# ACOUSTIC BEHAVIOR OF VAPOR BUBBLES 

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#### Abstract

In a microgravity environment vapor bubbles generated at a boiling surface tend to remain near it for a long time. This affects the boiling heat transfer and in particular promotes an early transition to the highly inefficient film boiling regime. This paper describes the physical basis underlying attempts to remove the bubbles by means of pressure radiation forces.


## 1 Introduction

At normal gravity, the effectiveness of boiling as a heat transfer mechanism relies in no small measure on the rapid removal of vapor bubbles from the heated surface. This process has a two-fold benefit, as it both aids in removing latent heat and in promoting microconvective motion near the surface. On the basis of this remark, one would be led to believe that boiling at reduced gravity would be very inefficient. Somewhat surprisingly, at small to moderate heat fluxes, this statement is only partly true as shown by several experiments (Siegel 1967; Clark 1968; Zell et al. 1989; Oka et al. 1992, and others). Two major differences between micro- and normalgravity boiling are however evident: (i) The bubble shape, size, and general dynamics is radically different; (ii) The critical heat flux is reduced severalfold at low gravity. As first shown by Siegel and Keshock (1964), a major heat removal mechanism in low gravity is the fact that a detaching bubble does not go far from the heated surface so that subsequent bubbles feed into it until the bubble leaves. At this point another bubble grows, detaches, is fed by smaller ones, and the cycle repeats. This process compounds with vigorous surface instabilities (Ervin et al. 1992) to produce a substantial heat transfer.

While the hovering of large bubbles near the nucleation site is beneficial at low to moderate heat fluxes, it is also at the root of the observed reduction in critical heat flux, where the heating surface is surrounded by a vapor blanket (Oka et al. 1992; Chung 1994). In order to increase the critical heat flux at low gravity it is therefore desirable to remove bubbles from the heated surface providing a substitute for buoyancy. Electric fields have been used for this purpose (Chung 1994), but it is too early to judge their effectiveness. The same author, Merte, and ourselves are also planning to use acoustic techniques.

The purpose of the present paper is to review the physical framework in which these acoustic bubble-removal techniques may be expected to operate.

The action of acoustic radiation forces on gas - rather than vapor - bubbles is well known (see e.g. Crum and Eller 1970; Crum and Nordling 1972; Crum 1971, 1975; Agrest and Kuzetsov 1972, 1973; Weiser and Apfel 1982; Barmatz and Collas 1985; Trinh and Hsu 1986; Holt and Crum 1992; Lee and Wang 1993a). For example, radiation forces are a major factor in acoustic cavitation where they promote violent translational motion and spatial reorganization of the gas that evolves from the liquid in an intense sound field. These and other aspects of pressure radiation forces have been extensively studied both experimentally and theoretically (see e.g. Yosioka and Kawasima 1955; Eller 1968; Foster et al. 1968; Wu and Du 1990; Löfsted and Putterman 1991; Lee and Wang 1993). In particular, Dr. E. Trinh (JPL) has carried out experiments on the Space Shuttle USML-1 demonstrating the action of these forces on drops and gas bubbles (Marston et al. 1993).

Gas bubbles are attracted or repelled by the pressure antinodes according as to whether they are driven below or above their resonance frequency. Furthermore, in the linear regime, neighboring bubbles repel each other when one is driven above and one below the natural frequency while they attract otherwise.

The resonance frequencies of bubbles thus play a major role for gas bubbles. While the situation may be expected to be similar for vapor bubbles, comparatively less work has been carried out on these systems. At
first sight, it may even come as an unexpected fact that a resonance frequency - that presupposes a restoring force - exists at all. To compound the surprise, it appears that two linear resonances exist in the case of vapor bubbles (Finch and Neppiras 1973; Khabeev 1976; Hsieh 1979; Marston and Greene 1978; Marston1979; Nagiev and Khabeev 1979). The effect of this secondary resonance on pressure radiation forces is unknown.

## 2 Vapor bubble resonances

In a certain sense, the process that provides vapor bubbles with stiffness is very different from that at work in the case of bubbles containing a permanent gas. In the latter case, it is the compression and expansion of the gas that provides a restoring force opposing the deformation. For a vapor bubble, on the other hand, the vapor pressure essentially follows the saturation line and it rises and falls in response to the heating and cooling of the bubble wall due to latent heat effects.

An alternative argument can be made, however, according to which the stiffnesses of gas and vapor bubbles are very similar. Indeed, they both depend on diffusive processes in the liquid, in the former case of mass, and of heat in the latter one. In this view, the critical difference is merely a consequence of the fact that the diffusivity for mass is typically two orders of magnitude smaller than that of heat.

An estimate of the order of magnitude of the fundamental resonance of a vapor bubble can be readily found by the following argument. Consider a vapor bubble the radius $R$ of which decreases by an amount $\Delta R$. This tends to cause the condensation of an amount of vapor (density $\rho_{V}$ ):

$$
\begin{equation*}
\Delta m_{V}=4 \pi R^{2} \rho_{V} \Delta R \tag{1}
\end{equation*}
$$

If the process occurs with a frequency $\omega$, the latent heat $L \Delta m_{V}$ liberated by the condensation increases the temperature of a shell of liquid of thickness $\sim \sqrt{D_{L} / \omega}$, where $D_{L}$ is the liquid thermal diffusivity, by an amount

$$
\begin{equation*}
4 \pi R^{2} \sqrt{\frac{D_{L}}{\omega}} \rho_{L} c_{L} \Delta T=L \Delta m_{V} \tag{2}
\end{equation*}
$$

with $\rho_{L}$ the liquid density and $c_{L}$ the specific heat. This heating of the bubble surface increases the saturation pressure by an amount $\Delta p=\left(d p_{v} / d T\right) \Delta T$, where the derivative is taken along the saturation line. A force tending to resist compression is generated in this way:

$$
\begin{equation*}
F=4 \pi R^{2} \Delta p=-k \Delta R \tag{3}
\end{equation*}
$$

where the following expression for "stiffness parameter" $k$ follows from the previous argument:

$$
\begin{equation*}
k=4 \pi R^{2} \sqrt{\frac{\omega}{D_{L}}} \frac{L \rho_{V}}{c_{L} \rho_{L}} \frac{d p_{V}}{d T}=4 \pi R^{2} \sqrt{\frac{\omega}{D_{L}}} \frac{\left(L \rho_{V}\right)^{2}}{c_{L} \rho_{L}} \tag{4}
\end{equation*}
$$

The Clausius-Clapeyron relation has been used in the second step. The added mass for a sphere in radial motion is given by $M_{A}=4 \pi R^{3} \rho_{L}$, and therefore (4) enables one to estimate the resonance frequency $\omega_{0}$ of the vapor bubble by $\omega_{0}=\sqrt{k / M_{A}}$ or

$$
\begin{equation*}
\omega_{0}^{2}=\sqrt{\frac{\omega}{D_{L}}} \frac{\left(L \rho_{V}\right)^{2}}{R c_{L} \rho_{L}^{2}} \tag{5}
\end{equation*}
$$

If $\omega \neq \omega_{0}$, this relation gives the position of the pole of the response function of $\Delta R(t)$ when the bubble is driven at the frequency $\omega$. By setting $\omega=\omega_{0}$, on the other hand, we find the natural frequency of the bubble as

$$
\begin{equation*}
\omega_{0}^{3} R^{2} \simeq 11.8 \frac{L^{4} \rho_{V}^{4}}{\rho_{L}^{3} c_{L}^{3} T^{2} k_{L}} \tag{6}
\end{equation*}
$$

where $k_{L}$ is the liquid thermal conductivity and a numerical constant has been introduced to account for the approximate nature of the derivation. With this adjustment, the previous argument gives results in reasonable


Figure 1: Graph of the resonance frequencies of vapor bubbles in water at $100^{\circ} \mathrm{C}$ as a function of bubble radius $R$. The dashed lines show the results given by the simplified arguments.
agreement with the more detailed ones of Marston (1979) as shown by the upper pair of lines in Fig. 1 for the case of water at $100^{\circ}$ and 1 atm .

For comparison, it may be recalled that the natural frequency of a gas bubble in adiabatic oscillation at a pressure $P_{\infty}$ is given by

$$
\begin{equation*}
\omega_{0}^{2} R^{2}=\frac{3 \gamma P_{\infty}}{\rho_{L}}, \tag{7}
\end{equation*}
$$

where $\gamma$ is the ratio of specific heats of the gas. This expression - and in particular its dependence on the bubble size - is very different from (6).

Figure 1 shows another remarkable fact, namely that in water, at $100^{\circ}$ and 1 atm , bubbles larger than about $30 \mu \mathrm{~m}$ possess two resonance frequencies. While the higher one is that predicted by the previous argument, the other one is much lower. The mechanics of this second resonance - that is one of the distinctive features of this system - can be described in physical terms as follows. At low frequency, inertia and damping effects are small and can be ignored. The main effects are the restoring force previously described and the surface tension force

$$
\begin{equation*}
-k \Delta R \simeq 4 \pi R^{2} \Delta\left(\frac{2 \sigma}{R}\right) \tag{8}
\end{equation*}
$$

where $\sigma$ is the surface tension coefficient. These two forces tend to oppose each other and, in suitable conditions, they can balance. This circumstance leads to an oscillating system forced by the sound field, but with a very small restoring force. The oscillation amplitude is then evidently large, and this is the second resonance. Proceeding as before, equating (3) and (8), and again adjusting a numerical constant, we find

$$
\begin{equation*}
\omega_{0} R^{4}=2.94 D_{L}\left(\frac{2 \sigma c_{L} \rho_{L} T}{\left(L \rho_{V}\right)^{2}}\right)^{2} \tag{9}
\end{equation*}
$$

This result is compared with the precise one of Marston (1979) in Fig. 1 (lower pair of lines).

## 3 Pressure radiation

In a sound field, an inhomogeneity whose response to the pressure perturbation is different from that of the surrounding fluid is subject to a pressure-radiation force (King 1934; Yosioka and Kawasima 1955; Gor'kov 1962; Nyborg 1967; Barmatz and Collas 1985; Trinh and Hsu 1986; Wu and Du 1990; Löfsted and Putterman 1991).

For the case of a bubble the component of this force due to the monopole (volume pulsation) is much stronger than that due to the dipole (translational oscillation) and is approximately given by (Eller 1968; Wu and Du 1990; Löftsed and Putterman 1991; Lee and Wang 1993)

$$
\begin{equation*}
\mathbf{F}=-\langle V \nabla p\rangle \tag{10}
\end{equation*}
$$

where the angle brackets denote the average over one period of the sound, $V$ is the bubble volume, and the gradient of the total pressure $\nabla p$ is evaluated at the position occupied by the bubble. In a linear approximation, if we write

$$
\begin{equation*}
p=p_{0}-p^{\prime}(\mathbf{x}) \cos \omega t, \quad V=V_{0}[1+\delta \cos (\omega t+\phi)] \tag{11}
\end{equation*}
$$

we find

$$
\begin{equation*}
\mathbf{F}=\frac{1}{2} V_{0} \delta \cos \phi \nabla p^{\prime} \tag{12}
\end{equation*}
$$

In a standing wave, for a bubble driven below resonance, $\cos \phi \geq 0$ and the force is in the direction of the pressure antinode, while the converse occurs above resonance.

The physical basis for this phenomenon is readily explained as follows. A bubble, being lighter than the host liquid, is subject to a buoyancy force in a direction opposite that of the local pressure gradient. Consider a bubble driven below resonance, that expands during the low-pressure phase of the wave and compresses during the high pressure phase. When it is in the expanded state, the acoustic pressure is negative and the bubble migrates toward the pressure minimum, i.e. the acoustic antinode. When the bubble is compressed, the acoustic pressure is positive and the bubble is driven towards the pressure node. Since, however, the force is proportional to the bubble volume, the migration during the antinode in the expanded state is stronger than the one in the opposite direction during the high-pressure phase, and the net effect is a drift toward the pressure antinode. A similar argument applied to a bubble driven above resonance shows that the drift is toward the pressure node in this case.

One can appeal to this intuitive argument to deduce the bubble behavior near the secondary resonance. Below this frequency, the pressure stiffness force (which increases with frequency, see Eq. 4) is smaller than the surface tension force, which implies that the bubble compresses during the expansion half-cycle of the sound and will therefore be driven toward a pressure node.

These considerations are based on linear theory. A rigorous analysis of pressure forces in the case of nonlinear oscillations is not available. On the basis of an approximate model, we have found that, for gas bubbles, the force can have a sign opposite to that expected on the basis of the linear theory in the neighborhood of a nonlinear resonance (Oguz and Prosperetti 1990). Whether this is indeed so, and whether the result also applies to vapor bubbles, is at present unkonwn.

## 4 Acoustic forces in boiling

The physical principles described in the previous sections indicate that it may be possible to enhance the removal of boiling bubbles from boiling surfaces under microgravity conditions by acoustic methods. A likely sequence of events may be the following.

Consider a standing sound wave in the presence of a heated boundary on which boiling takes place. Acoustically this boundary will behave nearly rigidly and therefore it will give rise to a pressure antinode in its
neighborhood. Consider then a growing vapor bubble. Initially it is small and its resonance frequency correspondingly high (Eq. 6), above that of the sound field. The pressure radiation force tends therefore to keep the bubble near the wall. When the bubble grows past the size such that it is resonant at the driving sound frequency, however, the radiation force changes sign and the bubble will tend to be pushed away from the wall toward a node of the sound field. Since the closest node will be at a distance of the order of $1 / 4$ of the wavelength from the wall, frequencies between 2 and 20 kHz , in water, would tend to remove the bubble at distances between $\sim 2$ and 20 cm from the wall. This is far enough that even a low-speed flow will be sufficient to carry the bubble away.

The previous chain of events can be influenced - positively or adversely - by acoustic streaming that could be a significant factor at the higher frequencies. Such flows have been extensively analyzed by Nyborg and other researchers (Nyborg 1965, Zarembo 1971, Lighthill 1978; Qi 1993), although Rayleigh - once again - was an early investigator of the phenomenon (Rayleigh 1898).

Another factor, the importance of which it is difficult to quantify a priori, is the effect of the "image" bubble. If the wall is acoustically rigid, the image bubble pulsates in phase with the real bubble and would tend therefore to attract it working against the primary pressure force of the sound field.

These considerations are based on a somewhat idealized model of the actual phenomena. The entire situation could be considerably more complex due to nonlinear effects, bubble distortion, and gas diffusion.

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