A Low Cost Approach to Simultaneous Orbit, Attitude, and Rate Estimation Using an Extended Kalman Filter

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An innovative approach to autonomous attitude and trajectory estimation is available using *only* magnetic field data and rate data. The estimation is performed simultaneously using an Extended Kalman Filter, a well known algorithm used extensively in onboard applications. The magnetic field is measured on a satellite by a magnetometer, an inexpensive and reliable sensor flown on virtually all satellites in low earth orbit. Rate data is provided by a gyro, which can be costly. This system has been developed and successfully tested in a post-processing mode using magnetometer and gyro data from 4 satellites supported by the Flight Dynamics Division at Goddard.

In order for this system to be truly low cost, an alternative source for rate data must be utilized. An independent system which estimates spacecraft rate has been successfully developed and tested using only magnetometer data or a combination of magnetometer data and sun sensor data, which is less costly than a gyro. This system also uses an Extended Kalman Filter. Merging the two systems will provide an extremely low cost, autonomous approach to attitude and trajectory estimation.

In this work we provide the theoretical background of the combined system. The measurement matrix is developed by combining the measurement matrix of the orbit and attitude estimation EKF with the measurement matrix of the rate estimation EKF, which is composed of a pseudo-measurement which makes the effective measurement a function of the angular velocity. Associated with this is the development of the noise covariance matrix associated with the original measurement combined with the new pseudo-measurement. In addition, the combination of the dynamics from the two systems is presented along with preliminary test results.

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INTRODUCTION

Most missions supported by NASA have an attitude and orbit determination requirement. In most cases, the attitude estimation is performed onboard the satellite. However, the orbit determination is performed primarily on the ground post pass. Efforts are underway to provide for spacecraft onboard autonomous orbit determination. However, these efforts are somewhat expensive and have limited availability which makes them less attractive to the multitude of missions being launched with very modest attitude and orbit requirements as well as modest budget. Up to this point their options were quite limited due to the expense of space qualified attitude determination hardware as well as the expense of ground based and GPS based orbit determination.

In this work, the EKF used to estimate the orbit and the attitude is expanded to include the estimation of the rates. The effective measurement used by the EKF now includes the difference between the observed and expected magnetic field and the derivative of this difference. A corresponding noise covariance matrix is developed. The combined filter dynamics are straightforward and requires input regarding external torques on the spacecraft. Results from simulated data are presented following the theoretical background of the EKF.

The resulting system is expected to provide low cost navigation, i.e. attitude, orbit, and rates, for low earth orbit satellites. The system relies on existing hardware, namely, magnetometers, sensors that are carried on virtually all low earth orbit satellites. There has been only 1 reported failure of a magnetometer for missions supported by NASA/GSFC. Sun sensors, if used, are also extremely reliable. Both sensors are currently available and most importantly, they are flight qualified. Comparable systems, such as GPS, are considerably more expensive and flight qualified receivers are not readily available. Any mission, whether commercial or government, with coarse accuracy constraints or desiring an inexpensive backup method for attitude, rate, and orbit estimation can use this system, provided the satellite has an onboard computer. The impact to the onboard processing will not be significantly more than current onboard processing and can easily be accomplished with current computing technology. Furthermore, utilizing onboard processing reduces the cost of ground operations. Overall, this is an innovative approach for a low
cost, coarse attitude and trajectory estimation system.

THEORETICAL BACKGROUND

Following is a summary of the EKF algorithm. The assumed models of the EKF are given as:

System Model:

$$\underline{X}(t) = \underline{f}(\underline{X}(t), t) + \underline{w}(t) \qquad (1)$$

Measurement Model:

$$z_k = h_k(X(t_k)) + \eta_k \tag{2}$$

Update Stage

The linearization of equation (2) results in

$$z_k = H_k \, X_k + \eta_k \tag{3}$$

where H_k is the measurement matrix of the new, combined filter. H_k is composed of sub-matrices which reflect the dependence of the effective measurement z_k on the state vector, X_k which contains the orbital elements, the attitude quaternion, and the angular velocity.

$$X_k^T = [a, e, i, \Omega, w, \theta, C_d, q, \omega]$$

where:

       a = semi-major axis
       e = eccentricity
       i = inclination
       Ω = right ascension of ascending node
       w = argument of perigee
       θ = true anomaly
       C_d = drag coefficient
       q = attitude quaternion
       ω = rotation rate

The measurement matrix is

$$H_k = \begin{bmatrix} H_o & H_a & 0 \\ 0 & 0 & [\underline{b} \times] \end{bmatrix} \tag{4}$$

Where H_o and H_a are the submatrices reflecting the dependence of the orbital components[3] and the attitude[4], respectively, on the effective measurement.

The effective measurement, z_k contains two elements, z_1 and z_2. The first is the difference between the measured and observed vector[1] and the second element is the difference in the derivatives of the measured and observed vectors[2]. Taking the derivatives of the observed and measured vectors brings in a dependence on the angular velocity through the formula:

$$D_q^I \dot{r} = \dot{\underline{b}} + \underline{\omega} \times \underline{b} \tag{5}$$

where: $\underline{\omega}$ = angular velocity vector
       $\underline{r}$ = reference magnetic field vector resolved in inertial coordinates
       D_q^I = transformation from inertial to body coordinates

$\underline{b}$ = observed magnetic field vector resolved in body coordinates

Incorporating the noise into the reference and observed magnetic field vectors, (5) can be written as

$$\dot{\underline{b}} - D_q^I \dot{\underline{r}} = [\underline{b} \times]\underline{\omega} + [\underline{\eta}_b \times]\underline{\omega} - \underline{\eta}_b' + D_q^I \underline{\eta}_r \qquad (6)$$

where: $\underline{\eta}_r$ = reference vector noise
$\underline{\eta}_b$ = measurement vector noise

$$\underline{\eta}_b' = (\eta_{1,k} - \eta_{1,k-1})/\Delta$$

$[\underline{b} \times]$ = anti-symmetric matrix composed of the elements of $\underline{b}$

The second element of the effective measurement is then formally defined as

$$\underline{z}_2 \hat{=} (\dot{\underline{b}} - D_q^I \dot{\underline{r}}) - [\underline{b} \times]\underline{\omega}_{est} \qquad (7)$$

Where $\underline{\omega}_{est}$ is the current estimated rate.

Assuming the noise from the reference vector to be zero, the noise terms in (6) can be combined into

$$\underline{\eta}_d = [-\underline{\omega} \times]\underline{\eta}_b - \underline{\eta}_b'$$

The measurement noise, $\underline{\eta}_b$ is augmented with the noise $\underline{\eta}_d$ into the noise vector, $\underline{\eta}$, of (3). In order to use this in the filter the covariance matrix R is computed as

$$R \hat{=} E\left\{ \begin{bmatrix} \underline{\eta}_1 \\ \underline{\eta}_d \end{bmatrix} \begin{bmatrix} \underline{\eta}_1^T \underline{\eta}_d^T \end{bmatrix} \right\} - E\left\{ \begin{bmatrix} \underline{\eta}_1 \\ \underline{\eta}_d \end{bmatrix} \right\} E\left\{ \begin{bmatrix} \underline{\eta}_1^T \underline{\eta}_d^T \end{bmatrix} \right\} \qquad (9)$$

If the magnetometer is calibrated such that the measurements have no bias $E\{\underline{\eta}_1\}$ and $E\{\underline{\eta}_d\}$ are zero and R becomes

$$R = \begin{bmatrix} E\{\underline{\eta}_1^T \underline{\eta}_1^T\} & E\{\underline{\eta}_1^T \underline{\eta}_d^T\} \\ E\{\underline{\eta}_d^T \underline{\eta}_1^T\} & E\{\underline{\eta}_d^T \underline{\eta}_d^T\} \end{bmatrix} \qquad (10)$$

The matrix $E\{\underline{\eta}_1^T \underline{\eta}_1^T\}$ is the noise covariance matrix for the magnetometer measurement. Based on the assumption that the $\underline{\eta}_{1,k}$ and $\underline{\eta}_{1,k-1}$ are independent, the $E\{\underline{\eta}_d^T \underline{\eta}_d^T\}$ becomes

$$E\left\{\underline{\eta}_d\,\underline{\eta}_d^{\,T}\right\} = DR_{TAM}D^T + (1/\Delta^2)R_{TAM} \tag{11}$$

where: Δ = the time difference between the current and the previous measurement

The matrix D is computed as

$$D = \left[\underline{\hat{\omega}} \times\right] + (1/\Delta)I \tag{12}$$

The noise covariance matrix then becomes

$$R = \begin{bmatrix} R_{TAM} & R_{TAM}D^T \\ -DR_{TAM} & DR_{TAM}D^T + (1/\Delta^2)R_{TAM} \end{bmatrix} \tag{13}$$

The update of the state vector and covariance matrix is performed using the standard EKF equations

$$\underline{x}_k(+) = \underline{x}_k(-) + K_k\underline{z}_k \tag{14}$$

$$P_k(+) = (I-K_kH_k)P_k(-)(I-K_kH_k) + R_k \tag{15}$$

where the gain matrix, K_k is computed as

$$K_k = P_k(-)H_k^T(H_kP_k(-)H_k + R_k) \tag{16}$$

The state vector, $\underline{x}$, given in (14) is the internal state used by the EKF. This form is used internally to estimate the angular error in the attitude (in addition to the other state vector elements) which is then converted to the quaternion given in the state, $\underline{X}$, above. This is the so-called 'multiplicative' approach[4].

The above derivation is valid for a magnetometer. For another sensor, such as a sun sensor, the following changes must be made. First, since another sensor is not influenced by the orbit, the measurement matrix in (4) is replaced with

$$H_k = \begin{bmatrix} 0 & H_a & 0 \\ 0 & 0 & [\underline{b} \times] \end{bmatrix} \tag{17}$$

where $\underline{b}$ is the measured vector. The effective measurement, $\underline{z}$, is based on the sensor measurements of the given sensor and is computed as for the magnetometer. The computation of R is as given in (13) with R_{TAM} replaced with the noise covariance matrix of the given sensor. Based on the results of Reference 4, the 3rd and 6th rows of H_k in (17) above and the corresponding rows and columns of R are removed. This is to prevent singularities from a line of sight sensor.

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Propagation stage

The propagation of the state estimate, based on equation 1 is performed as

$$\dot{\underline{X}} = f(\underline{\hat{X}}(t), t) \tag{18}$$

The updated estimate of the state vector, $\underline{\hat{X}}_k(+)$ is propagated from time t_k to t_{k+1} by a numerical solution of the continuous dynamics. The orbital dynamics are non-linear and describe a central force including both J2 effects and drag[3]. The differential equation which governs the propagation of the quaternion is linear and is dependent on the estimation of the spacecraft rotation rate[5]. The spacecraft dynamics are used to propagate the rate estimate. The dynamics model of the spacecraft is non-linear and the method of solution is given in Ref. 2.

The propagation of the covariance matrix is performed using the following

$$P_{k+1}(-) = A_k(\underline{\hat{X}}_k(+))P_k(+)A_k^T(\underline{\hat{X}}_k(+)) + Q_k \tag{19}$$

where Q_k is the spectral density matrix of $\underline{\varpi}(t)$ and A_k is the approximated transition matrix. A_k is computed using the following first order Taylor series expansion

$$A_k = I + F\Delta T \tag{20}$$

where ΔT is the time interval between reaction wheel data. The Jacobian F is derived for the orbital dynamics in Ref. 3, for the attitude dynamics in Ref. 4, and for the spacecraft dynamics in Ref. 2.

RESULTS

The initial testing of the above EKF was performed with simulated spacecraft data. The spacecraft in the simulation is the Rossi X-ray Timing Explorer (RXTE) satellite. Table 1 specifies the true state, the initial state used by the EKF, the initial covariance, and the initial RSS errors in position, velocity, attitude, and rates. The simulation is based on an actual spacecraft ephemeris, not a two-body propagation. The simulation contains 6 hours of data, which is equivalent to approximately 3.7 orbits. Magnetometer and sun sensor measurements were generated every 2 seconds, without any noise. The spacecraft is inertially pointed in the simulation; there are no attitude maneuvers. Therefore, the control data necessary for propagation of the rate estimate is nominal. The measurement noise for the magnetometer and sun sensor are 0.01 degrees and 50 milliGauss, respectively. The relatively large value chosen for the magnetometer measurement noise is a result of previous testing of the EKF for attitude and orbit estimation and only orbit estimation based on magnetometer data.

Figures 1 and 2 show the errors in the position. Figure 2 shows the position errors with an expanded vertical axis. After 1 orbit the maximum position error oscillates between ±40 km, with an average of approximately 0 km. Figure 2 also indicates that the orbit has not yet converged. Due to the low inclination of the RXTE orbit, the orbital elements take a considerable amount of time to reach steady state as shown in Reference 1. Figure 3 is a plot of the attitude errors about each of the spacecraft axes. The values oscillate between approximately ±1 degree. The average error is different for each of the three axes. Figure 4 shows the errors in the rate. The rate errors show a significant overshoot at the beginning and then slowly converge throughout the run. Errors in the rate contribute to the errors in the attitude estimate, this is evident in Figure 3. Additional tuning is necessary to determine if the final errors, and the size of the oscillations and initial overshoot can be reduced. Extending the length of the simulated data span is necessary.

Table 1. Truth Model and Initial Conditions

State Variable	Truth	Initial State	Initial Covariance	Initial Errors
a (km)	6956.7	7156.74	10000	RSS position error = 1948 km
e	0.00197	0.002074	0.0001	RSS velocity error = 2.1 km/sec
i (deg)	22.96	22.5	1	
Ω (deg)	109.74	110.735	10	
ω (deg)	220.04	225.036	100	
θ (deg)	18.19	28.19	100	
C_d	2.2	0.000	0	
q(1)	0	0.0454	.01[§]	RSS attitude errror = 8.8 deg
q (2)	0	0.0416		
q(3)	0	0.0454		
q(4)	1	0.99710		
ϖ_x (deg/sec)	0	0.0001	$(0.1)^2$	RSS rate error = 8.87 deg/sec
ϖ_y (deg.sec)	0	0.0001	$(0.1)^2$	
ϖ_z (deg/sec)	0	0.0001	$(0.1)^2$	

CONCLUSIONS

The initial test results indicate that the EKF has the potential to simultaneously estimate a spacecraft orbit, attitude, and rates. Final position errors less than 40 km, attitude errors less than 1 degree, and rate errors less than $7e^{-5}$ deg/sec resulted from the

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[§]This is the apriori covariance of the angular error in the quaternion, see Ref. 1. This value is used for each component of the error.

first test case based on the simulated RXTE magnetometer and sun sensor measurements. The initial test case consisted of clean, simulated magnetometer and sun sensor data covering approximately 3.7 orbits. Further tuning and additional data are necessary to reduce the final errors.

Future tests will be conducted on noisy, simulated data based on an inertial attitude. Maneuvers will then be inserted into the data to determine if the EKF can detect and follow the maneuver. Finally, the EKF will be tested with real spacecraft data, ideally from a number of missions such as the existing RXTE, Total Ozone Mapping Spectrometer, Gamma Ray Observatory, Transition Region and Coronal Explorer (TRACE) and the future Wide Field Infrared Explorer (WIRE) mission. The final test will consist of a real time test onboard a spacecraft.

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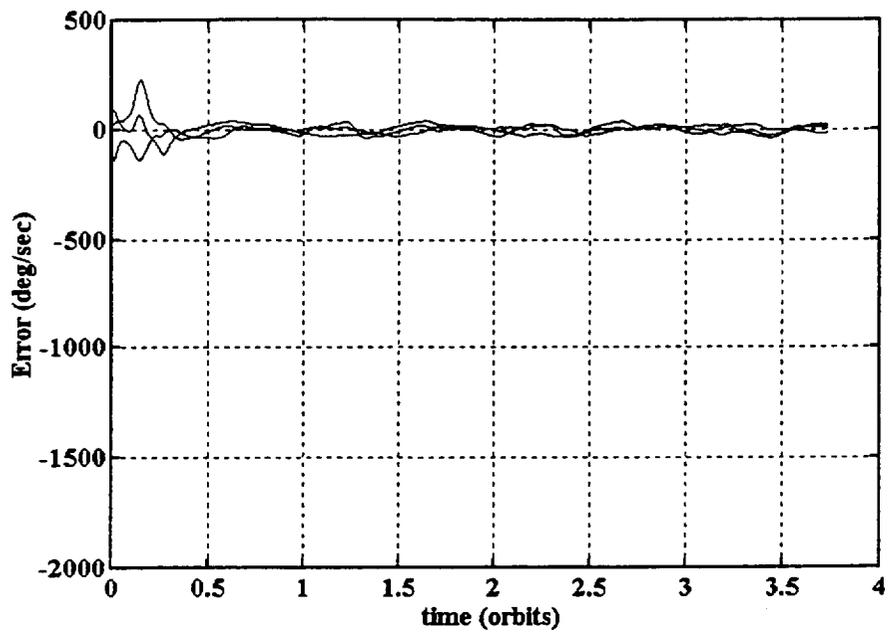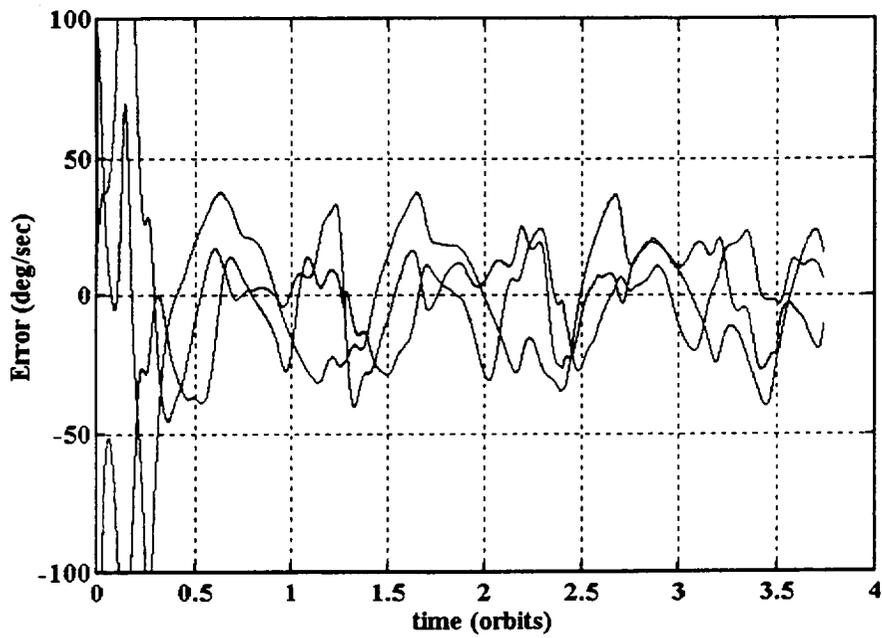
Figure 1. Position Error Components
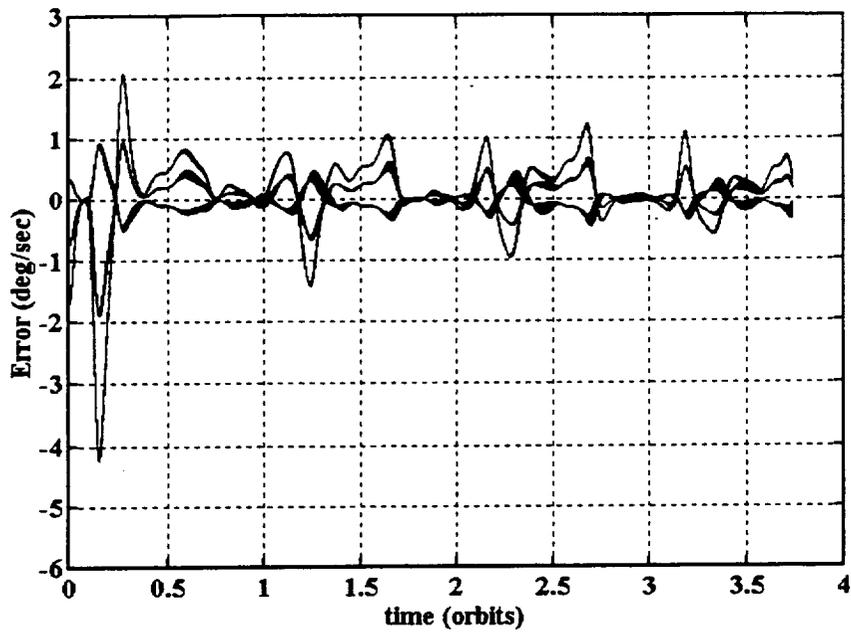

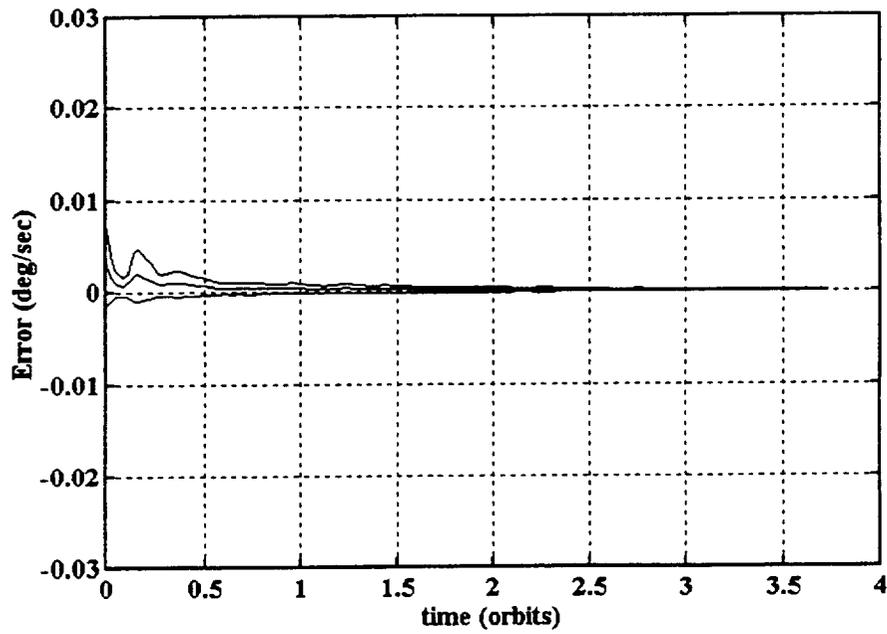**Figure 2. Position Error Components – Expanded Vertical Axis**

Figure 3. Attitude Error Components

Figure 4. Rate Error Components