

59-36

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**STRESS TUNING OF LASER CRYSTALS**

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## ABSTRACT

The topic of stress tunable laser crystals is addressed in this study with the purpose of determining the piezo-optic coefficients of a new laser material. This data was collected using a quadruple pass birefringence technique because of its high degree of sensitivity relative to the other methods examined including fringe shift analysis using a Mach-Zender interferometer. A green He-Ne laser was passed through a light chopper and Glan-Thompson prism before entering a crystal of Erbium doped Yttrium Aluminum Garnet (Er:YAG) (used in order to validate the experimental technique). The Er:YAG crystal is mounted in a press mechanism and the laser is quadruple passed through test specimen before being returned through the prism and the orthogonally polarized portion of the beam measured with a optical sensor. At a later stage, the Er:YAG crystal was replaced with a new crystal in order to determine the piezo-optic coefficients of this uncharacterized material. The applied load was monitored with the use of a 50 lb. load cell placed in line with the press. Light transmission readings were taken using a lock-in amplifier while load cell measurements were taken with a voltmeter from a 5 volt, 0.5 amp power supply. Despite the fact that an effective crystal press damping system was developed, size limitations precluded the use of the complete system. For this reason, data points were taken only once per full 'turn' so as to minimize the effect of non uniform load application on the collected data.

Good correlation was found in the transmission data between the experimentally determined Er:YAG and the previously known piezo-optic constants of non-doped crystal with which it was compared. The variation which was found between the two could be accounted for by the aforementioned presence of Erbium in the experimental sample (for which exact empirical data was not known).

The same test procedure was then carried out on a Yttrium Gallium Aluminum garnet (YGAG) for the purpose of establishing values of its unknown piezo-optic constant tensor using experimentally collected transmission data. Significant variation between the piezo-optic constants of YAG and YGAG crystals was found however, the excellent data correlation of separate experimental runs carried out on the YGAG sample demonstrates the validity of these results. The data collected during the stressing of the YGAG was of high quality, however the amount of data collected was somewhat limited by a fracture of YGAG specimen which undoubtedly altered the crystalline lattice structure and hence precluded any further testing.

## Introduction

A solid state laser is typically thought to operate at a particular wave length (such as the Helium Neon laser). In the case of those using lanthanide series atoms, however, the lasers have a slight tunability around the line center. Different wave lengths of light are employed in various applications because of the optical properties ( such as absorption or transmission) which a given applications requires. Light is transmitted and absorbed by different media at various wavelengths and often, a given laser will not operate at an ideal wavelength for a desired application. As one solution to this problem a laser may be essentially tuned using a variety of photoelastic methods. As a stress is applied, the predominant effect is a proportional change in the element overall dimensions, mainly a compression in the direction of the applied stress and an expansion perpendicular to it. Other such techniques have been employed which, as a second order effect, change the refractive index of the laser through direct manipulation of the laser crystal itself. One manifestation of this photoelastic effect involves the heating of the laser crystal in order to bring about thermal expansion to introduce a strain, which yields a change in the optical path length and hence the resulting change in the material through the thermo-optic constant. The topic of this report, however, involves the application of mechanical rather than a thermal stress (through the use of a screw press) which compresses and eventually strains the material, altering its refractive index, and consequently changing the values of the piezo-optic constants used to theoretically predict the effect of stress on the refractive index .

The experimental intention is to first establish an accurate method of incrementally stressing an erbium doped yttrium aluminum garnet and to replicate the previously determined values of the piezo-optic constants of the sample for validation purposes. Next, the same experimental procedure is carried out a second time on a material for which the piezo-optic constants are not known. Utilizing the validated photoelastic technique, a value of the transmission through the laser material per incremental load applied is obtained and evaluated with the use of a theoretical model in order to 'back-calculate' the values of the piezo-optic constants of this unknown material.

## THEORY

The purpose of the experimental research addressed in this report is to facilitate the tuning of the wavelength of a laser through the application of a direct mechanical stress in order to alter the crystalline lattice structure and hence, the performance characteristics of the laser itself. Theoretically, this change in the lasing material's optical properties is evident in a fourth rank tensor composed of the piezo-optic constants (denoted  $q_{ijkl}$ ). The below relationship relates these piezo-optic coefficients, the applied stress tensor and the relative dielectric tensor 'K' where repeated indices indicate summation:

$$\Delta(K^{-1})_{ij} = q_{ijkl} * \sigma_{ijkl}$$

The piezo-optic constants describe the effect of stress on the index of refraction of the material and it is the index of refraction which dictates many of the optical properties which a given material exhibits. In carrying out calculations on the photo elastic effect, the inverse of the relative dielectric tensor (called the relative dielectric impermeability tensor and expressed as 'B') is used in defining equations. The components of this 'B' tensor are the coefficients of the 'indicatrix'; a graphical representation of optical properties expressed as an ellipsoid of wave normals. The value of the semi-axis of these wave normals are the square roots of the dielectric constants which compose the 'B' tensor and coincide in orientation with the principal dielectric axis of the laser material as well. The unstressed material has an index ellipsoid equation similar to that of a sphere, however after a stress is applied, this 'indicatrix' equation changes significantly and correspondingly, alters the values of the refractive indices as well. Nevertheless, if shear stresses are minimal, the local material axes will remain oriented with the principle axes.

The elasto-optic constant (expressed as 'p') relates changes in the refractive index to strain and also yields information pertaining to the aforementioned piezo-optic constants of the material strain as well via the following relationship:

$$P_{ijkl} = q_{ijrs} * c_{rskl}$$

The materials to be tested are of a crystalline atomic structure therefore, in their least symmetric form can have up to 36 independent piezo-optic constants and 21 independent elastic constants. Due to the benefits of symmetrical lattice structure however, many of these values go to zero. A cubic material will typically possess only three such independent, non-zero constants and in the case of the isotropic materials, only two. Because the dielectric impermeability tensor, stress and strain tensors are all symmetric, a more simplified notation for these expressions is typically adopted as follows where 'c' is a component of the elastic stiffness tensor:

$$\Delta(B_m) = \Delta(K^{-1})_m = q_{mn} * \sigma_n$$

$$q_{mn} = P_{mr} * s_{rn}$$

$$P_{mn} = q_{mr} * c_{rn}$$

The theoretical basis for the experiment is the relationship which expresses the difference in optical path length between the two components of a beam of light incident on a crystalline specimen. The beam exits the crystal, having split into two components parallel and perpendicular to the optical axis and differing in optical path length as follows :

$$\Delta = d * ( n_o - n_e )$$

The length of the crystal is 'd' and  $n_o$  &  $n_e$  are the differing indexes of refraction of the two components. Expressed in terms of phase shift the above expression becomes:

$$\Delta \Phi = (2\pi / \lambda) * [l * ( n_o - n_e )]$$

Recall prior mention of the form of the refractive index ellipsoid and its sensitivity to the photo-optic effect. Assuming a cubic material, the equation of the 'indicatrix', which had been that of a sphere, is now of the following form as a result of the applied stress:

$$(1 / n^2 + q_{12} * \sigma_2) * x^2 + (1 / n^2 + q_{12} * \sigma_2) * y^2 + (1 / n^2 + q_{12} * \sigma_2) * z^2 = 1$$

Solving this equation for the index of refraction in the x and y directions :

$$n_x = n * (1 + n^2 * q_{12} * \sigma_2)^{-1/2} \quad \text{and} \quad n_y = n * (1 + n^2 * q_{22} * \sigma_2)^{-1/2}$$

If these values are inserted into the previous expression of phase difference through the specimen:

$$\Delta \Phi = ((\pi * n^3 * l) / \lambda) * ( q_{11} - q_{12} ) * \sigma_2$$

With an expression for the phase difference, the transmission of a polarizer plus wave plate plus analyzer assembly on a quadruple pass is expressed as:

$$T = S / S_o = \sin^2 (\Phi / 2) \cong \Delta \Phi^2 / 4$$

$$T = S / S_o \cong \{ [(4\pi * n^3 * l) / \lambda] * ( q_{11} - q_{12} ) * ( F / A ) \}^2 / 4$$

It is this expression which is used to validate the experimental method by matching the theoretically determined transmission of a material with known optical properties to that obtained by examining the coefficients of the curves fitted to the experimental data. In addition, this expression is then used to 'back-calculate' the piezo-optic coefficients of a new material using experimentally determined transmission values.

## Experimental Apparatus

Throughout the course of the experiment three different optical assemblies were utilized; a Mach-Zender interferometer as well as both a single and double pass variations of birefringence analyzers in order to obtain improved sensitivity and resolution. Throughout this series of experimentation however, several of the experimental components have remained constant.

The light source used for all three cases was a green Helium-Neon laser operating at a wavelength of 543 nanometers. In addition, the specimen stressing apparatus was a spring loaded crystal press. The base and threaded screw head are constructed of a stainless steel, however the frame was made of a plastic. The screw shaft itself was also constructed of steel with a spring loaded tip capable of applying approximately fifty pounds of force over 9-10 full turns. The screw shaft originally contacted a precision machined stainless steel piston with provisions for a sub-surface mounted thermocouple.

Over the early portion of the experiment it became evident that it would be necessary to characterize the performance of the press in order to draw some correlation between the number of turns and the actual load being applied as well as to establish what degree of uniform loading could be obtained using the basic assembly. In order to determine the load/turns relationship in effect, it was necessary to measure the compressive stress being applied at specified radial positions. A 50 pound load cell was enlisted for this task which used a four strain gauge arrangement in a Wheatstone bridge circuit measure load. The strain gauge measures change in dimension of a specimen (a load cell in this case) by passing a small current through a fine conducting wire which is attached securely to the surface of the loaded specimen. Half an amp of current from a five volt power source is put across two legs of the bridge (that is through the strain gauges) and is ultimately measured across the other two legs by a voltmeter. As a load is applied, the gauge and the specimen are deformed, resulting in a change in the resistance since the resistance of a conductor is a factor of , among other things, its cross-sectional area. The change in the circuit resistance is reflected in a voltage reading which may be converted directly into a load value, so long as the value of the shunted bridge is known as well. This value is used in the calibration of the load cell , in conjunction with additional manufacturer supplied calibration constants to account for changes which have occurred in the bridge circuit due to temperature variation, variation in electrical connection quality and so on. It is obtained by inserting a calibration resistor in parallel into the Wheatstone bridge circuit to obtain a shunt calibration value, from which the circuit sensitivity is found and, when multiplied by the transducer (load cell) output, is the applied load in pounds.

Early data readings lead to a suspicion that loading conditions were strongly dependent upon the azimuthal position of the screw shaft and readings obtained from the load cell confirmed this suspicion. The applied load, although linear in its overall trend, is oscillatory in nature about a linear mean value which strongly suggests non-uniform loading conditions. In order to remedy this situation, several damping mechanisms were introduced to the system. In order to eliminate variation in the load application point (suggested by the small circular pattern etched into the top piston face) a two piece piston / shaft assembly was fabricated with a ball spacer between them was utilized and a clear damping effect was observed in the resulting data. In addition, a hard rubber disc was also placed on the bottom face of the piston (which contacts the test specimen) which proved to have a substantial additional damping effect when used in conjunction with the two piece piston. Satisfactory data was obtained with this experimental assembly however for the actual experiment, the load cell was placed in line with the test specimen. This experimental

setup eliminated the need for a load / turns correlation but also precluded the use of the complete damping assembly as there was not enough room in the screw shaft to accommodate both the load cell and the shaft portion of the piston /shaft assembly. The shortened piston along with the rubber padding and the ball-bearing were used in all the experimental runs.

The Mach-Zender interferometer allows the projection of light interference fringes which, when a load is applied are caused to move. This shift in the interference fringes with applied load results in a series of nulls in output intensity which can be expressed in fringe shift per unit load with the below equation and hence, yield the desired piezo-optic constants. In the Mach-Zender the laser is passed through a beam splitter, resulting in two beams of like characteristics. One of the beams is then passed through the test specimen and the two beams are then made to interfere. Destructive interference fringes occur essentially when the 'peak' and a 'trough' of a two photon waves interact and produce a null. Since the two beams travels different path lengths, they are out of phase when they do interfere, the fringes are visible. As a load is applied a fringe shift occurs (a phase difference of  $2\pi$  corresponds to one whole fringe) but it is often difficult to apply a sufficient load to cause a shift of even one fringe without fracturing the test material. This proved to be true in our case and the Mach-Zender approach to interferometry had to be abandoned in favor of a system with greater sensitivity.

The second experimental arrangement employed light polarization as the measurement tool with which the birefringence was observed. The same He-Ne laser and crystal press were used, but this time in conjunction with a Glan-Thompson prism in order to facilitate examination of photoelastic effects using a birefringence measurement. The Glan-Thompson prism illustrated in figure #5 is made by diagonally cutting a single rectangular calcite crystal, grinding and polishing the cut inner faces and then bonding the calcite wedges together with a small gap. Calcite crystal is an anisotropic crystal which has the property of double refraction when struck by an incident beam. A single laser beam which enters the crystal will then leave it as two beams, not only separated, but also orthogonally polarized. The crystal essentially resolves the incident beam into two perpendicular plane polarized rays due to the crystals birefringent properties. The effect of the Glan-Thompson arrangement of two such prisms is to essentially eliminate one of two incident waves which pass through it. Since the air spacer which separates the two prisms has an index of refraction lower than that of the two emerging beams, the essentially undeflected beam of higher refractive index is removed by total internal reflection at the interface while the beam of lower index of refraction is transmitted.

The now polarized beam is passed through the specimen which if it is unstressed, does not alter the optical path length. The specimen itself acts as a waveplate, breaking the plane polarized light again up into two components. That component with vibrations parallel to the specimen optic axis moves faster through the specimen than that which vibrates perpendicular to it, thus resulting in a difference in optical paths as well as corresponding phase difference. The stressed specimen's optic axis has a single axis of symmetry with respect to crystal form, atomic structure and in this case, the applied stress as well.

The beam passed through the specimen, is reflected, and is passed through a second time. The benefit of this 'double pass' setup (or the 'quadruple pass' used in third and eventually final experimental set up) is to effectively double (or quadruple) the magnitude of the change in optical path length due to the application of the load to the specimen and hence affect a corresponding magnification in phase shift and optical sensor signal intensity. The beam is then directed back into the Glan-Thompson prism where the orthogonally polarized portion of the beam is reflected into an optical sensor with which the return beam intensity is measured. This sensor effectively converts the photon signal of the light into an electrical signal which is expressed as a voltage.

When used in conjunction with a light chopper and a lock-in amplifier, the beam of interest may be modulated at a given frequency by the chopper, and only that frequency signal received by the sensor will be reported by the amplifier.

## **EXPERIMENT**

In order to validate the experimental method, an Er:YAG laser crystal was tested initially. Incremental data readings were taken from zero to approximately fifty pounds - the maximum load of the press and the design limit of the load cell. Next, 'Zero' readings were then subtracted from both the transmission and applied load data and the load data was converted from a millivolt reading into pounds of force using the procedure outlined in appendix A. Using this data, transmission versus load graphs were generated and a theoretically predicted curve fitted to it. As described in the previous section, the transmission of the experimental system being used ( polarizer plus waveplate plus analyzer ) is given by a sinusoid squared relationship which may be mathematically modeled as a Taylor polynomial. Neglecting higher order terms and fitting the resulting fourth degree polynomial to the data set we are able to obtain an experimentally determined value of transmission, expressed in terms of the independent 'force' variable. The system has been predicted to behave as sine squared and so we expect a very small constant, linear and cubic terms and very large quadratic and quartic terms. Squaring the applied load data values allows us to fit the data to a simple quadratic and by dividing through by the experimentally determined maximum signal intensity ( $S_0$ ) and the load cell sensitivity, we obtain a result in terms of force squared.

The experimental procedure used in testing the Yttrium Gallium Aluminum Garnet (YGAG) crystal was the same as the used in the previous case however, the data reduction procedure was altered slightly. The overall specimen size was reduced by 75% while the overall length remained the same resulting in greater applied force per unit area. It is then reasonable to expect the resulting data to extend further into the quartic portion of the sine squared curve ( the 'peak' of the sinusoid curve if you will ) and the resulting curve fit to be dominated by the higher order quartic terms. For this reason the transmission data was first linearized by taking the arcsine of the square root of the transmission and then fitted directly rather than linearly ( i.e.  $y = b x$  rather than  $y = a + b x$  ). In addition, once the value of transmission has been found it is necessary to determine the YGAG material's refractive index as well. One may then solve for the piezo-optic constants of the YGAG crystal using the aforementioned theoretical determined equation for transmission and the experimentally determined data values.

## RESULTS

The following set of result are of the Er:YAG crystal which was tested for the purpose of validating the experimental method and compared with theoretical calculations. The percentage difference between the two results is most likely due to the fact that experimental analysis was performed on an Erbium doped YAG crystal while the theoretical calculations were carried out using the piezo-optic constants of a non-doped conventional YAG crystal.

**EXPERIMENTAL LASER TRANSMISSION :  $35.8542 \times 10^{-6} F^2$  (#)**  
**THEORETICAL LASER TRANSMISSION :  $30.38 \times 10^{-6} F^2$  (#)**  
**PERCENTAGE DIFFERENCE : 15.27%**

Listed below is the experimentally determined 'b' coefficient of the applied force over two separate runs) obtained in directly curve fitting the experimental data ( $y = bx$ ), along with the percentage difference between them. Also listed are the experimental YGAG piezo-optic constants ( $q_{11}$ - $q_{12}$ ) calculated using the data from these two runs along with their percentage difference as well. Finally, the percentage difference is given of the average value of the experimentally determined YGAG piezo-optic constants and that of the YAG crystal which was used previously in the validation of the experimental method.

**DIRECT FIT 'b' COEFFICIENT - RUN#1 :  $9.33 \times 10^{-3}$**   
**DIRECT FIT 'b' COEFFICIENT - RUN#2 :  $9.43 \times 10^{-3}$**   
**PERCENTAGE DIFFERENCE : 1.06%**

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**EXPERIMENTAL ( $q_{11} - q_{12}$ ) #1:  $0.1313 \times 10^{-12} Pa^{-1}$**   
**EXPERIMENTAL ( $q_{11} - q_{12}$ ) #2:  $0.1328 \times 10^{-12} Pa^{-1}$**   
**PERCENTAGE DIFFERENCE : 1.13%**

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**( $q_{11} - q_{12}$ ) YGAG / YAG PERCENT DIFFERENCE - 23.18%**

## CONCLUSION

The transmission data collected during the experimental test validation showed good agreement between the experimentally determined Er:YAG and the piezo-optic constants of non-doped YAG crystal with which it was compared. As stated previously, the presence of Erbium in the experimental sample is clearly a potential cause of the variation in results which was found between the two. A non-doped YAG was not available at the time.

The same test procedure was then carried out on a Yttrium Gallium Aluminum Garnet (YGAG) in order to establish its piezo-optic constant tensor. Significant variation of 23.2% between the piezo-optic constants of YAG and YGAG crystals was found however, the excellent data correlation of separate experimental runs carried out on the YGAG sample - a 1.13% percent difference - certainly bolsters the degree of confidence which can be placed in the validity of these results. Unfortunately, during later experimental runs a crack formed in the YGAG specimen and thus the amount of data collected for this particular specimen was somewhat limited.

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