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NONLINEAR BUBBLE INTERACTIONS IN ACOUSTIC PRESSURE FIELDS

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I. INTRODUCTION

The systems consisting of a two-phase mixture, as clouds of bubbles or drops, have shown many common features in their responses to different external force fields. One of particular interest is the effect of an unsteady pressure field applied to these systems, case in which the coupling of the vibrations induced in two neighboring components (two drops or two bubbles) may result in an interaction force between them. This behavior was explained by Bjerknes [1] by postulating that every body that is moving in an accelerating fluid is subjected to a "kinetic buoyancy" equal with the product of the acceleration of the fluid multiplied by the mass of the fluid displaced by the body. The external sound wave applied to a system of drops/bubbles triggers secondary sound waves from each component of the system. These secondary pressure fields integrated over the surface of the neighboring drop/bubble may result in a force additional to the effect of the primary sound wave on each component of the system. In certain conditions, the magnitude of these secondary forces may result in significant changes in the dynamics of each component, thus in the behavior of the entire system. In a system containing bubbles, the sound wave radiated by one bubble at the location of a neighboring one is dominated by the volume oscillation mode and its effects can be important for a large range of frequencies. The interaction forces in a system consisting of drops are much smaller than those consisting of bubbles. Therefore, as a first step towards the understanding of the drop-drop interaction subject to external pressure fluctuations, it is more convenient to study the bubble interactions.

Effects of the secondary Bjerknes forces are reported in a study concerning cavitation by Kornfeld and Suvorov [2]. They describe a strange zig-zag motion of the bubbles ("dancing bubbles") near the surface of an oscillating piston, driven at high frequencies. The phenomenon received only a qualitative explanation, based on the instability of the motion dominated by the inertia forces. Crum [3] studies experimentally the interaction between air bubbles immersed in water contained by an oscillating vessel. The vessel is driven at low frequencies (60 *Hz*) by a mechanical shaker. The ambient pressure is reduced below atmospheric conditions in order to increase the natural frequency of volume oscillations of the bubbles. He provides also an simple model for the relative motion of the bubbles, using a first approximation for the sound field radiated by one bubble at the location of the other one and neglecting the inertial forces. His theoretical results compare well with his experimental data at low oscillation frequencies. However, his algebraic model overpredicts the relative velocity between the bubbles, showing only the trends of variation of this quantity.

The inviscid theoretical approach to secondary Bjerknes forces of Pelekasis [4] and the computational results of Pelekasis and Tsamopoulos [5,6] produce useful data for a specific range of frequencies, pressure amplitudes and bubble sizes due to the admissible values of Bond and Weber numbers. They predict repulsive secondary Bjerknes forces if the frequency of the disturbance is located between the two natural frequencies for volume oscillations of the unequal sized bubbles and attraction forces for any other case, including the equal size bubbles interaction. The relative motion of the two bubbles is found to be dependent on the frequency of the external acoustic field. When the Bond number $Bo = (\rho \bar{a} R^2)/\sigma$, based on the average acceleration $\bar{a}$, the radius R of the larger bubble and the properties of the liquid (ρ the density, σ the surface tension coefficient), lies above a critical value, spherical-cap shapes are predicted to appear with the deformation confined on the side of the bubbles facing away from the direction of acceleration. However, their results are affected quantitatively and qualitatively by the absence of the viscous effects, especially important for the magnitude of the relative velocity and the time in which the two bubbles get to collide. This paper presents experimental results and theoretical predictions concerning the interaction and the motion of two levitated air bubbles in water in the presence of an acoustic field at high frequencies ($22–23kHz$).

II. EXPERIMENTAL SETUP

In order to obtain a stable position for the two bubbles within the tank (Figure 1), the dimensions, the quality and the amount of water used must be determined in such a way that a stationary sound wave is obtained inside the vessel; all these must be correlated with the signal frequency. The exact positions of the bubbles inside the tank are determined using an Edmund Scientific laser pointer ML-211 and a milimetric transparent grid. The oscillations are induced by a thin wall hollow cylinder piezoceramic transducer made of C-5400 material (Channel Industries) with the resonance frequency of $22.5kHz$. The maximum radius of the levitated bubbles with the present amplifier is $3.5mm$. The sinusoidal signal is generated by a Hewlett-Packard 33120A function generator and amplified by a Hewlett-Packard 6824A DC Power Supply Amplifier. The signal is checked with a Gould 4050 Oscilloscope (at the exit of the amplifier) against any kind of distortions. In these conditions, a stationary sound wave is formed inside the tank and the horizontal plane at approximately the middle of water depth corresponds to a pressure minimum. The bubbles are levitated at this location with different initial separations. Their motion is recorded and analyzed using a Kodak EktaPro 1000 Motion Analyzer, choosing the rate of frames per seconds between 60 and 1000 and adjusting the magnification with the Chinon - Hoya Zoom System of lens ($18 - 108mm$).

In the principal experiment, two air bubbles of various radii are injected using clean plastic syringes and needles at different distances apart and their motion recorded and played back frame-by-frame. Figure 2 gives photos of the video screen for a typical experiment, recorded at a rate of 1000 frames per second. The reticle provided by the EktaPro system is used to determine the position of the center of mass of each bubble and their radii; a simple calibration is performed for each session in order to determine the magnification on the screen.

In order to properly study their Bjerknes interaction, small bubbles are injected and afterwards their sizes are increased gradually until the mutual forces triggered the relative motion. At the high frequency used in experiments, it is impossible to measure with accuracy the amplitude of the volume oscillations of the bubbles, this quantity being determined indirectly from the equilibrium of forces in the vertical direction. Perfect levitation of the two bubbles is obtained easily over the entire central region of the vessel, no significant vertical motion of the two bubbles is observed after the initial injection oscillations died down. The quality and stability of the levitation are controlled through small changes of the level of the water inside the vessel (around $63\ mm$) and the intensity of the acoustic field (controlled from the function generator output voltage level with an accuracy of $\pm 1mV$), the frequency of the signal being maintained constant at $22.5kHz$. This paper only presents the attracting bubble results. Other modes of bubble interactions will not be discussed here.

III. MECHANISM AND DYNAMICS OF BUBBLE - BUBBLE INTERACTION

Consider now two gas bubbles of nominal radii R_{01} and R_{02}, which undergo volume oscillations: $R_1(t) = R_{01}(1 + \varepsilon_1 \sin\omega_1 t)$ and $R_2(t) = R_{02}(1 + \varepsilon_2 \sin(\omega_2 t + \varphi))$, levitated in an external stationary sound wave $p_A(z, t) = A_0 \sin(2\pi z/\lambda)\sin(\omega t) = A\sin(\omega t)$, where A will be a notation for the amplitude of oscillation at a fixed location in the liquid column. Volume oscillations of bubble 1 determine the acoustic pressure field around bubble 2 to be modified with the additional wave (spherical symmetry is assumed) [7]:

$$p_1(r, t) = y_1(t)p_A + y_1(t)[1 - (y_1(t))^3]((\rho U^2{}_1(t))/2)$$

$$+[1-(y_1(t))](U_1(t))([\rho U^2{}_1(t)-2p_A-R_1(t)(dp_A/(dR_1))]/c_0) \qquad (1)$$

where: $y_1(t) = (R_1(t))/r$, $U_1(t) = (dR_1(t))/(dt)$ and c_0=sound velocity in the liquid. The secondary Bjerknes force on the bubble 2 due to sound field radiated by bubble 1 will be the average taken over one period of the integral of this pressure field on the surface of bubble 2:

$$F_{12} = \langle \iint_{S_2} p_1(r,t)\hat{n}_2 dS_2\rangle = \langle \iiint_{V_2} \nabla p_1(r,t)dV_2\rangle \qquad (2)$$

Assume that the frequencies ω_1 and ω_2 are both equal with the external applied frequency ω (transient effects being very short); also, assume the bubbles having an in-phase oscillation $\varphi = 0$. With these assumptions and by neglecting all terms of the order 3 and greater in ε_i, equations (1) and (2) give us the following formulas:

$$|F_{ij}| = \frac{\pi}{3}R_{0i}^3R_{0j}^3\rho\omega^2\left[\frac{2A_i}{\rho\omega^2R_{0i}^2}(\varepsilon_i+3\varepsilon_j)+\varepsilon_i^2\right]\frac{1}{r^2} \qquad (3)$$

where $i, j = 1, 2$ $(i \neq j)$. The formulas give a better understanding on the parameters that govern the bubble-bubble interaction than the simplified model developed by Crum, whose results showed a pair of equal forces on the bubbles, depending directly only with frequency, radii and $1/r^{-2}$. The validation of our formulas will stand in comparing the experimental data (velocities of bubbles) with our model predictions and with Crum's experiments and model results.

The forces which are considered in studying the motion of the bubbles in the horizontal plan are the secondary Bjerknes forces given by (3) and the drag forces: $F_{Di}| = (\rho v_i^2 S_i C_{Di})/2$, where $Re_i = (\rho v_i(2R_{0i}))/2$ is Reynolds number for bubble i based on the instantaneous velocities v_i and nominal radii R_{0i}, and $S_i = \pi R_{0i}^2$ is the frontal area of the bubble. The drag coefficient is determined by the model proposed by Moore [8]: $C_{Di} = 48Re_i^{-1}(1-2.2Re_i^{-0.5})$. The forces generated by the gradient of pressure amplitude in the horizontal plane are neglected as the hydrophone measurements indicated a smooth distribution of the amplitude A .The differential equations governing the motion of the two bubbles are nonlinear, of second order with respect to time: $m_i\ddot{r}_i = |F_{ji}|-|F_{Di}|$, where m_i is the induced mass of bubble i which is equal to the half of the mass of the displaced liquid).This relation is transformed by changing the independent variable to $r(t) \equiv r_1(t)-r_2(t)$ and choosing the unknown functions to be the velocities of each bubble, v_i. The amplitude of the pressure signal will be considered the same around each bubble: $A_1 = A_2 = \rho\omega^2R_{01}^2\varepsilon_1^2 = \rho\omega^2R_{02}^2\varepsilon_2^2$ since $\omega_0^2 \ll \omega^2$ for the investigated frequencies. Further substitution results in the following system:

$$(v_1+v_2)v_i' = G_ir^{-2}-((72\upsilon)/R_{0i}^2)v_i[1-2.2(\upsilon/(2v_iR_{0i}))^{0.5}]$$

where: $G_i = 1.5R_{0j}^3\omega^2[\varepsilon_j(\varepsilon_j+2\varepsilon_i)]$, $i, j = 1, 2(i \neq j)$, $v_1' = (dv_1)/(dr)$, and $v_2' = -(dv_2)/(dr)$. The length scale and the velocity scale used to non-dimensionalize (4) are: $L = R_1$ and $U = (\sigma/(\rho R_1))^{0.5}$, where the larger bubble (index 1) is considered to be the left one. This nonlinear system is solved numerically with the initial conditions imposed by the initial velocities measured during each experiment: $v_1(r{=}d_0) = v_{10}$ and $v_2(r{=}d_0). = v_{20}$. The solution of the system is used to compute and plot the relative velocity of the bubbles $v(r) \equiv v_1(r)+v_2(r)$.

IV. RESULTS AND DISCUSSIONS

Figure 3 presents the results obtained by applying the model proposed by Crum and our model to our experimental data in order to compare the accuracy of both approaches. The comparison shows clearly that the simplified dynamics considered by Crum overpredicts the relative velocity even at large bubble separation. Neglecting the inertia forces and using a first order expression for the acoustic field radiated by one bubble in determining the secondary Bjerknes forces formulas are the principal causes for its limited accuracy. On the other hand, the new analytic model presently proposed gives results in good agreement with the experiments until the dimensionless distance between bubbles decreases below a minimum value: $r_m \cong 3$ as shown in Fig. 3. The differences between theory and experiment become significant for $r < r_m$, where the two neglected effects mentioned before (drag modification and acoustic interference) are important. At the moment of collision, secondary Bjerknes forces attain the maximum magnitude, which is less than 40% of the buoyancy on the larger bubble. Based on the proposed model, we designate two new dimensionless parameters which determine the time in which the bubbles reach the impact point: $\Gamma \equiv d_0^2/(R_{01}R_{02})$ (containing the geometrical conditions of the interaction) and $\alpha \equiv A/(P_0(z))$ (related to the acoustical conditions). Figure 4 presents the experimental data translated in dimensionless variables. These plots of the coordinates of center of mass of each bubble in time show that: $\Delta t \sim \Gamma^n \alpha^{-s}$ with positive values for n and s. The average values correlated from the experimental data are: $\bar{n} \approx 4/3$ and $\bar{s} \approx 1/2$.

V. ACKNOWLEDGMENTS

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VI. REFERENCES

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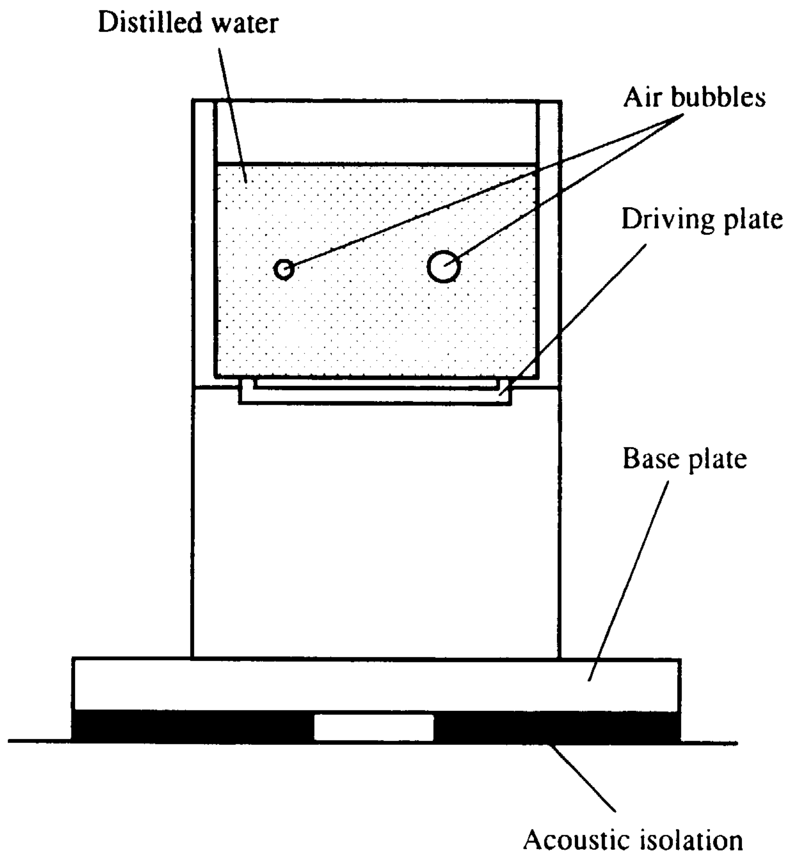

Fig. 1. The levitator.

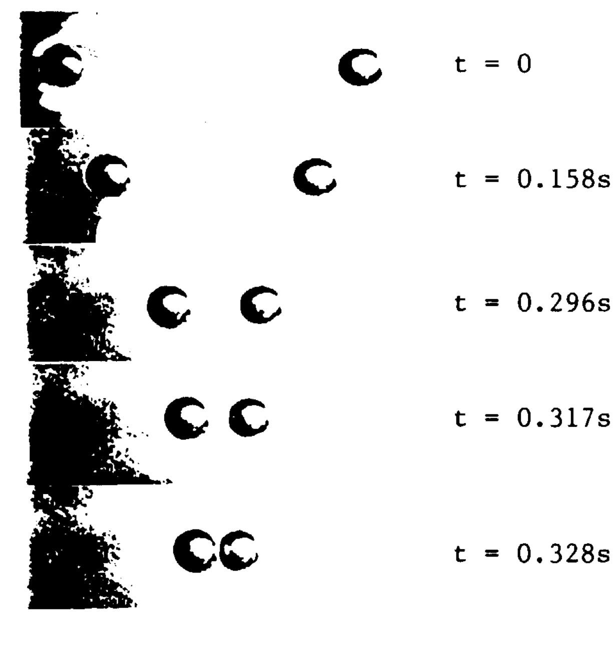

Fig. 2. Photos of a typical bubble motion under secondary Bjerknes forces interaction.

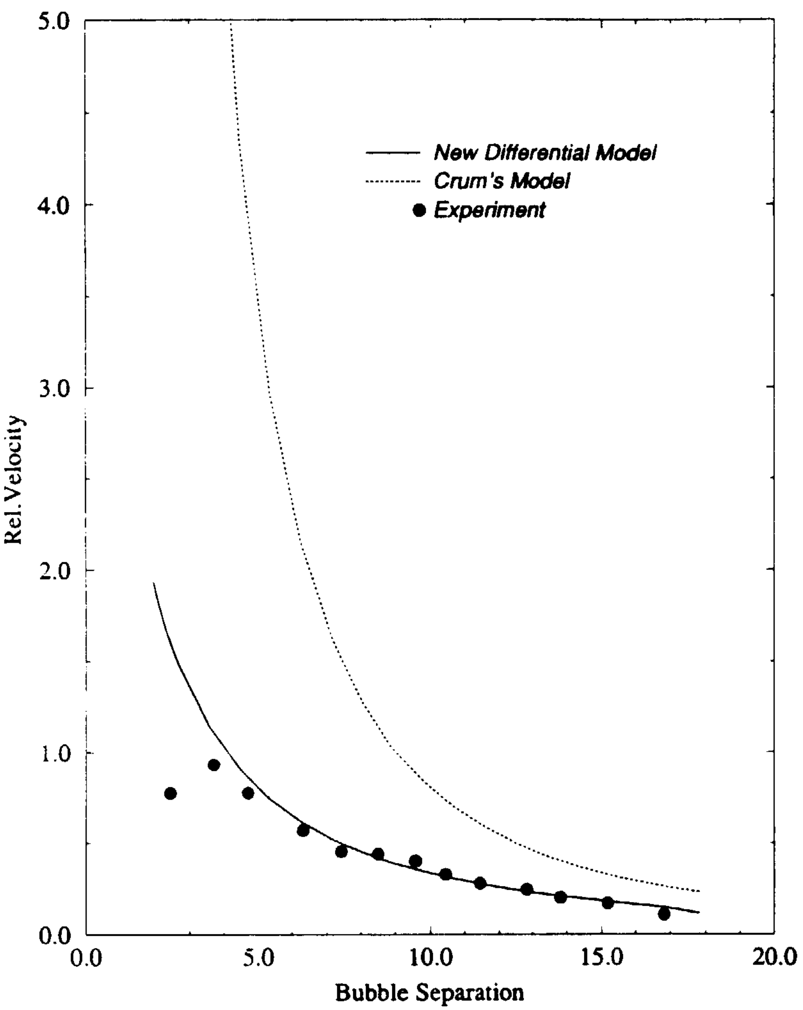

Fig. 3. Relative velocity of the bubbles. Comparison of new experimental data, Crum's model and new differential model: $\Gamma=390.6$ and $\alpha=0.49$

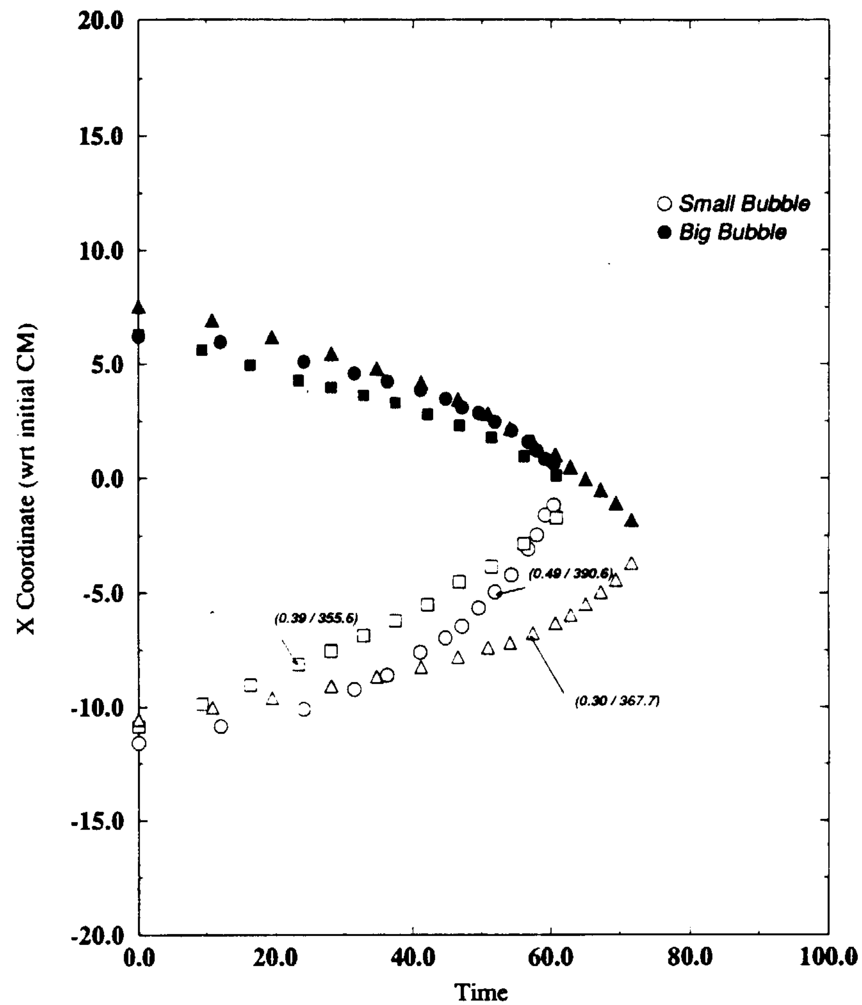

Fig. 4. History of x-coordinates of the centers of mass of each bubble α between 0.30 and 0.49