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*A FEASIBILITY STUDY ON A PARALLEL MECHANISM FOR EXAMINING THE  
SPACE SHUTTLE ORBITER PAYLOAD BAY RADIATORS*

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## ABSTRACT

This report summarizes the author's work as a 1996 NASA/ASEE Summer Faculty Fellow at Kennedy Space Center. The goal of the project was the development of the necessary analysis tools for a feasibility study of a cable suspended robot system for examining the Space Shuttle orbiter payload bay radiators. These tools were developed to address design issues such as workspace size, tension requirements on the cables, the necessary accuracy and resolution requirements, and the stiffness and moment requirements of the system. The report describes the mathematical models for studying the inverse kinematics, statics, and stiffness of the robot. Each model is described by a matrix. The manipulator Jacobian matrix characterized both the inverse kinematics and the statics of the robot. The manipulator Jacobian was also related to the stiffness matrix, which characterized the stiffness of the system. Analysis tools were then developed based on the singular value decomposition (SVD) of the corresponding matrices. It was demonstrated how the SVD can be used to quantify the robot's performance and to provide insight into different design issues.

## 1. INTRODUCTION

The Space Shuttle orbiter payload bay radiators are inspected before and after each mission in the Orbiter Processing Facility (OPF). These inspections are labor intensive and require access to overhead bridge crane buckets. The buckets are platforms that are attached to a telescoping tube which is attached to the overhead bridge crane trolley. Technicians ride in the buckets and visually inspect the radiator panels for defects. When defects are found, they need to be quantified and their location logged.

The inspection of the orbiter's radiators is an ideal candidate for automation. The use of robotics for the inspection of payload bay radiators will eliminate the use of the buckets, reduce coordination effort and paper work required, reduce inspection time, and increase personnel and equipment safety. Furthermore, a robotic system can prepare electronic maps of the radiator's surface and automate the generation of problem reports. Robotic concepts have been developed; however, these concepts need to be rigorously evaluated for technical feasibility.

McDonnell Douglas Aerospace has developed a series of unique tendon suspended robots under the trademark CHARLOTTE™ [1,2]. The robot translates and rotates on and about three axes by synchronously modulating the length of the suspending tendons. Charlotte™ was originally designed for attending Space Shuttle experiments and has flown on two Space Shuttle missions. Another version of Charlotte™ supports virtual reality simulations of extra-vehicular activities at the robotics laboratories of NASA's Johnson Space Center.

This report will consider the feasibility of using a cable suspended robot such as Charlotte™ for the radiator inspections. First, mathematical models for a cable suspended robot were developed. These models were used to study the inverse kinematics, the required cable tensions, and the stiffness of the robot. Analysis tools were then developed to address design issues such as workspace size, tension requirements on the cables, the necessary accuracy and resolution requirements, and the stiffness and moment requirements of the system. The report describes the mathematical models for studying the inverse kinematics, statics, and stiffness of the robot. Each model is described by a matrix. The manipulator Jacobian matrix characterized both the inverse kinematics and the stiffness of the robot. The manipulator Jacobian was also related to the stiffness matrix, which characterized the stiffness of the system. Analysis tools were then developed based on the singular value decomposition (SVD) of the corresponding matrices. It was demonstrated how the SVD can be used to quantify the robot's performance and to provide insight into different design issues.

## 2. MODELING THE ROBOT

### 2.1 INVERSE KINEMATICS

The robot is basically a box with cables attached to its eight vertices. Let the location of the  $i$ -th external cable connection be denoted by  $\mathbf{p}_i$  and the  $i$ -th onboard cable connection be denoted  $\mathbf{q}_i$ . The vector representing the  $i$ -th cable is

$$\mathbf{l}_i = \mathbf{p}_i - \mathbf{q}_i = \mathbf{p}_i - \mathbf{x} - \mathbf{Q}\mathbf{v}_i$$

where  $\mathbf{x}$  denotes the position of robot and the rotation matrix

$$Q = R_z(\phi)R_x(\theta)R_y(\psi) = \begin{bmatrix} c\phi c\psi - s\phi s\theta s\psi & -s\phi c\theta & c\phi s\psi + s\phi s\theta c\psi \\ s\phi c\psi + c\phi s\theta s\psi & c\phi c\theta & s\phi s\psi - c\phi s\theta c\psi \\ -c\theta s\psi & s\theta & c\theta c\psi \end{bmatrix}$$

represents the robot's orientation. A similar analysis can be found in [3].

## 2.2 INVERSE VELOCITY KINEMATICS

The inverse velocity kinematics relates the linear and angular velocities of the robot to the required rate of change in the cable lengths. This relationship is given by a manipulator Jacobian matrix  $L$  which is a function of the position and orientation of the robot. This matrix can be used to study the kinematics of the robot, e.g., identify kinematically singular configurations, determine optimal configurations, etc.

Let  $\mathbf{l}$  denote the vector of cable lengths. The inverse velocity kinematics for Charlotte™ was derived and is given by

$$\dot{\mathbf{l}} = L \begin{bmatrix} \dot{\mathbf{x}} \\ \omega \end{bmatrix}$$

where

$$L = \begin{bmatrix} -\hat{\mathbf{l}}_1^T & (\hat{\mathbf{l}}_1 \times Q\mathbf{v}_1)^T \\ -\hat{\mathbf{l}}_2^T & (\hat{\mathbf{l}}_2 \times Q\mathbf{v}_2)^T \\ \vdots & \vdots \\ -\hat{\mathbf{l}}_8^T & (\hat{\mathbf{l}}_8 \times Q\mathbf{v}_8)^T \end{bmatrix}.$$

## 2.3. STATICS MODEL

For the robot to be in equilibrium, the sum of the forces and the sum of the moments on the robot must be zero. Mathematically, this can be written as

$$\sum_a \mathbf{F}_a = \mathbf{0}$$

$$\sum_a \mathbf{M}_a = \mathbf{0}.$$

From these relations, it can be shown that the forces on the cables satisfy

$$\mathbf{A}\mathbf{F} = \mathbf{W}$$

where  $\mathbf{F}$  is the vector of cable tensions,  $\mathbf{W}$  is the vector consisting of the gravity term and moment of the robot, and  $\mathbf{A} = -\mathbf{L}^T$ .

The general solution of the force/moment equations is

$$\mathbf{F} = \mathbf{A}^+\mathbf{W} + (\mathbf{I} - \mathbf{A}^+\mathbf{A})\mathbf{z}.$$

where  $\mathbf{A}^+$  is the pseudoinverse of  $\mathbf{A}$  and  $\mathbf{z}$  is an arbitrary vector. This equation gives a two dimensional family of solutions. However, since there are cables involved, each component of the force vector must be nonnegative. A configuration will be called statically stable if such a

force vector exists. Intuitively, it would seem that the pseudoinverse solution would result in an unfeasible solution so the null space term  $(I - A^+A)z$  is necessary.

One may want to choose a solution that solves the following optimization problem:

$$\begin{aligned} & \text{minimize} \quad \|F\|_{\infty} \\ & \text{subject to} \quad F_i \geq B_i \end{aligned}$$

where  $B_i$  is some positive bound to insure that the cable does not become slack.

## 2.4 MODELING STIFFNESS

For the stiffness study, the cables were assumed to be under sufficient tension to be modeled as a spring. The stiffness for the  $i$ -th cable is given by

$$k_i = \frac{EA}{l_i}$$

where  $E$  is Young's modulus of elasticity,  $A$  is the cross sectional area of the cable, and  $l_i$  is the cable length. The stiffness matrix of the robot is given by

$$K = L^T K_D L$$

where  $K_D = \text{diag}(k_1, k_2, \dots, k_8)$ . The stiffness matrix is symmetric and positive semi-definite. A similar analysis can be found in [4].

## 3. SOME IMPORTANT KINEMATIC AND STATICS ISSUES

### 3.1 KINEMATICALLY STABLE CONFIGURATIONS

It is desirable that each motion of the robot result in at least one cable length getting longer and at least one getting shorter. Otherwise there is a motion that results in the cable lengths getting shorter. This could result in cables becoming slack. Kinematic instability is also related to the stiffness of the robot. We will say that a configuration is kinematically unstable if there is a motion that results in some of the cables becoming shorter with no cable becoming longer.

The following theorem, which characterizes kinematic stability by the left null space of  $L$ , was proven.

**THEOREM** A nonsingular configuration is kinematically stable if and only if there is a left null vector of  $L$  with the property that each of its components is positive.

This theorem is very useful for identifying kinematically unstable configurations and is easily implemented numerically. An example of a kinematically unstable configuration for the planar version of Charlotte™ is shown in Figure 1. Rotating the robot counter-clockwise with its center fixed results in each cable becoming shorter. One can sense from the figure that this configuration is not an equilibrium. The question of whether such configurations can be equilibrium points will be answered in the next subsection.

### 3.2 STATICALLY STABLE CONFIGURATIONS

An important question is the existence of a feasible force vector. We say that a configuration is statically stable if there is a force vector whose components are positive. Clearly, if there is a null vector whose components are positive, we can add enough of this vector to the pseudoinverse solution to obtain a force vector with positive components. Hence, we have the following.

**THEOREM** If there is a null vector of  $L$  with strictly positive components then the robot is in a statically stable configuration.

We thus have that a kinematically stable configuration is statically stable. However, a kinematically unstable configuration may possibly be statically stable depending on effects of gravity. For example, it is possible to hold the robot in static equilibrium while the robot is under all of the exterior cable connections. Such a configuration then is statically stable but not kinematically stable due to a pendulum effect. An example of a statically stable configuration which is kinematically unstable is the situation when the robot is lower than all eight cable connections. Below is an example of a statically unstable configuration for a planar version of Charlotte™.

**EXAMPLE** Consider the robot shown in Figure 1. Assume that the center of gravity is at the center of the robot and that the weight of the robot is 1 N. The outer cable connections are at (2,2), (2,-2), (-2,-2), and (-2,2) and the onboard cable connections are at (1,0), (0,-1), (-1,0), and (0,1). The  $\hat{\mathbf{i}}_i$  vectors are

$$\hat{\mathbf{i}}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \hat{\mathbf{i}}_2 = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \hat{\mathbf{i}}_3 = \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ -2 \end{bmatrix}, \text{ and } \hat{\mathbf{i}}_4 = \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

It then follows that the forces and moments satisfy

$$\frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 & -1 & -2 \\ 2 & -1 & -2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \mathbf{F} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

The general solution is

$$\mathbf{F} = \begin{bmatrix} 0.2 \\ -0.1 \\ -0.2 \\ 0.1 \end{bmatrix} + \alpha \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

where the first vector on the right is the pseudoinverse, the second vector is a null vector, and  $\alpha$  is an arbitrary constant. It is necessary to choose  $\alpha$  so that each component of  $\mathbf{F}$  is positive. This is clearly impossible as choosing  $\alpha$  to make the third component positive makes the second component more negative. One can see from the figure that this configuration is not kinematically stable as rotating the robot counterclockwise with its center fixed would correspond to having each of the cables getting shorter.

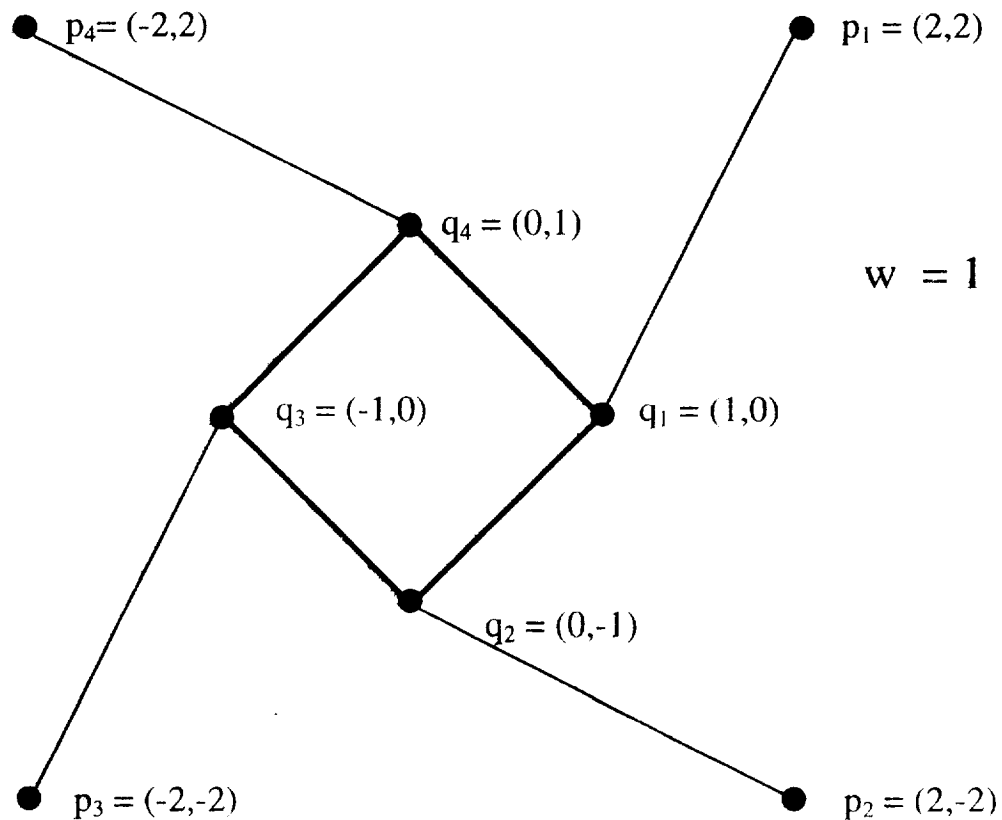


Figure 1 A configuration that is kinematically and statically unstable.

#### 4. ANALYSIS USING THE SINGULAR VALUE DECOMPOSITION

The singular value decomposition (SVD) is a matrix decomposition that provides insight into the input/output relationship of a matrix [5]. For example, it identifies which input vectors result in large output vectors and which ones result in small output vectors. Applied to the manipulator Jacobian, the SVD tells us the amount of change in cable lengths necessary to produce a given robot motion. This is important for analyzing the amount of resolution/accuracy required for the cable lengths. From the SVD we can determine the robot motion that results in the least change in the cable lengths. This tells us the direction in which the robot is most flimsy.

The SVD determines the null space of a matrix. This then tells us whether or not a configuration is statically stable. This is useful information for determining workspace limitations. Stiffness is also an important design issue. The SVD of the stiffness matrix tells us the directions in which



the robot is most stiff and least stiff and gives us the values of stiffness in these directions. This information can help in the choice of cable material.

Several programs using MATLAB were developed using the SVD to evaluate performance capabilities of the robot. The programs and results were provided to the NASA colleague.

## 5. RESULTS AND DISCUSSIONS

The goal of this the project was the derivation of the mathematical models for a cable suspended robot and the development of the analysis tools to evaluate the performance of the robot. The analysis tools were based on the singular value decomposition and are applicable to a large class of cable suspended robot systems. Computer software in the form of .m files was written to use a widely available commercial program called MATLAB. These programs will provide a basis for the evaluation and the design of a cable suspended robot system for the Space Shuttle orbiter payload bay radiators.

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