WAVES IN RADIAL GRAVITY USING MAGNETIC FLUID

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ABSTRACT

We are beginning laboratory experiments using magnetically active ferrofluids to study surface waves in novel geometries. Terrestrial gravity is eliminated from the dynamics, and the magnetic body force felt by ferrofluid in the presence of a magnetic field gradient is used to create a geopotential field which is a section of or an entire sphere or cylinder. New optical, electromagnetic and ultrasonic diagnostic techniques are under development to initially study capillary-gravity wave propagation and interaction in such geometries.

INTRODUCTION

Terrestrial laboratory experiments studying various fluid dynamical processes are constrained, by being in an Earth laboratory, to have a gravitational body force which is uniform and unidirectional. Therefore fluid free-surfaces are horizontal and flat. Such free surfaces must have a vertical solid boundary to keep the fluid from spreading horizontally along a gravitational potential surface. In the absence of terrestrial gravity, surface tension forms fluid masses into spherical balls without solid boundaries, as demonstrated on the Space Shuttle in the Drop Physics Module [ref. 1, for example]. A fundamentally different problem is the behavior of fluids with a *body* force rather than a surface force that generates the spherical geometry. We are beginning a suite of experiments on a wide range of phenomena that use magnetic body forces in so-called "ferrofluids" to generate unique configurations for physical modeling.

The linear and nonlinear characteristics of traveling or standing waves on a liquid surface are a classic problem in fluid dynamics. Capillary-gravity waves represent an interaction of fluid inertia with the local gravitational *body* restoring force *and* free-surface boundary curvature *surface* tension [2]. It is of great theoretical interest to contrast wave behavior with and without lateral boundaries. Lateral boundaries have a strong effect on the spatial structures (fronts, pulses, solitons, etc.) observed in nonlinear propagating wave systems [3, for an extensive review]. With a spherical magnetic body force a straight circumferential channel formed by boundaries at ± latitudes is an experimental system for free-surface capillary-gravity waves which has periodic boundary condition in one direction. Similarly, an entire spherical surface is boundary-less and periodic in two-dimensions. For an overfull channel, the sidewall boundary contact line becomes pinned and the additional cross channel curvature increases the wave speed [4]. In an equatorial channel geometry, the contact lines are of the same length (as opposed to in an annular geometry) and this symmetry makes comparison to theory easier. The additional longitudinal curvature further increases the wave speeds.

If the gravitational force oscillates in time, then standing capillary gravity waves are excited. This is normally accomplished by vibrating a fluid container vertically. These "Faraday waves" have become, along with Rayleigh-Benard convection, a canonical system for studies of non-equilibrium pattern formation,

spatio-temporal evolution, and chaos [3]. The spherical geometry, with parametric oscillation obtained by oscillating our magnetic field, is a very clean system for investigating such parametrically excited waves, without complications of boundary shapes and contact angles which can determine patterns and mode competition dynamics even very far from the boundaries [5].

BACKGROUND

Ferrofluids are dilute suspensions of magnetic dipoles, for example magnetite particles of order 10 nm diameter, suspended in a carrier fluid. A surfactant coating keeps the particles separate enough that thermal Brownian motions in the fluid are sufficient to overcome both gravity and particle-particle attraction to keep the dipoles in suspension. For flows in which external magnetic field variations and the bulk fluid motions are slow compared to the time for the magnetic fluid particles to rotate ($\sim 10^{-6}$ s), the fluid magnetization, $\vec{M}$, is parallel to the applied magnetic field, $\vec{H}$. If there is a gradient in $H \equiv |\vec{H}|$, then there will be a systematic body force owing to the slight but persistent correlation of field strength and pole sense. The body force terms in the Navior-Stokes equations become:

$$\rho \frac{d\vec{u}}{dt} + \cdots = \cdots + \rho \nabla \Phi + M \nabla H .$$

where $\nabla \Phi = \vec{g}$ as usual. The superparamagnetic ferrofluid response is complicated by the fact that M is a function of the applied field unlike ρ for the gravitational case. (M increases linearly with H for small H and saturates at constant M for large H.) Ferrofluids became commercially available in the 1960's and have been used in several engineering applications [6].

One of the first scientific applications of the ferrofluid magnetic body force in an experiment large enough for inertial forces to be important was initiated by one of the investigators (DRO). Equatorially trapped Kelvin and Rossby waves, which are important in large-scale atmospheric and oceanic fluid dynamics, were studied in the laboratory for the first time. This laboratory experiment demonstrates the basic technique to be used in the present study. A sketch and photograph from this study are shown in Figure 1. The apparatus was cylindrically axisymmetric, and the body force on the water-based ferrofluid depends only on the field strength and gradient from an interior magnet. A cylindrical magnet was embedded in a smooth, solid, plaster/plastic spherical core. The geopotential field and its gradient, the body force, were made nearly spherical by careful choice of magnet shape and size in relation to the solid core, and by immersing the "planet" and its ferrofluid ocean in immiscible silicone oil of the same density. Thus the earth gravity was removed from the dynamics of the ferrofluid/oil interface and the only dynamically active force was the radial magnetic gravity. The entire apparatus rotated, and waves were forced by exterior magnets. In the photograph in Figure 1b, an example very large height perturbation is forced by an additional, stationary, hand-held magnet at left. Upon release such a bulge propagated in the co-rotating or eastward direction (as a Kelvin wave) or in the counter-rotating or westward direction (as a Rossby wave) along the equator, trapped there by the overall rotation and spherical geometry.

The fluid height on the equator as a function of longitude was observed in profile by viewing the limb of the black ferrofluid spheroid against a white background. Video frames of the waves were grabbed and then contoured and converted to polar coordinates. The radial displacement amplitude, $r - r_0$, at each longitude, ϕ, where $r_0(\phi,)$ is the undisturbed radius, is contoured in Figure 2. A side view of the waves showed that the amplitude was trapped to near-equator latitudes, as expected, for both the eastward Kelvin waves and westward Rossby waves in this rapidly rotating experiment. The time/longitude contour plot of equator height in Figure 2a shows a Kelvin wave propagating eastward at constant speed more than twice around the spheroid. In Figure 2b a westward Rossby wave propagates much slower. The theoretical linear Rossby wave speeds are 1/3, 1/5, 1/7, etc. of the Kelvin wave speed and those slopes are shown in Figure 2b

for reference. These experiments were the first laboratory study of this class of waves which have been recently observed with satellite altimetry in the Pacific ocean [7] and appear to a integral component of the equatorial climate system, especially the El Niño - Southern Oscillation [8]. Because of the high cost of ferrofluids, these are some of the first experiments in which inertial effects are important. These large experiments (~ a few liters) were possible by using new inexpensive ferrofluid at large dilution.

FUTURE WORK

A larger version of the spherical geometry experimental setup in Figure 1 is being constructed. The size increase and use of fluids with smaller viscosity will give more than an order of magnitude reduction in viscous damping. With this apparatus we will first investigate capillary gravity waves at the interface of a neutral two-fluid system in a zonal channel. In particular we will study the effects of pinned contact lines on nonlinear waves and the dynamics of nonlinear waves leading to contact line rupture. Pinned contact lines used for stabilizing stationary and rotating liquid columns in float-zone crystal growth experiments are sensitive to residual accelerations in the microgravity environment of a spacecraft [9]. Although many studies have been made of the effect of g-jitter on the stability of cylindrical fluid columns [10, 11], few studies directly concerned with the problem of pinned contact line rupture are available. Weidman, *et al.*, [12] have investigated the *static* rupture of pinned rotating menisci formed by overfilling annular grooves in a rotating disk. We will investigate the *dynamic* rupture of contact lines pinned to the boundary of a circular basin at the North pole or pinned to the continuous edges of an equatorial cylindrical zone (Figure 3a) using both sinusoidal variation of the magnetic radial gravity (using an interior electromagnet) or large amplitude traveling waves, in both stationary and rotating experiments. Pinned contact lines which are stable at the onset of infinitesimal hydrodynamic instability, will rupture at sufficiently large g-jitter amplitude, or during interference of two colliding or over-taking large-amplitude waves in the periodic geometry. Experimentally, these waves can be cleanly generated by magnets external to the fluids.

Studies of standing capillary-gravity waves in laterally bounded containers, forced by parametric oscillation of gravity, have undergone a recent resurgence. We will initiate a new class of ferrofluid experiments on a sphere or in an equatorial channel to study pattern formation with periodic boundaries and no contact angle impurities. On the entire spherical surface or in a polar basin formed by a high-latitude circle boundary, symmetry-breaking effects of rotation on traveling and standing waves patterns will be studied. Our initial analysis indicates that we will have sufficient electromagnetic driving strength in the frequency range 0.01 to 10 Hz, to generate Faraday wave patterns by parametric excitation with up to a few tens of waves in a circumference on the sphere or somewhat fewer fitting around inside a polar basin. The spherical system has the periodic boundaries of the standard theoretical development [5] but which have not been previously possible experimentally.

Ferrofluid experiments offer the potential for conducting laboratory studies of thermally driven oceanic flows over a substantial part of a sphere. As opposed to Earth atmospheric motions, in which sphericity, or the so-called Beta effect, is generally **not** a dominant term in the vorticity budget, slow deep overturning oceanic motions are highly constrained by the dynamical influence of planetary vorticity advection. Such effects cannot be studied in the terrestrial laboratory, when the all important case of continuous thermal stratification is considered, because of the strong gravity induced cells that have no oceanic analogs. Previous GFFC-type [13] experiments are also ill-suited to oceanic flow modeling because they are too viscous (thin gaps). We will address the key **technical** issues in building a relatively simple but effective microgravity based experiment to enable laboratory study of continuously stratified spherical flows in complex basins spanning a latitude range of at least plus or minus 60 degrees (Figure 3b). We realize that such an experiment must be conducted in microgravity, and our terrestrial demonstration experiment will be

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contaminated by terrestrial gravity induced flows. The main technical issues to explore are: 1) how to get a large enough magnetic buoyancy frequency in an apparatus of sufficient size, 2) how to stratify and force motion in the experiments, and 3) how to visualize the flows effectively.

ACKNOWLEDGMENT

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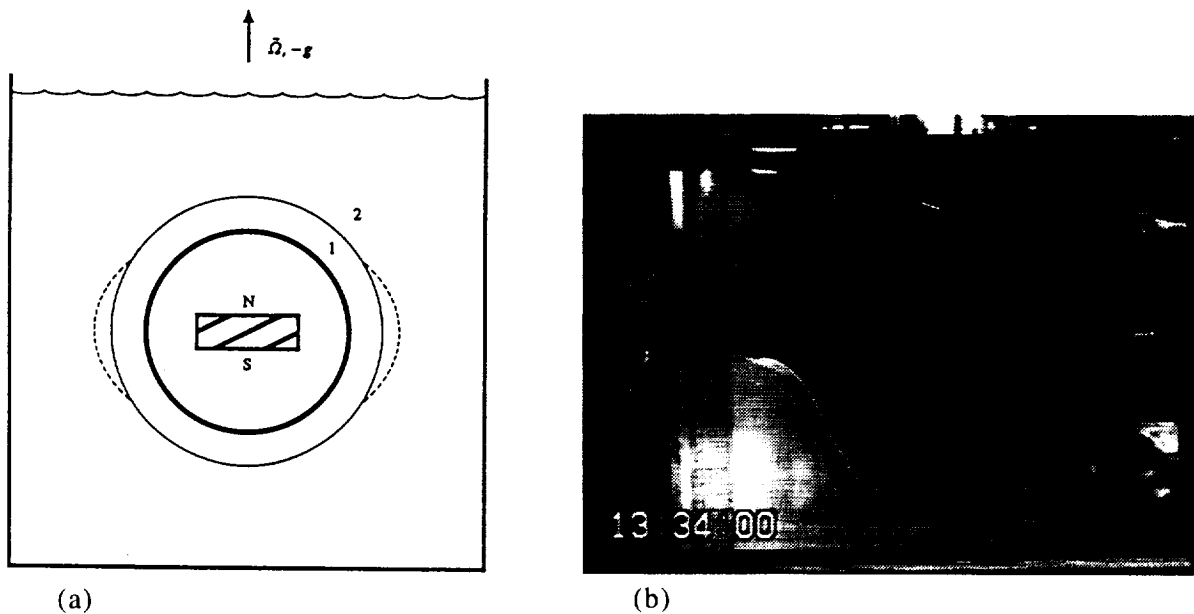
(a)   (b)

Figure 1. Side view schematic cross section (a) and photograph (b) of cylindrically symmetric experiment. A cylindrical tank holds a silicone oil-freon mixture (2) surrounding a water-based ferrofluid layer (1) on a plaster/plastic ball covering a strong permanent magnet. $\rho_1 \cong \rho_2$. The entire apparatus rotates at angular velocity, Ω. In the photograph, an external magnet forces a large amplitude wave.
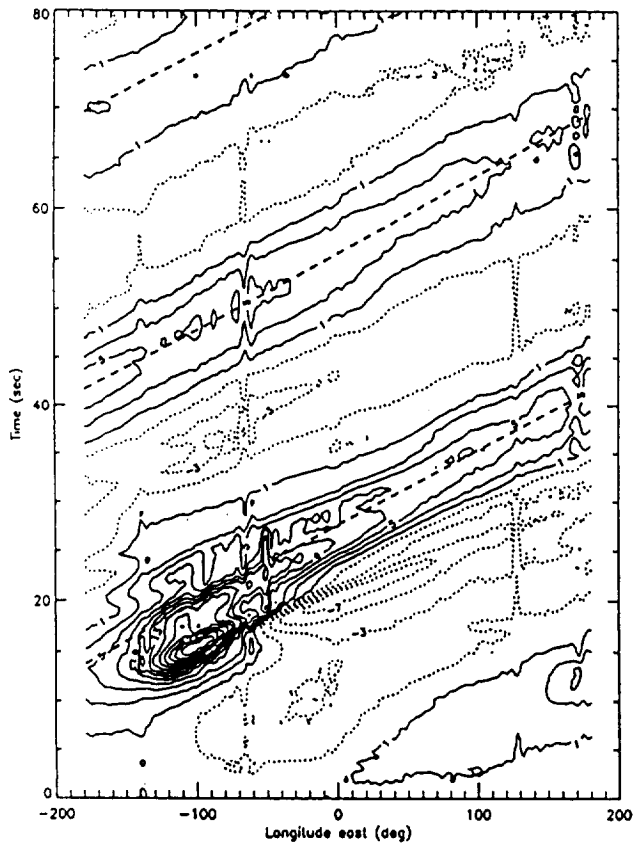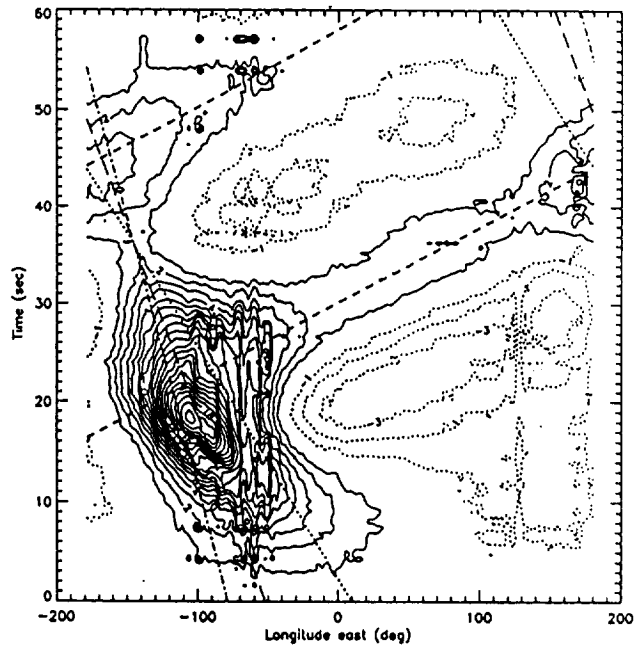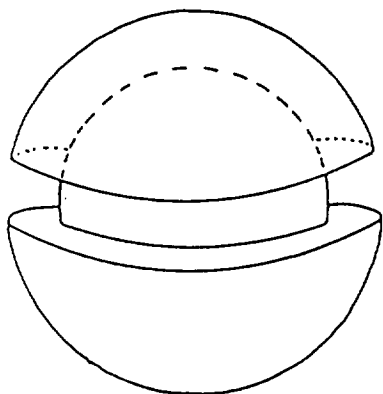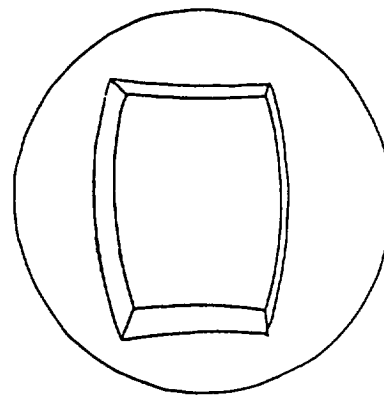
Figure 2. Contours of displacement height, $r - r_0$, near the equator as a function of longitude and time. a) Eastward forcing at about the Kelvin wave speed. b) Westward forcing at 0.44 times speed of a). The eastward line, - - -, has constant speed, c. The lines · · ·, — — —, and - · · - correspond to westward speeds which are 1/3, 1/5, and 1/7 of c, respectively.

Figure 3. Schematic of (a) equatorial channel and (b) ocean basin experiments.