

Dynamics of Granular Materials

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Granular materials exhibit a rich variety of dynamical behavior, much of which is poorly understood. Fractal-like stress chains, convection, a variety of wave dynamics, including waves which resemble capillary waves, $1/f$ noise, and fractional Brownian motion provide examples. Work beginning at Duke will focus on gravity driven convection, mixing and gravitational collapse. Although granular materials consist of collections of interacting particles, there are important differences between the dynamics of a collections of grains and the dynamics of a collections of molecules. In particular, the ergodic hypothesis is generally invalid for granular materials, so that ordinary statistical physics does not apply. In the absence of a steady energy input, granular materials undergo a rapid collapse which is strongly influenced by the presence of gravity. Fluctuations on laboratory scales in such quantities as the stress can be very large—as much as an order of magnitude greater than the mean.

I. Introduction

In this paper, I briefly given an overview of important aspects of granular flows. I then focus on recent experiments on gravity-driven convective flows. I conclude by indicating future directions.

Granular materials¹⁻³ are collections of macroscopic particles or grains typically having inelastic interactions and often surrounded by a fluid, such as air or water. The interactions between granular materials are governed chiefly by the local elastic and frictional forces between particles, or between particles and a wall. The surrounding fluid may also play an important role in the dynamics of the system. For instance, when granular materials are shaken, then can “convect” in a manner which is reminiscent of ordinary convection, and the surrounding gas can profoundly affect the nature of the flow. Although models for granular materials are often predicated on the assumption that the particles are spheres, in many practical situations, the particles are decidedly nonspherical. This means that both the local and collective properties of granular dynamics will have features associated with particle shape.

One might expect that granular materials behave like a gas or fluid, but on a larger scale. This expectation is usually not met for a variety of reasons. Granular interactions are inelastic. If a system starts with with nonzero kinetic energy, it very rapidly loses that energy— in a finite length of time. In the process, clustering occurs, and the system becomes inhomogenous.⁴ Here, gravity almost surely plays a key role—causing collapse in a finite time even for a single particle bouncing on a smooth surface. At this point, it is very difficult to sort out the role of gravity from clustering effects which would occur in the absence of gravity.

Non-transient granular flow requires the constant input of energy, from gravity, shaking, or other sources. Often, the ergodic hypothesis of statistical mechanics fails: the system comes nowhere near sampling the possible microstates for given macroscopic parameters. One of the complications of many granular flows is that part of the system is in a frozen state

while nearby regions are moving. Finally, the interaction between grains and a boundary is typically very different for a granular material versus a conventional viscous fluid. In the latter case, the appropriate boundary condition at the wall is that of "sticking"—the fluid is at rest relative to the wall. But, there is no such condition for granular materials—grains can and do slip along a wall. In addition, it is possible to transfer stresses from deep within a granular material to the walls through friction and the mechanism of stress chains. A dramatic consequence of this is that columns of granular materials do not demonstrate hydrostatic head effects, such as those seen in a column of water.

Models of granular dynamics typically try to capture one of two extremes. In one extreme, the grains are compacted and remain in enduring contact with each other and the walls. In the other extreme, the grains move about rapidly and collide randomly like the molecules of a gas. The experiments described here pertain chiefly to the second regime in which energy is provided through shaking.

There are also many technical applications of granular materials, ranging from the commonplace transport of coal and grain to the high-tech handling of pharmaceutical powders, fluidized beds, and the preparation of sinters for advanced materials. The total cost involved in the handling of these materials is enormous.⁵ A modest improvement in our understanding of granular flow properties could lead to significant savings.

The current understanding of the statistical properties and dynamics of these materials lags far behind the understanding of the dynamics of conventional fluids. Current models, although useful and sophisticated, all show weaknesses which invite alternative models. In no case is there a model which is as firmly established as the Navier-Stokes equations of Newtonian fluid mechanics.

II. Shaken Granular Materials

Work beginning at Duke will focus on shaken granular materials, on mixing of these materials, and on gravitational collapse. These aspects of granular flows are all highly sensitive to gravitational fields. Indeed, various aspects of these phenomena are so dominated by the presence of gravity that only in a very low- g environment will it be possible to sort out the different aspects of the relevant physics.

Because the work at Duke is only beginning, I will chiefly note here some of the basic phenomena involved in shaken granular materials. I will also note a novel coarsening effect which we have recently discovered in the Duke lab. I will then comment briefly on future directions of study.

The observation of granular convection can be dated at least to the time of Faraday, who noticed organized motion of powders sprinkled on vibrating plates. Broadly, in a gravitational field, g , grains subject to vertical vibrations which have accelerations exceeding g , undergo a kind of large scale convective flow, in which a heap may also form. However, the qualitative nature of these flows can depend on several parameters, and until recently, the relative importance of these parameters was not clear. Indeed, several important control parameters for these flows were only recently identified. Several groups,^{6,8,7,9-11} including the one at Duke, have been recently involved in characterizing granular convection. In general, the detailed mechanisms responsible for granular convection are only partially clarified, and much work remains before we can say that this system is understood.

The recent work at Duke has been focused on mapping out a “phase diagram” for granular convection. This means identifying and characterizing important dynamical states, followed by determining the relevant control parameters, and the ranges of these parameters over which the states occur. For the present discussion, I assume that the material is contained in a box of width W , height, $H \gg W$, and length $L \gg W$. In most of the Duke experiments, the shaker has an annular geometry, for reasons having to do with the wall friction. The material generally cannot fill the container, since otherwise, no convection occurs; hence, the actual fill height of the container is $h < H$. The grains are assumed to have a typical grain diameter, d . In general, the grains need not be spherical; indeed grain “roughness”, can play a key role in the overall convective flow, although this is not generally appreciated. In most experiments, the grains are surrounded by air at 1Atm , but in general, the gas pressure is P . By assumption, the containing box is shaken in a purely vertical direction with a sinusoidal displacement, $z = A \cos(\omega t)$. One of the most important control parameters is then the dimensionless peak acceleration $\Gamma = A\omega^2/g$. In general, granular convection can only occur for $\Gamma > 1$; for smaller Γ 's, the material will relax slowly to a uniform height.

Two primary mechanisms have been proposed for the origins of convection under uniform shaking (The case of nonuniform shaking is clearly different.) These mechanisms are friction with the sidewalls, and lift effects associated with the surrounding gas. Recent molecular dynamics (MD) studies and experiments have helped to clarify the role of friction. Roughly, the idea is that when the granular material is shaken, it exists in both a compacted state and a dilated state during different phases of each shake. Typically, the compacted state occurs when the material is being accelerated upward by the shaker, whereas the dilated state occurs as the shaker accelerates downward. It is during the dilated stage that the grains are most free to move. The expectation is that friction with the vertical walls will then drag the outermost grains down more strongly than the inner grains, resulting in a convection loop which circulates downward at the vertical sidewalls and upward in the interior. This is indeed the case for smooth spherical particles, but curiously, not the case for rough particles of comparable size. In the rough case, circulation consists of an upward motion of the grains towards the top of a heap, followed by strong downward avalanching along the slope of this heap. And, the frictional convection mechanism does not suggest the formation of a significant heap. This heap is, nevertheless, a common feature of the flow.

The surrounding gas pressure can also play a significant role, particularly if the particles have $d \leq 1\text{mm}$. Recent experiments at Duke⁹ in which the pressure is carefully controlled over $0 \leq P \leq 1\text{Atm}$, have shown that the heaping/convection is strongly suppressed when P falls below about 10Torr . The Duke observations settled an ongoing dispute^{6,7} about the role of gas pressure on granular convection. The physical mechanisms which lead to the reduction of heaping/convection at 10Torr are still under study at Duke. Gas effects can also be very dramatic for relatively small d and large Γ , where a bubbling instability occurs.⁹

Interestingly, the height h of the granular layer is also important in determining the state of shaken granular materials. If h , is small, say less than 1 cm, the convective flow does not occur. Rather, there occur a variety of different parametric wave states.^{7,11,9} If the layer is tall enough that the convection and heaping occurs, then these parametric states still likely occur. However, a different kind of wavy instability, discovered at Duke, dominates. These are traveling waves which propagate up the slope of the heap.

A recent aspect of work at Duke has been studies to characterize the fluctuations for shaken granular materials. This is interesting because it may be possible to relate these fluctuations to kinetic theory models,¹² in which the fluctuations are characterized by a granular temperature—a quantity which is analogous to, but not to be confused with, the ordinary temperature for thermodynamic systems.

Another issue of interest is the pattern forming mechanism which leads to the formation of a single heap for granular convection. We have found that this process has two apparent aspects. In some cases—i.e. for certain particle sizes, shaker amplitudes, etc. the evolution of the heaping/convection state occurs via the relatively early formation of only a few (one or two, typically) large heaps which then merge into a single heap. However, we have recently found that in some circumstances, the initial instability is to a relatively small wavelength spatial oscillation which then coarsens, so that in the end there is only a single heap. An example of this novel state is given in Figure 1.

III. Conclusions and Directions

The discussion above gives a brief overview of some of the key phenomena associated with granular convection. This system is particularly relevant here because of the dominance of gravity. It seems clear, however, that the availability of a low gravity environment would lead to significant new insights into granular dynamics. In particular, shaken granular systems would appear more like granular “gases”, and convection might or might not occur. In particular, low-g access would mean that we could test recent theories of inelastic collapse which are not easily tested in an earth-bound environment. In addition, granular materials also exhibit interesting mixing and segregation phenomena in a gravitational field. Segregation by size can occur if there is a distribution of grain sizes.^{13,14,10,17} Typically, larger particles rise to the top, even if they are heavier than the smaller particles. Important mechanisms for segregation include geometric effects¹⁶ (small particles fall more easily into the “cracks”) and convection.^{10,15} This kind of segregation can be either useful or detrimental in commercial applications. A particularly interesting prospect would be the study of size effects in a low-g environment.

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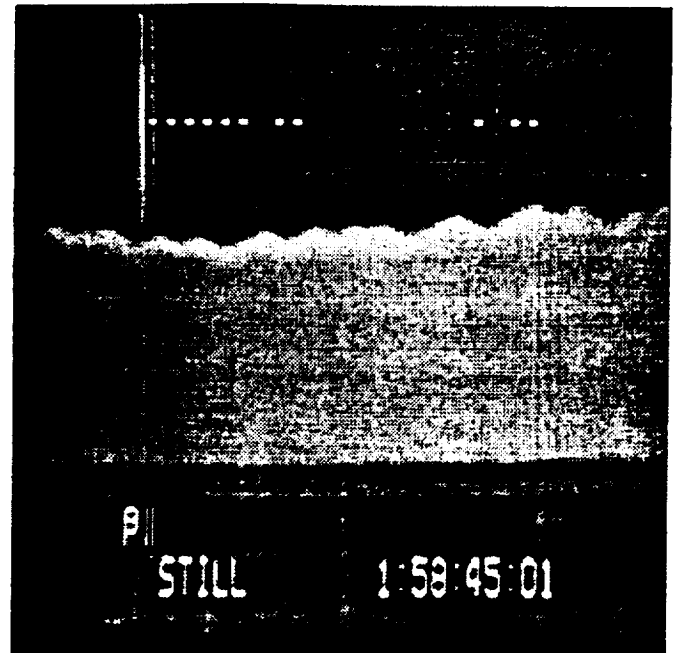
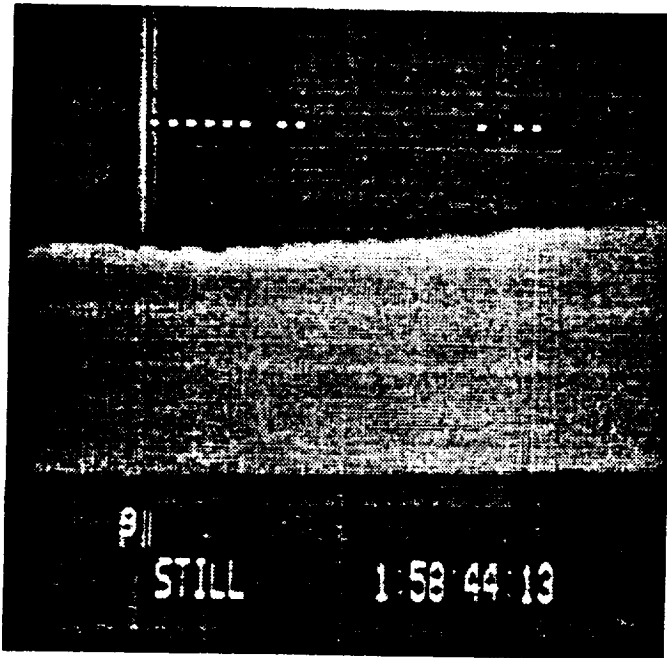
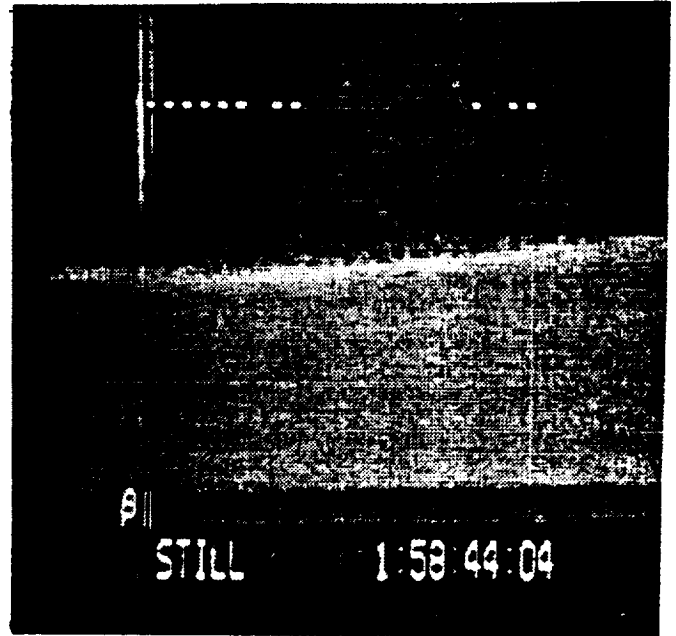
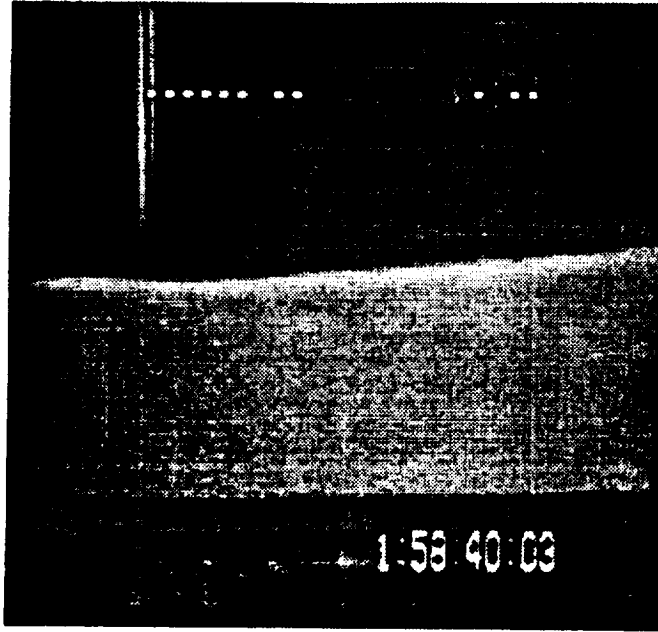


Figure 1: A time sequence showing the initial formation of the wavy disturbance. Here $f = 10.16\text{Hz.}$, $a = 5.13\text{mm.}$. The grains are glass, spherical particles of diameter $d = 0.15\text{mm.}$ The container has an annular geometry, with an gap width $w = 4.5\text{mm.}$ The container is open to the surrounding air.