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61

CRITICAL VELOCITIES IN OPEN CAPILLARY FLOW

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INTRODUCTION

This paper describes the proposed research program on open capillary flow and the preliminary work performed theoretically and in drop tower experiments. The work focuses on the fundamental physical understanding of the flow through capillary bound geometries, where the circumference of the cross section of the flow path contains free surfaces. Examples for such a flow configuration are capillary vanes in surface tension tanks, flow along edges and corners and flow through liquid bridges. The geometries may be classified by their cross section areas, wetted circumferences and the radii of curvature of the free surfaces. Some possible geometries are depicted in Fig. 1. In the streaming float zone the flow path is bound by a free surface only. The ribbon vane is a model for vane types used in surface tension tanks, where a structure in proximity to the tank wall forms a capillary gap. A groove is used in heatpipes for the transportation of the condensed working fluid to the heat source and a wedge may occur in a spaceborne experiment where fluid has to be transported by the means of surface tension.

The research objectives are the determination of the maximum volume flux, the observation of the free surfaces and the liquid flow inside the flow path as well as the evaluation of the limiting capillary wave speed. The restriction of the maximum volume flux is due to convective forces (flow velocity exceeding the capillary wave speed) and/or viscous forces, i.e. the viscous head loss along the flow path must be compensated by the capillary pressure due to the curved free surface. Exceeding the maximum volume flux leads to the choking of the flow path, thus the free surface collapses and gas ingestion occurs at the outlet.

The means are ground-based experimental work with plateau tanks and in a drop tower, a sounding rocket flight and theoretical analysis with integral balances as well as full three dimensional CFD solutions for flow with free surfaces. Due to the hydrostatic pressure on earth the open capillary flow path dimension (perpendicular to the flow direction) cannot exceed the capillary constant $l_c = \sqrt{\rho g/\sigma}$ and experiments with larger dimensions require a low gravity environment. The efforts will culminate in a definitive flight experiment with well chosen geometries for fundamental and reliable results.

FLOW BETWEEN PARALLEL PLATES

Motivation from surface tension tank technology

The capillary rise in open cross sections is used in surface tension tanks to transport and position the propellant under the condition of microgravity. Usual designs of surface tension tanks have a refillable reservoir from which the propellant is supplied to the engines. The purpose of the capillary vanes is to fill the reservoir. The filling of vanes in a compensated gravity environment has been investigated by DREYER et al. [1]. For a new generation of surface tension tanks a design without a refillable reservoir is developed. The propellant is withdrawn directly from the capillary vanes and not from the reservoir. This yields higher volume fluxes through the vanes. Data on the flow through open capillary vanes are rare and not verified by experiments. The limits of the flow rate due to choking and the stability limits of the free surface are not known for different types of flow geometries with open cross section.

In a paper by JAEKLE [2] the design and the analysis of vanes in surface tension tanks are considered. The performance verification of capillary vanes relies completely on analysis. Testing on ground is not possible due to the hydrostatic pressure which prevents the establishment of the liquid volume in the gap by surface tension only. In the analysis section JAEKLE mentioned the similarities of the flow through vanes with flow in flexible tubes, open channels and compressible duct flow. Each of these flows is governed by similar equations and has a limiting or choking velocity.

For the use of capillary vanes in surface tension tank designs the critical velocity must be known. Exceeding this critical velocity leads to the ingestion of gas and thus to a malfunction of the thruster. Experimental verifications of the work of JAEKLE are not available. To achieve a better knowledge of the critical flow velocities in capillary vanes, experiments under reduced gravity conditions are performed funded by the European Space Agency (ESA) and the German Space Agency (DARA).

Theoretical approach

Figure 2 shows a schematic drawing of the flow between two parallel plates with free surfaces at the sides. If a constant volume flux Q is applied, the radius of curvature R of the free surface changes in flow direction. The cross section area A is a function of R and changes in flow direction s.

The assumptions are the following: the problem is stationary, the flow is incompressible and isotherm, the properties density ρ, viscosity μ and surface tension σ are constant at the given temperature T. The free surface at the sides of the capillary vane has one radius of curvature R in the plane perpendicular to the flow path. The second radius of curvature is neglected. The flow is one-dimensional in the direction of the vane axis, we apply a momentum balance in one direction. We consider an average value of the flow velocity in the direction of the vane axis s over the cross section area A. The cross section area A, the average velocity v and the pressure p change only gradually along the flow path.

Differentiating the mass balance we have

$$dv = -\frac{Q}{A^2}\, dA \ , \tag{1}$$

with the average flow velocity v in s-direction. We apply a Bernoulli equation in the direction of the vane axis with an additional friction term:

$$-\tau_w W ds - A dp - \rho g A dz = \rho A v dv \ . \tag{2}$$

The terms of the LHS of Eq. (2) are the friction force, the pressure force and the weight due to the gravitational acceleration (g is the gravity acceleration), respectively. The RHS of Eq. (2) is due to the convective acceleration occurring with the change of the cross section A. The friction term arises from the wall shear stress τ_w multiplied by the wetted perimeter W and the differential path length ds. The effect of an additional friction drop due to the change of the velocity profile within the entrance length is neglected in the following calculations. The pressure force arises from the surface tension times the radius of curvature of the free surface. The area $A(R)$ and the derivative of the area with respect to the radius of curvature dA/dR can be calculated from the system geometry. Finally the differential equation for the change of the radius of curvature with respect to the path length dR/ds reads

$$\frac{dR}{ds} = \frac{\dfrac{k_f \nu}{32}\dfrac{W^2}{Q} + \dfrac{A^3 g}{Q^2}\dfrac{dz}{ds}}{\dfrac{dA}{dR} - \dfrac{\sigma}{\rho}\dfrac{A^3}{Q^2 R^2}}, \tag{3}$$

with a friction factor k_f. The change of the radius of curvature with the flow length setting $g = 0$ is shown in Fig. 4 for the parameter: $Q = 10.5$ ml/s, plate distance $a = 5$ mm, plate breadth $b = 30$ mm, Silicon Fluid 1.0 cSt. The minimal radius of curvature for the parallel plates equals $a/2$. For given parameters such as flow rate, geometry, etc. the maximum possible path length can be calculated from Eq. (3). Setting the denominator to zero one gets a velocity for the capillary wave speed.

Using proper scales such as lengths a and b, velocity $\sqrt{2\sigma/\rho a}$ and gravity acceleration on earth g_0 one gets the dimensionless equation

$$\frac{d\bar{R}}{d\bar{s}} = \frac{\dfrac{k_f}{32}\mathrm{Oh}\dfrac{\bar{W}^2}{\bar{Q}} + \mathrm{Bo}\dfrac{\bar{A}^3 \bar{g}}{2\bar{Q}^2}\dfrac{dz}{ds}}{\dfrac{d\bar{A}}{d\bar{R}} - \dfrac{\bar{A}^3}{\bar{Q}^2 \bar{R}^2}}, \tag{4}$$

with the OHNESORGE number $\mathrm{Oh} = \dfrac{\mu}{\sqrt{2\rho a \sigma}}$ and the BOND number $\mathrm{Bo} = \dfrac{\rho a^2 g_0}{\sigma}$. The non-dimensional area $\bar{A}$ contains the ratio $\Lambda = a/b$. The parameter range of the experiments is $\Lambda = 0.1 \ldots 0.3$, $\mathrm{Oh} = 0.0015 \ldots 0.005$ and $\mathrm{Bo} = 0$ with respect to the application in surface tension tanks and the drop tower limitations.

Results of drop tower experiments

The aim of the experiments is to verify the Eq. (3) for a parameter range which is comparable to the application of this equation in surface tension satellite tanks. Experiments have been performed in the drop tower Bremen. A test vane (flow length 100 mm) is positioned in the experiment container perpendicular to the free liquid surface under normal gravity conditions. During the free fall of the experiment the vanes fills itself due to the capillary force up to the top. At the top of the vane a constant volume flux is withdrawn. The maximum volume flux at a given length of the vane and the shape of the free surface at the sides will be evaluated and compared to the solutions of Eq. (3). Figures 5 and 6 show two videoprints of the drop tower experiment with the same parameter as Fig. 4. Figure 5 shows the parallel plates with the optical axis perpendicular to the plate plane. The flow is from the bottom to the top. A subcritical volume flux is applied to the withdrawal device at the top. The radius of curvature decreases in flow direction. No gas ingestion at the outlet occurs as expected for the supercritical case. The curves of the innermost point in Fig. 3 can be compared with the contour of the free surface in Fig. 5. Theory is limited up to the minimal contour, for the upper part of the vane (see Fig. 6) the second radius of curvature (which was neglected) plays an important role. Figure 6 shows the free surface at the sides in vicinity of the outlet. The contour of the free surface is clearly visible. The radii of curvature (in the direction of the vane and perpendicular to it) are evaluated with digital image processing. The corresponding experiments for supercritical conditions are in preparation.

Conclusions

Eq. (3) gives a good estimate to adjust the volume flux to sub- and supercritical values. Within a drop tower experiment one certain volume flux can be adjusted. The resultant flow is either sub- or supercritical. Thus, only an upper and a lower bound for the maximum volume flux can be evaluated. A continous increase and decrease of the volume flux would be desirable.

Stationary conditions cannot be achieved. Due to the start of the withdrawal after the filling of the capillary, the adjustment of free surface at the side to its final position needs more time than available during the free fall (4.74 s). But the drop tower is a unique tool to optimize the experimental setup and to get preliminary results for long-term microgravity facilities. As a next step a sounding rocket mission funded by ESA is scheduled for 1997. The calculation of the flow field and the resulting pressure with a finite element CFD code (FIDAP) is underway. The additional pressure drop due to the development of the flow field behind the entrance of the vane is taken into account. Furthermore, a complete analysis of the flow between parallel plates with free deformable surfaces at the sides is in preparation.

STREAMING FLOAT ZONE

Since the advent of research under microgravity conditions, strong interest has been in investigations on liquid columns. Microgravity renders possible the establishment of large liquid columns. They are most convenient for studying static and dynamic effects of liquid surfaces under microgravity conditions. Axisymmetry considerably simplifies treatment by theory (LANGBEIN [3], [4]).

The scientific questions, which have to be solved by means of large liquid columns under microgravity conditions, are (i) the stability of liquid columns in rest and under the influence of rotation, (ii) the behavior of liquid columns under the influence of constant volume flows, (iii) the dynamics of breakage with particular regard to the volume distribution to the various fragments, inclusive of satellite drops, (iv) the resonance frequencies of liquid columns with respect to axisymmetric and non-axisymmetric surface deformations, (v) the onset of non-axisymmetric flows, (vi) the sensitivity of liquid volumes to random, periodic or pulse-type accelerations, (vii) stationary Marangoni convection in liquid columns, which either are heated to different temperatures at the bottom and the top or else are heated along the periphery by means of a ring heater, (viii) the onset of oscillatory Marangoni convection, i.e. the critical Marangoni number, when heating is intensified, and its hysteresis, when heating is reduced and the onset of turbulence

Within the scope of the present proposal we are interested in particular in the behavior of liquid columns under the influence of constant volume flows. If the column is established between two coaxial circular tubes, one finds a quadratic velocity profile (Hagen-Poiseuille flow) inside the tubes, whereas in the region of the floating zone one has the free-slip condition along the surface. There the flow is going to speed up. Distant from the orifices one can assume a uniform flow velocity. The length of the float zone on the other hand is limited by the Rayleigh instability. This relates the investigations to floating zone stability.

An increase in flow velocity acts like a decrease in pressure, such that the diameter of the column will shrink at the inlet and will widen at the outlet. Therefore, under 1g conditions one may expect a stabilizing effect of an upward flow and a destabilizing effect of a downward flow. Further objectives of interest are the beginning of oscillatory flows (critical numbers), and their relations to resonant modes. Do stationary non-axisymmetric flows arise or do they revolve around the axis?

FLOW THROUGH LIQUID FILLED WEDGES

The dynamics of penetration of liquids into solid wedges under microgravity conditions has been and will be treated experimentally and theoretically. The case of good wetting between the liquid and the wedge, which entails fast wetting, is being investigated in drop tower experiments. The limit of slow penetration (bad wetting) has been studied in the IML-2 experiment DYLCO. The theory of liquid penetration into a wedge has been developed. It is based on the principle that the local flow in wedge direction is determined by the local gradient of capillary pressure. The limit of long times, when the curvature in wedge direction becomes negligible in comparison to the curvature perpendicular to it, allows for a similarity solution in position over the square root of time, such that the tip of the liquid meniscus proceeds proportional to $\sqrt{t}$ also. The numerical solution of the full fourth order differential equation confirms this asymptotic result and the experimental findings as well.

The theory of liquid penetration is based on the assumption that in the planes perpendicular to the wedge the liquid surface is given by a circular section with radius $R(z)$, z being the coordinate in the wedge direction. This principle recently has been successfully applied to the calculation of static liquid surfaces in polyhedral containers (LANGBEIN [5]). It has been shown that the liquid surface evolves exponentially from the cylindrical shape along the wedges to the spherical shape of the meniscus near the corners. The differential equation of liquid penetration into the wedge is obtained by requiring that the change in liquid volume on the side to be filled must equal the flow through the cross-section at z. This volume flow is generally proportional to the squared area of the cross section times the pressure gradient. The yielding equation is a fourth order partial differential equation for $R(z,t)$, which in general has to be solved numerically. On the other hand, it is similar in character to a diffusion equation with the modification that different powers of $R(z,t)$ appear on its two sides.

The work on the liquid penetration into a wedge is necessary for the investigation of the flow through such a configuration. The theory will be expanded on that case and appropriate experiments will be carried out.

ACKNOWLEDGMENTS

The funding of the drop tower flights by the German Space Agency (DARA) and the European Space Agency (ESA) is gratefully acknowledged. The work on the dynamics of liquid in edges and corners has been supported by the DARA under contract number 50 WM 9432.

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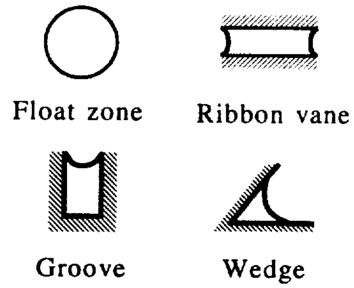

Figure 1: Different types of open capillary flow geometries. The flows is perpendicular to the paper plane.

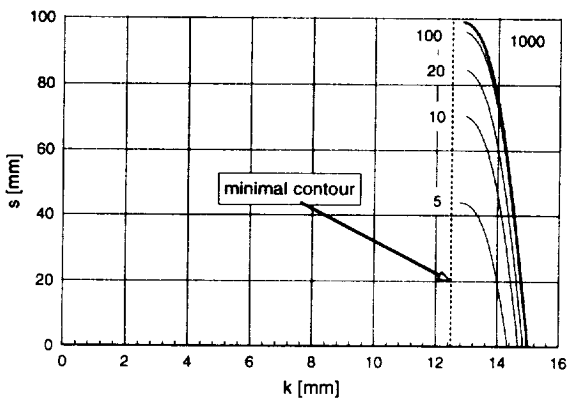

Figure 3: Plot of the innermost point of the free surface versus path length for the comparision with Fig. 5 (a = 5 mm, b = 30 mm).

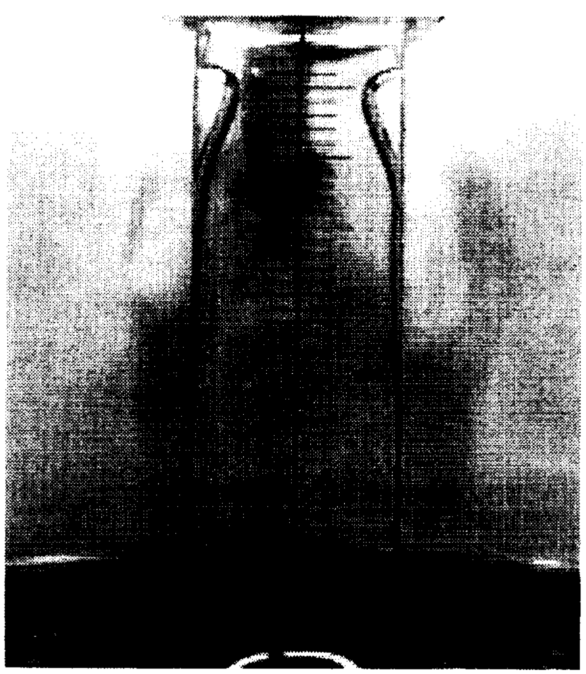

Figure 5: Videoprint of a subcritical flow between two parallel plates. The optical axis is perpendicular to the plates, the flow is from the bottom to the top (same parameter as Fig. 4).

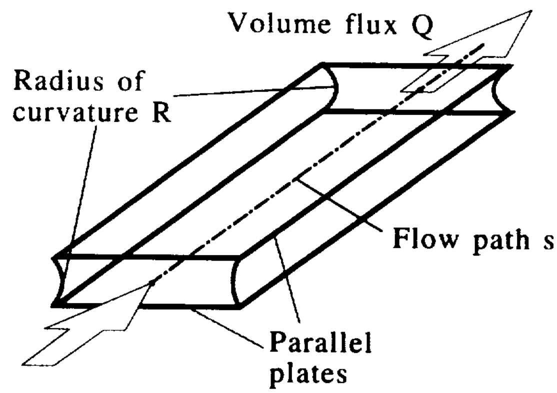

Figure 2: Schematic drawing of the flow between two parallel plates with a free surface at the sides. A constant volume flux is applied, the radius of curvature changes in flow direction.

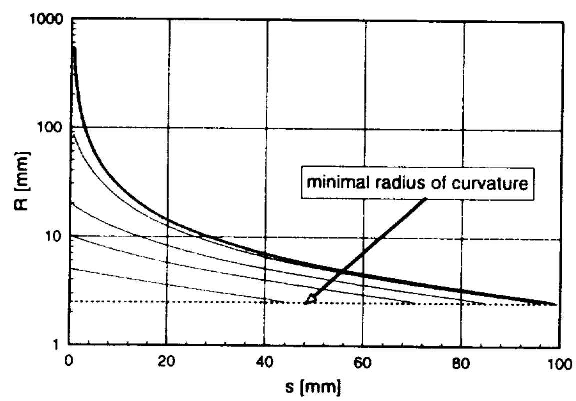

Figure 4: Plot of the radius of curvature versus the path length for different initial radii with Q=10.5 ml/s. The dashed line marks the minimal radius of curvature. The liquid is Silicon Fluid 1.0 cSt, plate distance 5 mm, plate breadth 30 mm.

Figure 6: Detail from the top of the vane below the inlet. The meniscus is stable, no gas ingestion occurs. The free surface is clear visible.