STUDIES IN THERMOCAPILLARY CONVECTION OF THE MARANGONI-BENARD TYPE

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ABSTRACT

The effects of an imposed nonplanar, oscillatory shear upon the onset of Marangoni-Benard convection, as predicted by linear theory, in a layer of liquid with a deformable free surface were reported upon by Or and Kelly (ref. 1) for small amplitude oscillations. Depending upon the operating conditions, either stabilization or destabilization might occur. The aim of the current paper is to report results for finite amplitude imposed oscillations so that the actual amount of stabilization or destabilization can be determined for prescribed operating conditions.

INTRODUCTION

The small amplitude analysis of Or and Kelly (ref. 1) predicts that the finite wavelength mode of thermocapillary instability, which tends to be the critical mode on earth, can be stabilized by imposing an oscillatory, nonplanar shear on the fluid layer. However, the same shear has a destabilizing influence upon the long wavelength mode associated with surface deformation, which tends to be the most unstable mode under microgravity conditions. Although the small amplitude analysis is useful for an initial approach to the problem in view of the many nondimensional parameters involved, a fully numerical approach is necessary in order to predict how much stabilization or destabilization is obtained for finite amplitude oscillations. Therefore, an expansion of the solution was made in terms of a Fourier representation in a plane parallel to the bounding wall and Chebyshev functions in the direction normal to it (using the Tau method). This procedure yields a set of ordinary differential equations with time-periodic coefficients that is then investigated by use of Floquet theory in order to determine the stability boundaries. Only one figure showing some preliminary results was shown in 1994 (Fig. 5 of ref. 2); much more extensive results have since been obtained and are now reported.

PROBLEM DESCRIPTION

A layer of Boussinesq fluid with mean thickness h rests above a solid horizontal plate that can perform simple harmonic motion along each of the axes defining the plane of the wall. A difference in phase (δ) between the two oscillations creates an oscillatory nonplanar shear field, as defined for the basic state by the well-known solution of Stokes. The layer is heated at the isothermal wall and, since viscous dissipation is neglected, the temperature of the basic state is governed by the steady conduction equation and so varies linearly with distance normal to the wall. The surface of the layer, in general, is allowed to deform in accordance with the free surface boundary conditions, and heat transfer there is characterized by a Biot number (Bi).

The formulation of the problem has been given by Or and Kelly (ref. 1) for arbitrary amplitude oscillations and so is not repeated here. The numerical problem consists of solving eqs. (22) and (23) of ref. 1 subject to the boundary conditions given there as eqs. (24) - (28). For lack of space, neither these equations nor details of the numerical solution are given here. The governing nondimensional parameters will be described below.

RESULTS

Besides the phase angle δ, the other nondimensional parameters governing the basic state's velocity are the nondimensional frequency $\beta = (\omega h^2 / 2\nu)$, where ω is the dimensional frequency and ν is the fluid's kinematic viscosity, a Reynolds number (Re) that involves the magnitude of the wall's velocity in one direction, and a parameter λ that is a ratio of the magnitude of the two components of the wall's motion. For $\delta = \pi/2$, the effects of the nonplanar aspect of the fluid motion are the greatest, and λ is a parameter that governs the pattern selection, which will not be discussed here.

Additional nondimensional physical parameters entering into the equations are the Prandtl number Pr = ν / κ (κ being the thermal diffusivity), the Rayleigh number $Ra = \alpha g \Delta \overline{T} h^3 / \nu\kappa$ (a being the coefficient of thermal expansion, g gravity, and $\Delta \overline{T}$ the temperature difference across the layer in the basic state), the Marangoni number $M = \gamma \Delta \overline{T} h / \rho \nu \kappa$ (γ being proportional to the rate of change of surface tension with temperature), the Bond number Bo = $\rho g h^2/\sigma$ (σ being the surface tension) and the Crispation number $C = \rho \nu \kappa / \sigma h$. We shall assume that Ra << 1 so that the effects of buoyancy can be neglected.

As $\beta \to 0$, the shear in the basic flow vanishes whereas, for $\beta >> 1$, the thickness of the Stokes layer is much less than h. In either case, no change occurs in, say, the critical value of M_c relative to the case Re = 0. Hence, an optimal value of β exists which has a value between one and two, as shown in Fig. 1.

Some typical values for a case typical of conditions on earth (Bo = 0.15) are shown in Figure 2. For the given conditions, the finite wavenumber mode is critical when Re = 0. Fig. 1 indicates that this mode can be stabilized substantially as Re increases. However, the nearly vertical solid line near Re = 190 indicates that a limit to the stabilization exists. This line is the stability boundary for long wavelength disturbances which are much more stable at low values of Re. We therefore conclude that stabilization of the system is limited by a pronounced destabilization of the long wavelength mode at sufficiently large values of Re. This destabilization is due to the action of the unsteady shear at the free surface and has been discussed for the isothermal case by Yih (ref. 3) and Or and Kelly (ref. 4).

As the value of g diminishes relative to the value on earth (g_0), the nearly vertical line in Fig. 2 moves to the left and both the maximum amount of stabilization and the Reynolds number at which this is achieved diminish. These cut-off values are shown in Fig. 3 as a function of g/g_0. In particular, as $g / g_o \to 0$ stabilization becomes impossible and, instead, destabilization occurs for all Re > 0. A sequence of neutral stability boundaries for Bo = 0.01 is given in Fig. 4 as Re increases. In Figs. (4a) and (4b), M_c increases with Re because the finite wavenumber mode is critical and tends to be stabilized by the oscillations. However, the long wavelength mode ($k \to 0$) is also becoming less stable. When Re = 70, the long wavelength mode is already critical and destabilization of the layer occurs. For Re = 100, the layer is unstable even when the wall is cooled relative to the ambient.

Because the destabilization of the long wavelength mode is associated with the interfacial instability discussed first by Yih (ref. 3), the isothermal case was explored to a greater extent than done by Yih. As Fig. 5 indicates, more than one unstable, long wavelength region of instability occurs as β increases, but the regions are separated from each other. However, when finite wavelength disturbances are also considered, the corridors of stability are eliminated. Current research is aimed at clarifying the connection between these new finite wavenumber modes existing for the isothermal case with the finite wavenumber modes already established for the thermocapillary case.

REFERENCES

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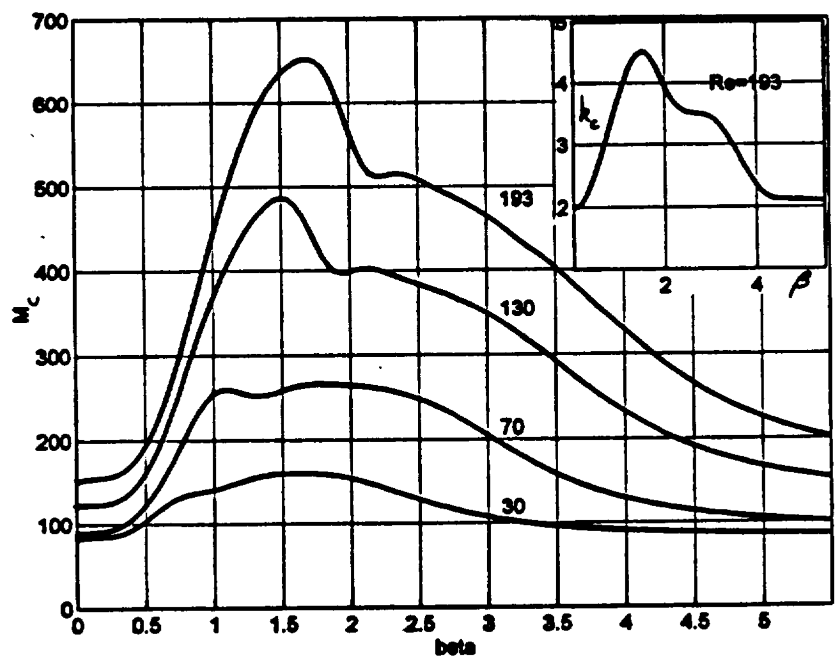

Figure 1: Critical Marangoni number versus nondimensional frequency for various Re; Pr = 7, Cr = 2 x 10^{-6}, Bo = 0.15, Bi = 0.1. Insert shows variation of critical wavenumber with frequency for one Reynolds number.

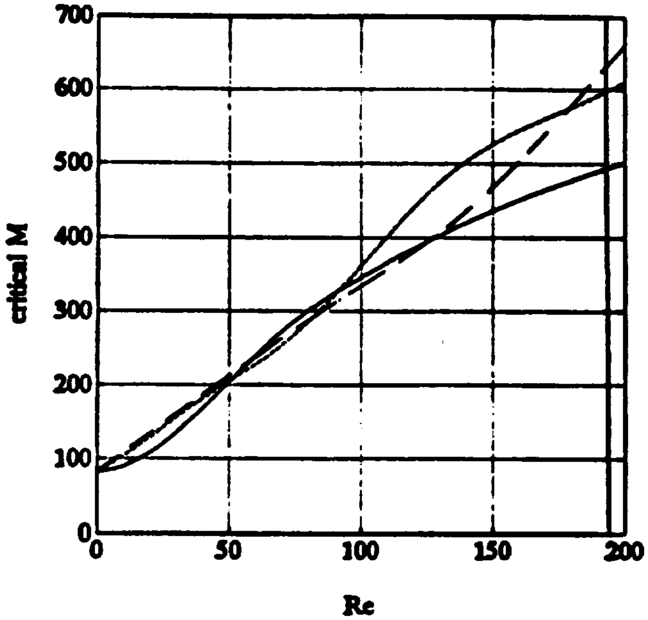

Figure 2: Critical Marangoni number as a function of Reynolds number for $\beta = 1.1$ (solid), 1.4 (short dashes) and 1.7 (long dashes); $Pr = 7$, $Cr = 2 \times 10^{-6}$, $Bo = 0.15$, $Bi = 0.1$.

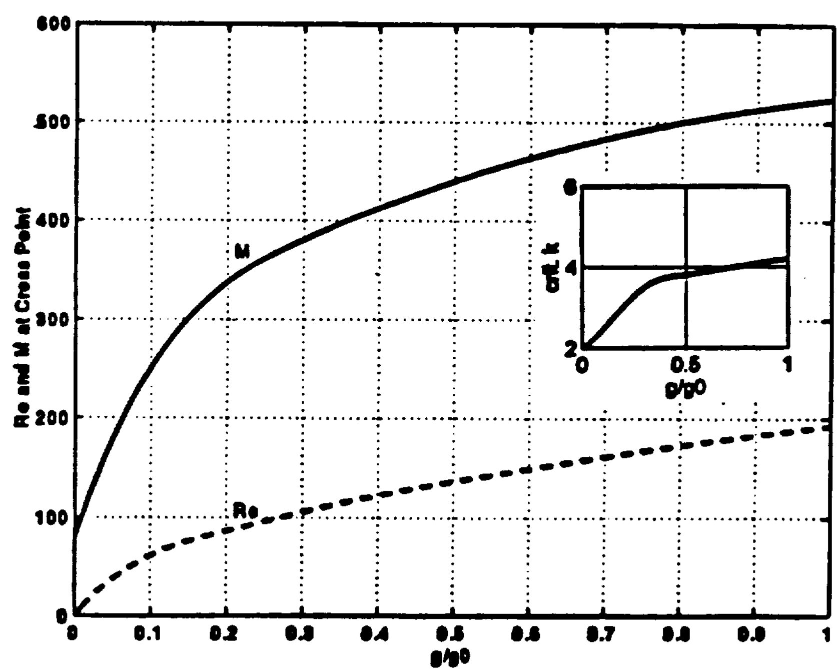

Figure 3: The cut-off values of Reynolds number (lower curve) and Marangoni number (upper curve) for maximum stabilization at $\beta = 1.1$ as a function of g/g_0; $Pr = 7$, $Cr = 2 \times 10^{-6}$, $Bo = 0.15$ at $g = g_0$, $Bi = 0.1$, $\beta = 1.1$.

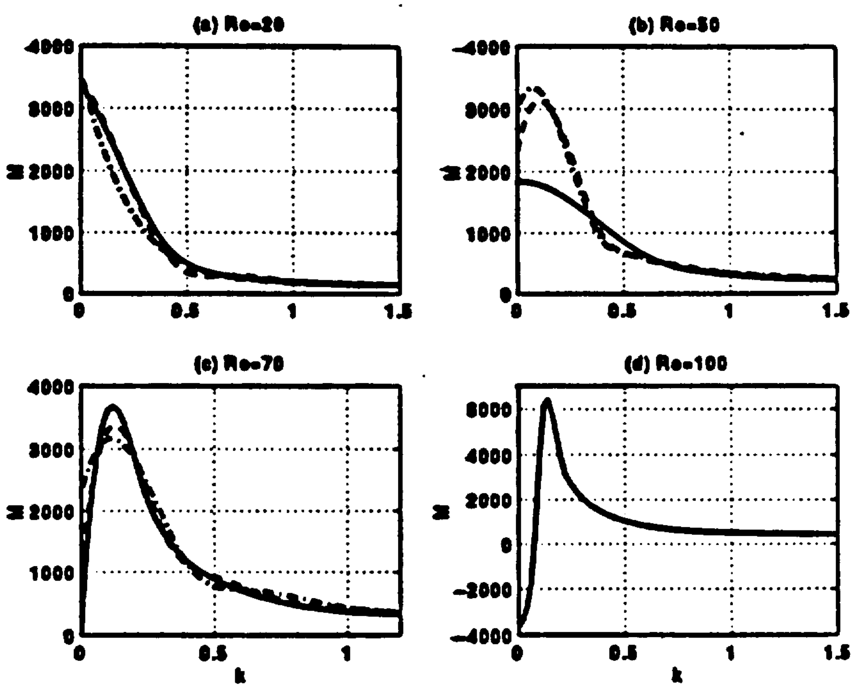

Figure 4: Critical Marangoni number as a function of wavenumber for various Reynolds numbers for β = 1.1 (solid), 1.4 (dashes), 1.7 (dash-dot); Pr = 7, Cr = 2 x 10^{-6}, Bo = 0.01, Bi = 0.1.

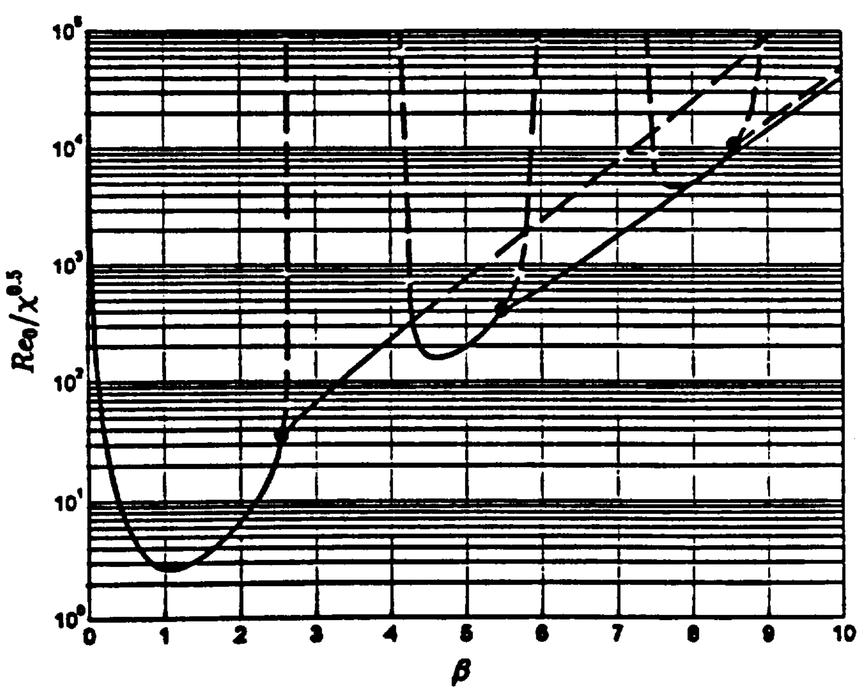

Figure 5: Scaled critical Reynolds number as a function of frequency for the isothermal case with Bo = 0.1; $\chi = gh^3/2\nu^2$. Black dots mark points at which finite k neutral curves bifurcate from the $k \rightarrow 0$ neutral curves shown as loops.