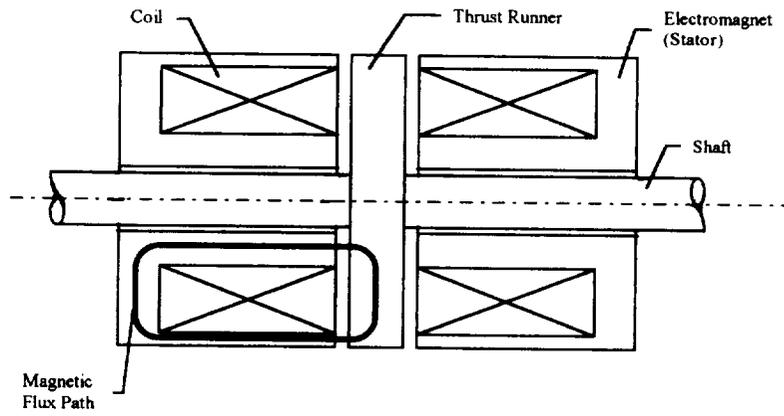
Figure 1. Double-Acting Electromagnetic Thrust Bearing
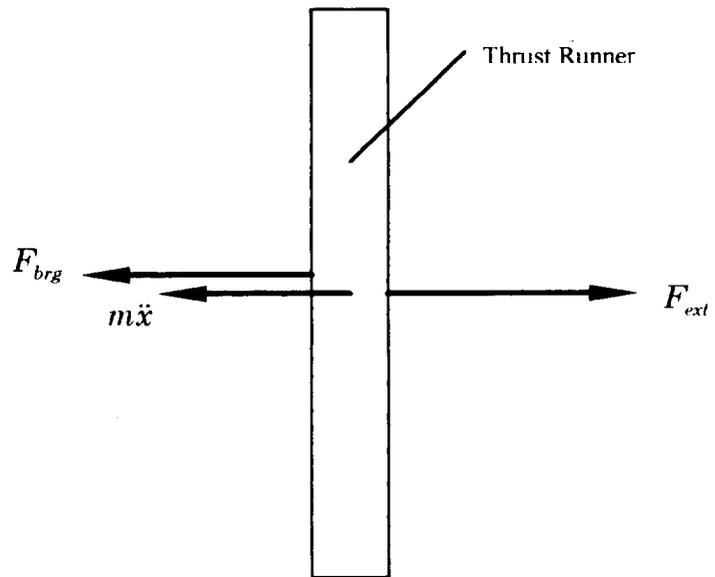
Figure 2. Free-Body Diagram of Thrust Runner for a
Magnetic Thrust Bearing
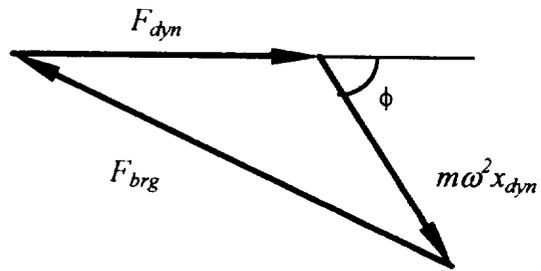
(a) Subcritical Operation
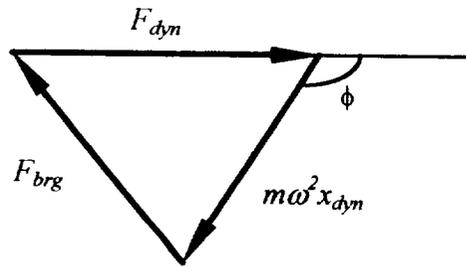
(b) Supercritical Operation

Figure 3. Phasor Diagrams for Motion of Thrust Runner

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