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USING SOLAR RADIATION PRESSURE TO CONTROL L2 ORBITS

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The main perturbations at the Sun-Earth Lagrange points L1 and L2 are from solar radiation pressure (SRP), the Moon and the planets. Traditional approaches to trajectory design for Lagrange-point orbits use maneuvers every few months to correct for these perturbations. The gravitational effects of the Moon and the planets are small and periodic. However, they can not be neglected because small perturbations in the direction of the unstable eigenvector are enough to cause exponential growth within a few months.

The main effect of a constant SRP is to shift the center of the orbit by a small distance. For spacecraft with large sun-shields like the Microwave Anisotropy Probe (MAP) and the Next Generation Space Telescope (NGST), the SRP effect is larger than all other perturbations and depends mostly on spacecraft attitude. Small variations in the spacecraft attitude are large enough to excite or control the exponential eigenvector. A closed-loop linear controller based on the SRP variations would eliminate one of the largest errors to the orbit and provide a continuous acceleration for use in controlling other disturbances.

It is possible to design reference trajectories that account for the periodic lunar and planetary perturbations and still satisfy mission requirements. When such trajectories are used the acceleration required to control the unstable eigenvector is well within the capabilities of a continuous linear controller. Initial estimates show that by using attitude control it should be possible to minimize and even eliminate thruster maneuvers for station keeping.

INTRODUCTION

Current numerical trajectory analysis does not take full advantage of optimization tools and analytical methods for designing transfer trajectories and gravity assists to the Sun-Earth L1 and L2 Lagrange points. The effects of perturbations due to the variation in solar radiation pressure and the periodic effects of the Moon should be used in selection of the nominal L2 orbit; . By redefining the nominal orbit instead of trying to control these perturbations later, we reduce deviations by several orders of magnitude. The authors are prototyping analytical tools in Mathematica and MATLAB to design and select nominal orbits of this type. The new tools are being designed so they can easily be incorporated into the next generation of operational flight dynamics tools.

These methods have applications for any Lagrange-point mission and implications for phasing loop and lunar gravity assist trajectories. The case studies used for this analysis focus on the MAP mission. The tolerance already available in the pointing of the MAP sun shield can be used to produce variations in the SRP. Although the resulting control force is small it is the same order of magnitude as the perturbations that remain after applying the new methods. This should allow for station keeping without requiring any propulsion maneuvers. This paper provides some of the mathematical background required for targeting to L2 and describes how it can be applied to identify the relationships between the targeting variables and goals for the MAP mission.

MAP MISSION DESIGN CRITERIA

The mission requirements for the MAP trajectory design involve the fuel budget (110 m/s), launch vehicle dispersions (as large as 10 m/s) and the required launch window (any day of the month). In addition, constraints from the power, thermal and communication systems need to be taken into account.

The criteria for the L2 orbit design are defined using the Earth's shadow and the distance from the Earth-Sun vector. The spacecraft must stay out of the Earth's shadow to prevent loss of power from the solar arrays but still stay close enough to the Earth-Sun line to satisfy communication requirements and to keep thermal radiation from the Earth and Moon from bypassing the sun-shield. For MAP the angle between the Map-Earth vector and the sun-line must be between 0.5° and 10° which translates to a minimum distance of 13,000 Km and a maximum distance of 260,000 Km from the Earth-Sun line.

The minimum distance requirement rules out the option of going directly to the L2 point exactly and staying there. The types of orbits that can maintain this distance without escaping are halo orbits and Lissajous orbits. Lissajous orbits have different oscillation periods for the ecliptic-plane and out-of-plane motion. The resulting Lissajous pattern (Figure 1) inevitably passes through the shadow every 7.5 years. Halo orbits, on the other hand, are periodic and avoid the shadow problem by using non-linear effects of large amplitudes in the ecliptic plane. For MAP the maximum distance of 260,000 Km rules out halo orbits, leaving Lissajous patterns as the only choice. The eventual closing of the Lissajous pattern is not a major concern for MAP since the projected mission length is less than 3 years.

TRANSFER TRAJECTORIES

Direct transfers to L2 are very sensitive to small launch vehicle (LV) dispersions. Figure 2 shows that a very small error in orbit determination or in ΔV at perigee leads to very large errors in the approach to L2. Errors as small as 3 cm/sec are large enough to cause significant effects in the final orbit that must be corrected within a month or two of arrival. Dispersion errors can not be immediately corrected since it takes time to detect them. By the time the error has been detected, the spacecraft has climbed out of the Earth's gravity and the orbit is no longer as sensitive to the ΔV of correction maneuvers. The magnification of the initial error becomes critical for realistic dispersion errors since an uncorrected injection error of 3 m/sec would miss the desired orbit at L2 and diverge from the desired trajectory in less than a month.

Direct insertion from parking orbit into large halo orbits can be achieved in special cases[1] (Figure 3). However, the final orbits have very large amplitudes that violate the 260,000 Km requirement. Achieving a tighter Lissajous pattern would require large insertion maneuvers at L2 even with no dispersion errors. For smaller Lissajous patterns with a reasonably short (6 months) flight time this option does not seem promising. The MAP trajectory design has opted to use a combination of phasing loops and a lunar flyby in order to save fuel ,eliminate the need for an insertion maneuver at L2 and avoid the sensitivity of direct transfer[2].

The lunar flyby provides a significant gravity assist that reduces the total launch energy (C_3) and eliminates the need for an insertion maneuver to achieve a small Lissajous pattern near L2. Lunar flyby orbits of this type require precise targeting to achieve the correct timing and orientation. They are also very sensitive to LV dispersions and small variations in the orbital elements at perigee. These variations can cause the spacecraft to miss the Moon altogether or to fly by too close and achieve escape velocity from the Earth-Moon system. In addition to the sensitivity problems, direct injection into a lunar flyby trajectory would restrict the launch window to a few days each month.

PHASING LOOPS

Phasing loops provide an elegant solution to both the launch window and LV dispersion problems. The injection burn into the phasing loop can be performed at almost any time of the month. The first loop is designed to purposely miss the Moon. The first apogee also provides an opportunity to raise perigee (if necessary) and adjust the orbit inclination at a very low ΔV cost. Moreover, by the time of the next perigee, orbit determination (OD) should be good enough and the correction compensating for the initial dispersion would not require too much ΔV.

The trajectory is still very sensitive to small variations in ΔV and to the initial conditions on the launch date. However, this sensitivity is now an advantage, allowing us to use very small maneuvers to achieve our goals. We can wait for accurate OD data and use it to determine an optimal trajectory that is designed specifically for the known launch date and LV dispersion errors. The optimization goal for the phasing loop stage is to achieve the precise timing and orientation required for a successful lunar flyby.

To get an initial estimate of the timing, location and orientation of the flyby trajectories we can integrate the equations of motion backward from L2 and determine when, where and from which direction the spacecraft approaches lunar orbit. A rough initial approximation that does not include lunar gravity effects (Figure 4) shows that the spacecraft crosses the lunar orbit twice each month at a specific location and orientation. This rough approximation shows the location of the two lunar flyby opportunities within a few degrees. While the back-propagation results obtained by neglecting lunar gravity can not be used for approximating the actual flyby, they do provide us with some design information for the mission. The timing of the flyby is determined by the intersection of the spacecraft and lunar orbits since the Moon has to be close enough to provide a gravity assist. The asymptote for a B-plane analysis can also be determined with this method.

We can use the approximate flyby times to choose the number and periods of the phasing loops. The path followed by the spacecraft several days after the flyby is nearly independent of the lunar gravity. The arrival time at L2 is very sensitive to the exact timing of the flyby. The exact arrival time at L2 is not a major concern for most missions. We can therefore afford to make small adjustments in the timing of the flyby as long as the departing asymptote is adjusted to assure that the spacecraft arrives at the desired trajectory.

The final orbit parameters at L2 are extremely sensitive to the choice of departing asymptote from the lunar flyby. Numerical studies using Swingby* show that small adjustments of the incoming asymptote (a day before the flyby) and at periselene are sufficient to determine the entry point on the Lissajous pattern or even to reverse its direction. However, the qualitative results of these numerical investigations are limited by the fact that very small numerical errors in the initial conditions grow exponentially with time so that numerical targeting schemes can not be used effectively for more than 5 months. Successive targeting with appropriate cost functions that measure the deviation from a nominal orbit at L2 can be used to achieve better results[3]. The definition of these cost functions relies on a linear approximation of the orbit at L2.

Meaningful definitions of the goals at L2 can be phrased in terms of the size of the unstable eigenvector, the size of the two oscillations (A_Y and A_Z), the phase difference between the oscillations ($\Delta\phi$), the common phase at insertion (ϕbar) and the arrival time. Controlling the unstable eigenvector is critical for maintaining an orbit at L2. However, the design of the phasing loops can not effectively perform this task. A station keeping method (periodic thrust maneuvers or induced SRP variations) must be used regardless of the phasing loop strategy. The size of the two oscillations A_Y and A_Z must satisfy the requirements for minimum and maximum angle from the sun line but are allowed to vary within that range. The phase

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* Swingby is the Mission Analysis tool developed and used by the Flight Dynamics Analysis Branch and Goddard Space Flight Center. Swingby was used for SOHO, WIND, Clementine and Lunar Prospector trajectory design and maneuver calibration.

difference $\Delta\phi$ determines where the spacecraft is on the Lissajous pattern. Targeting to an opening Lissajous pattern (Figure 1b) assures that the spacecraft will not pass through the shadow for at least 3 years (as long as the unstable eigenvector is controlled).

The allowed variation in the three parameters A_Y, A_Z and $\Delta\phi$ is limited by mission requirements. However, the size of the stable eigenvector (related to the arrival time at L2) and ϕbar (the position on the Lissajous orbit) do not affect any mission requirements and do not need to be controlled. In designing the phasing loops and lunar flyby, A_Y, A_Z and $\Delta\phi$ should be used as targeting goals instead of numerical estimates that rely on plane crossings available in tools like Swingby. This makes it possible to satisfy mission requirements without placing unnecessary restrictions on the targeter which make it converge on non-optimal solutions. This targeting method, when combined with accurate modeling of SRP and lunar perturbations can reduce errors to the point that intentional variations of the MAP sunshield attitude are sufficient to perform station keeping. Any deviations from desired performance can be controlled through the attitude control system and propulsive maneuvers are not required.

LINEAR ANALYSIS AT L2

The formulation of the non-linear equations of motion is done in a rotating coordinate system centered at the Earth-Sun barycenter with the x-axis along the Sun-Earth line, the z-axis toward the ecliptic north and the y-axis chosen to complete an orthogonal right handed system. Normalized units for this system are 1 AU for length and Year/(2π) for time. However, for practical calculations it is often convenient to express length as a fraction of the distance from L2 to Earth and time in a multiple of the time constant (τ) or periods ($2\pi/\omega_{XY}$ and $2\pi/\omega_Z$) that correspond to the eigenvalues of the linear system.

Figure 5 shows a cross section along the XY plane (ecliptic) of the non-linear potential in the Rotating Libration Point (RLP) coordinate system centered at L2. We can see that a linear approximation about the Lagrange point is valid for a Lissajous pattern (the ellipse) with a maximum excursion of 5° from the Earth-Sun line. For larger orbits, the non-linear effects become more significant but the approximation still gives reasonable results even for a 10° excursion.

The linearized system has one unstable mode (eigenvector) that grows exponentially with an eigenvalue of $1/\tau = 2.5*(2\pi/\text{year}) = 1/(23 \text{ days})$. This means that small errors and the effects of perturbations in the direction of the unstable eigenvector will double every 16 days ($\tau\ ln\, 2$). The system also has a stable eigenvector that decays at the same rate. Initial displacements in the direction of the stable eigenvector will shrink to a half in 16 days as well.

In addition to the stable and unstable eigenvectors, the system also has two pairs of imaginary eigenvalues. One of the pairs ($\omega_Z = 1.99$ periods per year) corresponds to a pure oscillation in the z direction (the corresponding eigenvectors have no x and y components). The fact that all other eigenvectors have no z component indicates that as long as the linear approximation is valid, motion in the ecliptic plane and the out-of-plane oscillations are completely independent. The ellipse in Figure 5 traces the trajectory of a pure oscillation in the ecliptic plane which corresponds to the second pair of eigenvalues ($\omega_{XY} = 2.07$ periods per year).

SOLAR RADIATION PRESSURE (SRP) EFFECTS

As mentioned earlier, SRP is the main perturbation to spacecraft orbits in the near vicinity of the Earth-Sun L2 Lagrange point. Figure 6 shows the MAP spacecraft with it's large, 5 meter diameter, sunshield. It is easy to compensate for the average MAP SRP value ($0.2\ \mu m/s^2$)* and for the first order effects of the Moon

* Based on:
Mass = 850 Kg
Reflectivity = 1.9
Solar pressure = $5\ \mu N/m^2$

at L2 by moving the spacecraft a few hundred Kilometers closer to the Sun. However, very small variations in the orientation of large sun-shields can excite (or control) the unstable eigenvector and thereby cause (or prevent) escape from the vicinity of L2.

The effect of predicted errors in MAP attitude determination (0.02°) and attitude control (0.1°) on the x component of the SRP vector are expected to be negligible*. However, a small bias in the spin axis of the spacecraft can generate a small constant acceleration in the y or z directions. The angle between the x-axis and the spacecraft-sun line depends on the size and phase of the orbit and introduces a known bias with a corresponding acceleration in the y or z direction. Known accelerations can be accounted for in the orbit design, however, with an attitude pointing tolerance of 0.25°, the variation in acceleration can be as high as: $0.2\ \mu m/s^2 * \sin(0.25°) = 0.9\ nm/s^2$.

An acceleration of $0.9\ nm/s^2$ in the z direction is too small to have an effect on a periodic oscillation with magnitude measured in thousands of Kilometers. However, the same acceleration in the y direction contributes $0.3*\tau*0.9\ nm/s^2 =\sim 0.3*(3.5\ Km/\tau) =\sim 1.2\ Km/\tau$[†] toward the derivative of the state component that grows exponentially in the direction of the unstable eigenvector. To compensate for this derivative the spacecraft would have to be 1.2 Km away from the nominal trajectory in the opposite direction. . If an unpredicted error of $1.2\ Km/\tau$ is allowed to accumulate and we can not prepare for this displacement, we should expect the error to double every 16 days. If we assume eight maneuvers with a fuel budget of 8 m/s for stationkeeping, maneuvers are limited to $1\ m/s =\sim 2000\ Km/\tau$ and the time between maneuvers can not exceed 170 days ($\tau\ ln[2000/1.2]=7.4\tau$).

On the other hand, a closed-loop onboard controller with access to the ACS-reference input can purposely generate a bias of up to $0.2\ \mu m/s^2 * \sin(0.25°-0.1°) = 0.5\ nm/s^2$ to control deviations from the nominal trajectory. The controller would saturate for position errors larger than 2 Km (with no velocity error) or velocity errors of 1 mm/s ($2\ Km/\tau$, with no position error). Designing our nominal trajectories with sufficient detail to have deviations on the order of several hundred meters should allow us to maintain the orbit at L2 without saturating the onboard controller. This strategy would eliminate propulsion maneuvers for the duration of the mission.

SYSTEM INTEGRATION

The results in this paper show a strong interaction between navigation, orbit determination and attitude control. In order to use a closed loop controller based on solar radiation pressure, the corresponding spacecraft systems would need to be integrated. The integrated system should communicate state parameters and use these parameters to improve the accuracy of the OD and control commands.

It is clear that both the OD and navigation need to have access to accurate and timely attitude data. In addition, the closed loop controller should be able to command the ACS to perform minor adjustments to the nominal direction of the spin axis.

Sun shield area $=\sim 20\ m^2$
Effective area $= 20*\cos(22.5)$
Average acceleration $= \quad 1.9 * 5\ \mu N/m^2 * 20\ m^2 * \cos(22.5°) / 850 =\sim 0.2\ \mu m/s^2$

* Since the MAP spacecraft makes a full rotation about an axis pointing away from the Sun every hour, the average SRP acceleration is directed away from the Sun. If the spin axis points exactly away from the Sun and the spin motion is symmetric, there would be no acceleration perpendicular to the spin axis. Any bias in the spin axis would cause a perpendicular force in the y or z direction (RLP coordinate system).

† The derivative was calculated in time units normalized so the time constant τ of the unstable mode is 1.

FUTURE ANALYSIS

The variation in SRP due to small changes in attitude should be included in the OD process. Most current OD tools assume a spherical spacecraft with constant cross-sectional area and do not model the forces as normal to the surfaces exposed to the SRP. In order to perform accurate control, the controller must have accurate data. Orbit determination methods need to be improved, to match the accuracy of the analysis. One way to do this is to design a Kalman filter that will perform OD in real time as part of an integrated onboard ACS and navigation system.

Future analysis should address the open questions regarding attitude control and the possibility of handling the windmill effect. The strategy of designing nominal momentum wheel states to match the expected perturbations created by the difference between the center of gravity and center of pressure can have similar results to those presented here. This option should be studied in more detail since the possibility of using small variation in attitude to control the remaining deviations may work in this case as well. The full advantage of eliminating station keeping maneuvers can not be realized if propulsion maneuvers are still required for momentum dumping. Solar radiation pressure also affects the total momentum of the spacecraft and it may be possible to use a similar strategy to eliminate the need for momentum dumping.

CONCLUSIONS

The methods discussed in this paper can be applied to any Lagrange-point mission, with implications for phasing loop and lunar gravity assist trajectories. The importance of accurate SRP modeling in the design of the nominal trajectory has been demonstrated and a direct application for the MAP mission has been shown. Using attitude variations as small as 0.25° can affect the SRP enough to control the orbit at L2. This seems like a promising method for minimizing or eliminating propulsion stationkeeping maneuvers for MAP. If the onboard controller is successfully flown on MAP as a technology demonstration, NGST would be able to use it as proven technology.

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