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## INTERNAL WAVES IN CVX

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### ABSTRACT

Near the liquid-vapor critical point, density stratification supports internal gravity waves which affect 1-g viscosity measurements in the CVX (Critical Viscosity of Xenon) experiment. Two internal-wave modes were seen in the horizontal viscometer. The frequencies of the two modes had different temperature dependences: with decreasing temperature, the higher frequency increased monotonically from 0.7 to 2.8 Hz, but the lower frequency varied nonmonotonically, with a maximum of 1.0 Hz at 20 mK above the critical temperature. The measured frequencies agree with independently calculated frequencies to within 15%.

### PURPOSE OF CVX

CVX will measure shear viscosity very close to the liquid-vapor critical point of xenon in microgravity. At critical density and close to the critical temperature  $T_c$ , the viscosity  $\eta$  is predicted to diverge according to

$$\eta \sim (T - T_c)^{-y} ,$$

where the exponent  $y$  has the same value for all fluids. Microgravity will allow this divergence to be observed directly in a pure fluid for the first time. This divergence is similar to those which have been seen in 1-g for static properties such as compressibility or heat capacity. However, two limitations have suppressed 1-g measurements of viscosity to a blunted increase of only 20% above the background viscosity.

### LIMITATIONS ON THE VISCOSITY MEASUREMENT

The first limitation is the fluid itself. The characteristic exponent is small:  $y \simeq 0.04$ . In addition, the amplitude of the viscosity increase is also small: no effect is noticeable above  $T_c + 3$  K (for a fluid with  $T_c \approx 300$  K).

The second limitation is gravity, which has two manifestations in this experiment. The first, well-known, manifestation is the density stratification caused by the enormous compressibility near the critical point. The interesting region

having densities near the critical density  $\rho_c$  narrows to a thin layer near the sample's midplane. Thus, the precision of the measurement of the critical point enhancement of viscosity is limited in Earth's gravity.

The second manifestation, the excitation of internal waves, is the subject of this paper. In 1-g measurements, the stratification supported internal wave modes within the viscometer's bandwidth of operation. Because these internal-wave modes were not included in the model of the viscometer, they limited the measurement's accuracy. The viscometer has been demonstrated to have the required accuracy both in carbon dioxide below the critical density [1] and in xenon at the critical density far above the critical temperature. However, when both the temperature and the density were close to the critical point of xenon, the presence of internal waves prevented such a demonstration.

### INTERNAL WAVES

Internal waves are always possible in the presence of a stratified density profile. See, for example, Lighthill's book [3]. In a closed container, the stratification typical near the critical point can support internal gravity waves whose modes are reminiscent of the "sloshing" modes of a cup of water. Experimentalists have previously remarked on the existence of internal-wave modes near the critical point [4]. Only a few papers have been published on this subject, mostly in Soviet journals [5, 6, 7]. These papers were concerned with exploratory questions such as the existence of gravitational waves or the dispersion relation at the critical temperature  $T_c$ . Thus they could not help in identifying the observed internal-wave modes or in measuring and calculating their frequencies.

### THE CHARACTERISTIC FREQUENCY

The characteristic frequency for internal waves is the Brunt-Väisälä or buoyancy frequency, defined by

$$\omega_{BV} \equiv \left[ \frac{g (d\rho/dz)}{\rho} \right]^{1/2},$$

where  $g$  is the gravitational acceleration, and  $\rho$  and  $d\rho/dz$  are the fluid density and its derivative with respect to height  $z$ . In general,  $\omega_{BV}$  is a local quantity depending on the height  $z$  of the fluid element. A closed container of stratified fluid will have a spectrum of modes, none at a frequency higher than the maximum value of  $\omega_{BV}$  within the container.

Gravity-induced stratification just above the critical point of a pure fluid is a special case of a continuous density profile. Far above the critical point,

$\omega_{BV}$  is much less than 1 Hz and is nearly uniform with height. As the critical point is approached,  $\omega_{BV}$  increases, so that for xenon the frequency  $\omega_{BV}/2\pi$  is approximately 1 Hz at  $T_c + 60$  mK. Closer to  $T_c$ , the height dependence of  $d\rho/dz$  causes  $\omega_{BV}$  also to depend on height. Thus, in the thin region where  $\rho \simeq \rho_c$ ,  $\omega_{BV}$  diverges as  $T_c$  is approached. The resulting distribution of  $\omega_{BV}$  leads to a spectrum of internal-wave modes whose eigenfrequencies do not diverge.

## THE DISPERSION RELATION

The equation of motion for the internal waves in the stratified xenon comes from the inviscid Navier-Stokes equation and the conditions of continuity and incompressibility. (At frequencies of order 1 Hz, the fluid oscillations are adiabatic, and incompressibility is a good approximation.)

The simplest example of internal-wave modes is a small, periodic disturbance of an exponentially stratified fluid, namely one whose unperturbed density profile  $\rho_0(z)$  is

$$\rho_0(z) = \rho_{00}e^{-\alpha z} ,$$

where  $\alpha$  and  $\rho_{00}$  are independent of height  $z$ . The fluid is contained in a rectangular box whose dimensions are given by

$$\begin{aligned} -a/2 &< x < +a/2 \\ -b/2 &< y < +b/2 \\ -L &< z < +L ; \end{aligned}$$

where  $z$  is the vertical direction.

For the exponential density profile, the associated eigenvalue problem can be analytically solved (e.g. see [3]), yielding the eigenfrequencies  $\omega$  given by the dispersion relation

$$\omega^2 = \frac{\alpha g \left[ (j\pi/a)^2 + (k\pi/b)^2 \right]}{(\alpha/2)^2 + \left[ (j\pi/a)^2 + (k\pi/b)^2 + (l\pi/L)^2 \right]} .$$

This dispersion relation has the unusual feature that  $\omega$  decreases upon increasing the vertical index  $l$ .

Xenon's density profile is not exponential near the critical point. However, above  $T_c + 60$  mK, the nearly-linear profile could be approximated by an exponential profile. Thus this model was useful for revealing the symmetry of the internal-wave modes, which was important in deciding which modes coupled to the oscillating screen.

Close to  $T_c$ , the density profile is significantly nonlinear, and numerical solution of the equation of motion is necessary to obtain the dispersion relation from the eigenvalue problem consisting of the fluid's equation of motion and its boundary conditions. In collaboration with applied mathematicians at NIST, this problem was solved for density profiles both above and below  $T_c$ .

## APPARATUS

The viscometer consisted of an overdamped, oscillating screen immersed in the xenon. Changes in viscosity are inferred from changes of the oscillator's transfer function, the frequency-dependent ratio of the oscillator's displacement to its applied torque.

The oscillator was constructed by cutting an  $8 \times 19$  mm rectangle out of a larger piece of delicate nickel screen while leaving attached two wire extensions that formed the torsion fiber. The torsion fibers were attached to a stiff yoke with Pb-Sn solder. The yoke was centered between four electrodes parallel to the screen and separated by a 7.6 mm gap. The complete assembly was sealed into a cylindrical copper cell.

After characterization of the oscillator in vacuum, xenon was loaded into the viscometer cell to within 0.2% of the critical density  $\rho_c$ . After loading, the cell was placed into a thermostat consisting of three independently-controlled, concentric aluminum shells.

The measurements were made with prototype electronics at NIST. A commercial spectrum analyzer generated oscillating source voltages which, after modification, were used to apply torques to the oscillator. The oscillating screen and the fixed electrodes also formed a capacitance bridge that was operated at 10 kHz to detect the screen's displacement. The bridge's signal was detected by a lock-in amplifier, whose output was continuously measured by the spectrum analyzer.

Further details may be found in Reference [2].

## MEASUREMENT TECHNIQUE

The viscometer was designed for ultimate use in the Space Shuttle's microgravity, where the effect of internal waves will be negligible. To verify that internal waves were responsible for the deviations in 1-g, the frequencies of the modes were measured as a function of temperature. The measurements were made with the cell's axis horizontal and the oscillator's torsion axis vertical.

Our measurement technique assumed the internal-wave mode was a massive, high-Q oscillator coupled weakly to the low-mass oscillating screen. To bring the fluid into steady oscillations, the screen was driven at a frequency  $f_{drive}$  for at

least 10 cycles. Then the drive was turned off, and the screen's residual motion was recorded. If  $f_{drive}$  was near an internal-wave frequency, subsequent transient oscillations were visible, and the frequency of the internal wave was assigned after examination of the transient waveform's spectrum.

## COMPARISON

The xenon was contained within a simple cylinder; however, the internal geometry was complicated by the electrodes and the oscillator itself. The calculation of the frequencies of the internal wave modes relied on the approximation of the container's internal geometry as a rectangular box.

The viscometer's internal symmetry restricted the oscillator from coupling to most of the internal-wave modes. This restriction was useful due to the small spacing between modes. For example, even with the three modal indices restricted to only 0, 1 or 2, the frequencies calculated for 9 of the 16 possible modes can fall within a range of only 6%. Without the coupling restriction, the overlap of many modes would have complicated the interpretation of the measurements.

The frequencies computed for the modes with  $(jkl)$  equal to (111) and (112) are plotted in Figure 1. In these computations the density profiles, calculated using the parameters given in T[8], were the actual profiles expected from xenon's equation of state. The calculated and measured frequencies agree to within 15%.

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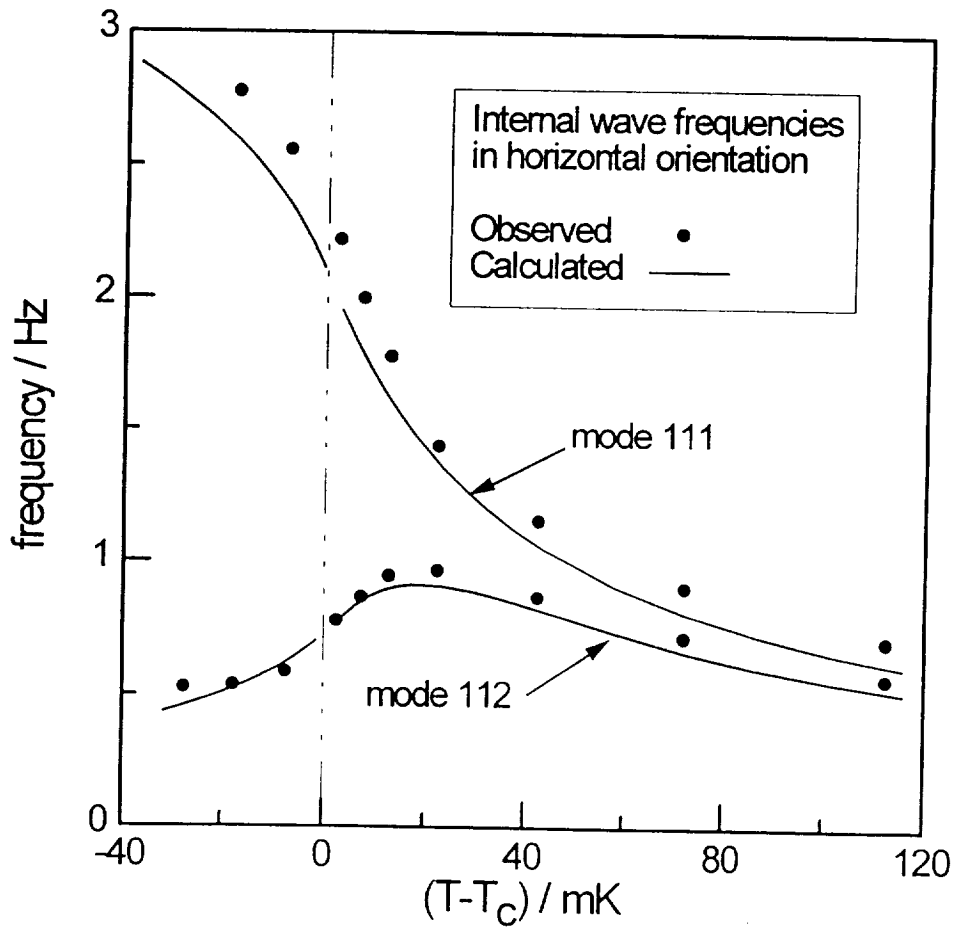


Figure 1. Internal-wave frequencies, measured (circles) and calculated (lines). The cell's axis was horizontal and the screen's torsion axis was vertical, so that the screen's motion was horizontal. The numerical calculations used the actual profiles derived from xenon's equation of state.