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Thermoacoustic Effects at a Solid-Fluid Boundary: the Role of a Second-Order Thermal Expansion Coefficient

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Abstract

Analytical and numerical studies are to be carried out to examine time-averaged thermal effects which are induced by the interaction of strong acoustic fields with a rigid boundary (thermoacoustic streaming). Also of interest is the significance of a second-order thermal expansion coefficient that emerges from this analysis. The model problem to be considered is that of a sphere that is acoustically levitated such that it is effectively isolated in a high-intensity standing acoustic field. The solution technique involves matched asymptotic analysis along with numerical solution of the boundary layer equations. The objective of this study is to predict the thermoacoustic streaming behavior and fully understand the role of the associated second-order thermodynamic modulus.

1 Introduction

Much interest has arisen in the past decade or so, in the study of fluid transport phenomena in the presence of an acoustic field (for instance, see review by Trinh [17]). In particular there has been a desire to understand the fluid mechanics and heat transfer processes in a microgravity environment where the acoustic field is the dominant force field in the system. The work has been motivated by the need to understand the behavior and influence of these transport phenomena for a series of microgravity experiments that have been conducted and planned. These experiments have been devised to serve a variety of purposes ranging from basic science studies, to investigations of containerless materials processing in a microgravity environment. These experiments involve the use of acoustic levitation (as one of the many different levitation techniques) to carefully isolate and study the phenomena in question in an effectively containerless manner. Of equal interest and importance has been the study of these fluid dynamical phenomena from the viewpoint of measurement of properties of the materials used in these levitation experiments. It is a combination of the above features, i. e. the study of novel transport phenomena coupled with the possible measurement of a second order thermodynamic modulus, which will form the focus of this study.

2 Background

This study centers around acoustic streaming and thermoacoustic transport effects induced by a strong acoustic field, and the significance of a material property (a thermodynamic modulus) of the host fluid supporting the acoustic field. Such effects are typically encountered in primarily two fields of application requiring the interaction of high-intensity acoustics with rigid boundaries, namely, acoustic levitation, and thermoacoustics.

There exists a large body of literature on the applications of acoustic levitation and no attempt will be made to provide a review at this stage. However it is noted that the intense acoustic fields used for levitation give rise to steady streaming flows in the host fluid due to nonlinear effects related to the presence of rigid boundaries such as those of the sample and the container walls. Such flows have also been studied with an interest in predicting the heat transfer rates they induce. Based on the original work of Riley [10], Davidson [2], and Lee & Wang [8], Gopinath & Mills [6, 7] have made some predictions on the resulting heat transfer behavior in the context of containerless materials processing in a microgravity environment.

In contrast, the associated thermoacoustic transport effects have received little or no attention in this context. In particular there is a need to understand the second-order time-averaged thermal effects that may be induced by the interaction of the strong sound fields with rigid boundaries, *even in the absence of any externally imposed driving temperature potential.* This time-averaged thermal phenomenon has been aptly named *thermoacoustic streaming* in a review by Rott [12], using a direct analogy with *acoustic streaming*. Notable early work in this field includes that of Rott [11] and Merkli & Thomann [9], although all for internal duct geometries. Swift [15] also provides a fine review in a tutorial article, although again in a different context, i. e. for so-called thermoacoustic engines. However the role of this phenomenon in the context of acoustic levitation, and its impact on heat transport is still unclear.

3 Research Plan

The research plan involves the analytical/numerical examination of a model problem in which high-intensity acoustic fields are used for levitation in a microgravity environment so as to isolate and study thermoacoustic streaming effects, and thereby also, investigate the surprising role of an associated second-order thermodynamic modulus which emerges in these effects. Equally important in this study would be the role of the acoustic streaming motion itself, in its capacity to convect the heat generated by these second-order thermoacoustic effects. Based on the findings from this theory, suggestions will be made for possible future experiments for a more detailed study of these phenomena and for measurement of this thermodynamic modulus. It is emphasized for clarity that the above material property which arises is that of the host liquid, and not of the levitated sample (which merely serves as a suitable rigid boundary to induce these effects). A very preliminary analysis of but a portion of this problem has been attempted by Gopinath [5], which however is incomplete and needs to be treated in far greater detail.

3.1 Analytical Formulation

The model problem to be studied will comprise of a sphere levitated so as to be effectively isolated in a high intensity plane standing acoustic field. The sphere will be initially treated as rigid with a large thermal inertia so as to decouple the fluid mechanics aspects from the thermal energy aspects. Starting with the governing Navier-Stokes equations formulated in axisymmetric spherical coordinates, a scale analysis will first be conducted to identify the dimensionless parameters of importance since this problem exhibits numerous parameter scales.

Thus far, most studies of this problem have relied on the incompressible flow assumption to render tractable the governing equations based on the solenoidality of the vector velocity field. Here too, this simplifying assumption will be made (at first) to allow complete attention on the thermoacoustic effects which are the key effects being studied. In other words, on the basis of the different length scales of the problem, this assumption requires that the radian wavelength be much larger than the sphere radius, which is in turn taken to be much larger than the oscillatory amplitude of the fluid particle in the acoustic field. The latter condition further allows the flow to be considered attached to circumvent the complex situations arising from separated flow behavior. With these simplifying conditions there is now the added advantage of being able to incorporate the influence of the axial location of the rigid sphere in the standing sound field. All studies on acoustic streaming in such a situation have understandably focused on the sphere located at the velocity antinode in the standing field where these streaming effects are maximized (and not at the velocity node where they are essentially nonexistent). However the thermoacoustic streaming effects to be considered in this study are perfectly capable of being significant at both locations and the variation over this range is one of the aspects that will be investigated.

Once the basic behavior is established, later studies will include cases of larger sample diameters and/or ultrasonic levitating frequenices for which the sample size is comparable to the radian wavelength. The flow may no longer be considered incompressible and the strategy under these conditions is also briefly noted below.

3.2 Solution Methodology

For the effectively incompressible flow conditions, the vector velocity field can be conveniently expressed as the curl of a vectorized stream function, $\vec{\psi}$. In this manner the vector equations of fluid motion can be converted into a higher order scalar equation for the stream function in standard fashion. The energy equation, in its most general form, is also suitably non-dimensionalized using the appropriate scales determined earlier. For the more general compressible flow situation, the vector velocity field is expressed in addition as the gradient of a scalar potential, ϕ, as

$$\vec{u} = \nabla\phi + \nabla\times\vec{\psi}$$

and this is incorporated into developing the governing equations in terms of ϕ and $\vec{\psi}$.

It is identified that for the high frequencies being treated, a matched asymptotic expansion technique can be successfully used. The fluid domain being studied can be decomposed into a thin inner Stokes layer region adjacent to the wall of the sphere, with the remainder of the domain making up the outer region. For the inner region, the primary oscillatory flow behavior can be readily determined from a leading order solution of the governing equations. This can then be used with the vector equation of motion in its primitive form, to determine the leading order pressure field in the inner region. This procedure can be extended to the next higher order correction which would then yield a description of the acoustic streaming motion originating in the inner region, due to the nonlinear interactions of the first harmonic contributions from *both* ϕ and $\vec{\psi}$.

The solution of the energy equation would also add new and interesting results in the study of such streaming phenomena. Just as the time-averaged fluid motion formed the focus in studies on acoustic streaming, it is the time-averaged thermal effects which will form the focus in this study of thermoacoustic streaming. It may be recalled that in the small sphere limit, the density fluctuations in the fluid are negligible in the continuity equation and hence allow the incompressible flow assumption. However preliminary analysis confirms that these density fluctuations are not negligible in the energy equation and are fully capable of making a significant contribution to the time-averaged thermal energy exchange mechanisms in the fluid. Such a deduction which could be easily overlooked is only possible because of the non-dimensionalized form of the governing equations which clearly shows the scales and magnitudes of the different contributions. It is emphasized that the sphere is not subject to any externally imposed heating/cooling mechanism. All the time-averaged heat transfer effects to be studied are purely a manifestation of the thermoacoustic streaming effect arising from the interaction of the strong acoustic fields with the sphere boundary.

A detailed analysis of the energy equation to determine all the time-averaged contributions to the thermoacoustic streaming effect requires that the density fluctuations in the fluid be first determined. This is possible from a suitable equation of state of the fluid which additionally requires a knowledge of the pressure field in the fluid. The (isobaric) thermal expansion coefficient of the host fluid, β, (defined in the usual manner such that $\beta T \equiv 1$ for an ideal gas) is

$$\beta = -\frac{1}{\varrho}\left(\frac{\partial \varrho}{\partial T}\right)_p ,$$

and is found to have an important role in the time-averaged heat exchange to be investigated. In addition, analysis of these contributions shows that a heretofore ignored second-order thermal expansion coefficient also plays a key role in this time-averaged thermal energy exchange. This property arises in time-averaged contributions from pressure fluctuations and is in fact a thermodynamic modulus defined as

$$\xi = \frac{T}{\varrho\beta}\left(\frac{\partial^2 \varrho}{\partial T^2}\right)_p - 1$$

so that for an ideal gas, $\xi \equiv 1$. The significance of a related thermodynamic property was alluded to by Allen et al. [1, Eq. 4] and used by Swift [16] in their exploration of liquids as working substances in Stirling/Malone type engines. However no detailed work has been done in this area and possible interest in the use of liquids as working substances in thermoacoustic engines would warrant a more careful look into this property. It may be pointed out at this stage that similar higher order thermoacoustic properties of fluids have been investigated before, although in an entirely different setting; the recent papers by Sharma [13, 14] and the references therein provide a good indication of the interest in this type of fundamental work.

As for the boundary conditions in the present problem, as stated earlier the simplifying assumption of large sphere thermal inertia will be initially used to obtain a proper handle on the problem. This will be subsequently relaxed to allow a thermal interaction between the sphere and the fluid. This step would simulate the actual physics more realistically and allow for finite heat capacity effects of the sphere to be incorporated. As for the solution procedure, this change requires that the governing equations of motion in the fluid now also be coupled to the energy equations in both the fluid and the sphere. The primary temperature oscillations in the fluid need to be first determined from a leading order solution of the energy equation. In addition, the complete unsteady form of the heat conduction equation has to be solved to determine the temperature distribution in the sphere, and has to be also properly matched to the temperature distribution in the fluid to maintain interfacial continuity of temperature and heat flux.

The above discussion dealt with the solution methodology and the streaming effects only in the inner Stokes layer region. Alhough this region is very narrow, the mechanics therein are very crucial since the large velocity gradients in these Stokes layers are responsible for the origin of the thermoacoustic streaming effects. These effects result in a time-averaged temperature distribution being induced which in turn determines the steady temperature distribution in the rest of the fluid domain external to this narrow region. This behavior occurs in much the same way as the generation of the time-averaged acoustic streaming velocity distribution which in turn functions as a slip velocity in driving the steady flow in the outer region.

The mechanics of the inner region has to be appropriately coupled to that of the outer region. For the flow behavior, it would be necessary to ensure a proper matching of the stream function and the velocity potential between the inner and outer regions. In the absence of the thermoacoustic streaming effect, the temperature could have been treated as a passive scalar field and could have also been coupled in a similar manner. In the present case however, the pressure and fluid density variations are also important and have to be carefully matched in a consistent manner. For the large streaming Reynolds being studied, the outer region is expected to have a boundary layer structure which will be numerically determined subject to the matching conditions from the inner region. This procedure is fairly well established for such flows as reported earlier by Gopinath & Mills [6] and Gopinath [3, 4]. Of particular interest in the outer region will be the influence of the steady acoustic streaming fluid motion on the steady thermoacoustic streaming temperature distribution. This feature will determine the steady convective heat transfer rate in the outer region and needs to be computed numerically from a boundary layer form of the governing equations. This behavior of the influence of the time-averaged flow on the time-averaged heat transport will describe the overall transport effect due to these combined streaming phenomena.

3.3 The Case of Drops/Bubbles

The case of the levitated sample being a compressible medium, such as a drop/bubble, would represent a more generalized version of this problem. For simplicity, the sample will be assumed to be spherical (for sufficiently small diameters and high surface tension), with the acoustic field sufficently well controlled so as to not induce any shape oscillatory modes in the drop/bubble. For such cases, careful matching of the fluid velocities and stresses across the spherical interface is required, along with maintaining the continuity of temperature and heat flux. Based on progress with the case of the rigid sphere, this is the final problem that will be dealt with to establish more general results for these thermoacoustic effects.

4 Conclusions

This study provides a controlled setting to explore some fundamental thermoacoustic streaming transport behavior in conjunction with a study of the significance of a second-order thermodynamic modulus. It is hoped that with such a controlled study, it would be possible to obtain a good grasp of the nature of the thermoacoustic streaming phenomenon and identify the precise role of the above mentioned thermodynamic modulus. It is projected that the current theoretical analysis would also yield ideas for possible future experiments to explore these transport phenomena and measure this thermodynamic modulus. It is possible to envisage experiments which would involve the use of heat sensing instrumentation inside the levitated sphere to detect the temperature and heat transfer exchange rate with the sphere. This experimental data could then be correlated with the theoretical results which would serve as reference, to make inferences about the roles of the different heat exchange mechanisms.

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