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OPTIMAL INTEGER RESOLUTION FOR ATTITUDE DETERMINATION USING GLOBAL POSITIONING SYSTEM SIGNALS

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In this paper, a new motion-based algorithm for GPS integer ambiguity resolution is derived. The first step of this algorithm converts the reference sightline vectors into body frame vectors. This is accomplished by an optimal vectorized transformation of the phase difference measurements. The result of this transformation leads to the conversion of the integer ambiguities to vectorized biases. This essentially converts the problem to the familiar magnetometer-bias determination problem, for which an optimal and efficient solution exists. Also, the formulation in this paper is re-derived to provide a sequential estimate, so that a suitable stopping condition can be found during the vehicle motion. The advantages of the new algorithm include: it does not require an a-priori estimate of the vehicle's attitude; it provides an inherent integrity check using a covariance-type expression; and it can sequentially estimate the ambiguities during the vehicle motion. The only disadvantage of the new algorithm is that it requires at least three non-coplanar baselines. The performance of the new algorithm is tested on a dynamic hardware simulator.

INTRODUCTION

The utilization of phase difference measurements from Global Positioning System (GPS) receivers provides a novel approach for three-axis attitude determination and/or estimation. These measurements have been successfully used to determine the attitude of air-based,[1] space-based,[2-3] and sea-based[4] vehicles. Since phase differences are used, the correct number of integer wavelengths between a given pair of antennas must be found. The determination of the integer ambiguities can either be accomplished by using "static" (motionless) or "dynamic" (motion-based) techniques. The ambiguities essentially act as integer biases to the phase difference measurements. Once the integer ambiguities are resolved, then the attitude determination problem can be solved.[5]

The static method finds a solution that minimizes the error residual at a specific time by searching through an exhaustive list of all possible integers and rejecting classes of

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solutions when the residual becomes too large.[6] Refinements can be made to the solution by restricting the search space with knowledge of a-priori information, such as the maximum tilt the baseline should encounter.[7] Static methods generally rely on solving a set of Diophantine equations.[8] The appeal of these methods is that they provide an "instantaneous" attitude solution, limited only by computation time, and are well suited for the short baselines. However, the minimum residual does not guarantee a correct solution in the presence of noise.[1] In fact, it is possible that static methods can report a wrong solution as valid, especially when some of the calibration information, such as line bias, is incorrect. This lack of integrity can cause significant problems if the sensor output is used to control a high bandwidth actuator, such as gas jets on a spacecraft. Another consideration is that static methods sometime require that the antenna array must be within a defined angle (typically 30 degrees) of a reference attitude, which is often true for ground-based applications, but is less likely for space-based applications. Also, structural flexibility in the baselines may lead to erroneous solutions. All of the aforementioned limitations imply that static methods, while attractive because of their fast solutions, are not totally acceptable for general purpose applications.

The other technique for resolving integer ambiguities involves collecting data for a given period of time and performing a batch solution, in which the integer terms remain constant over the collection period. This technique relies on the fact that a certain amount of motion has occurred during the data collection, either from vehicle body rotation or GPS line of sight motion. The main disadvantage of this technique, compared to static approaches, is that it takes time for the motion to occur, which may be on the order of several minutes. Another consideration is that a potentially significant amount of memory is required for the storage of the batch data collection. But, motion-based techniques also have significant advantages over static methods. Most importantly, motion-based techniques are inherently high integrity methods because there are numerous checks that can be implemented into the solution before it is accepted. These include using statistical checks applied to error residuals, matrix condition number checks, and using the closeness of the computed floating-point "integers" to actual integers as a check. The probability of an erroneous solution being reported as valid can be made as small as desired by appropriately setting the thresholds on these integrity checks. For these reasons, motion-based techniques have been more widely used for on-board applications.

Traditional motion-based techniques of integer ambiguity resolution rely on the fact that either GPS line of sight motion or vehicle motion dominates the changes in differential carrier phase measurements. Cohen[9] developed an algorithm, known as "quasi-static" integer resolution, that can be used when the GPS line of sight motion and the vehicle rotation both account approximately evenly for the differential carrier phase measurement changes. This algorithm can be adapted to almost any vehicle motion, slow or fast, simply by varying the sample rate and the data collection time. The quasi-static method solves a collection of differential phase measurements for a single attitude estimate and then considers perturbations to the initial estimate at each measurement epoch to produce a time varying batch solution to the data. Although this is a widely used algorithm, there are certain disadvantages. First, an a-priori attitude estimate must be given. Second, the algorithm is an iterative batch estimator that may produce erroneous estimates, depending on the accuracy of the a-priori attitude estimate. Finally, if a large number of samples in the data collection are required to observe the motion, large-order matrix inversions may be required. Another method (Ref. 10) performs a minimization on three Euler-angle attitude parameters independent of each other, followed by determining the integers. This approach has been shown to provide better convergence than Cohen's method and works well for non-coplanar baselines; however, singular conditions can exist at various attitude rotations and a significant amount of vehicle motion may be necessary for a solution.

In this paper, a new motion-based algorithm is derived. The main advantages of the new algorithm over the prior methods include: (i) it resolves the integer ambiguities without any a-priori attitude knowledge, (ii) it requires less computational effort, since large matrix inverses are not needed, and (iii) it is non-iterative. The only disadvantage of the new algorithm is that it requires at least three non-coplanar baselines. The algorithm is first shown as a batch solution, and then shown as a sequential solution. A covariance expression is also derived which can be used to bound the integer solution so that a sufficient integrity check for convergence can be developed. This is extremely useful in the sequential formulation, since the solution can be found as the motion occurs, rather than taking a batch solution at a specific data collection interval. For these reasons, the new algorithm provides an attractive method for real-time ambiguity resolution.

The organization of this paper proceeds as follows. First, the concept of the GPS phase difference measurement is introduced. Then, a brief review of Cohen's quasi-static method is shown, and limitations and computational aspects of this algorithm are discussed. Next, the new motion-based algorithm is derived. The conversion of the GPS sightline vector into the body frame is first reviewed. Then, the batch solution used to resolve the integer ambiguities is derived, followed by the sequential solution. Finally, the new algorithm is validated by using an actual GPS receiver with a hybrid dynamic simulator to simulate the vehicle motions of a low-altitude Earth-orbiting spacecraft.

GPS SENSOR MODEL

In this section, a brief background of the GPS phase difference measurement is shown. The main measurement used for attitude determination is the phase difference of the GPS signal received from two antennas separated by a baseline. The wavefront angle and wavelength are used to develop a phase difference, as shown in Figure 1. The phase difference measurement is obtained by

$$b_l \cos\theta = \lambda(\Delta\phi - n) \tag{1}$$

where b_l is the baseline length (in cm), θ is the angle between the baseline and the line of sight to the GPS spacecraft, n is the number of integer wavelengths between two receivers, $\Delta\phi$ is the phase difference (in cycles), and λ is the wavelength (in cm) of the GPS signal. The two GPS frequency carriers are L1 at 1575.42 MHz and L2 at 1227.6 MHz. As of this writing, non-military applications generally use the L1 frequency. The phase difference can be expressed by

$$\Delta\phi = \underline{b}^T A \underline{s} + n \tag{2}$$

where $\underline{s} \in R^3$ is the normalized line of sight vector to the GPS spacecraft in a reference frame, $\underline{b} \in R^3$ is the baseline vector (in wavelengths), which is the relative position vector from one receiver to another, and $A \in R^{3\times3}$ is the attitude matrix, which is an orthogonal matrix with determinant 1 (i.e., $A^T A = I_{3\times3}$). The measurement model is given by

$$\Delta\tilde{\phi}_{ij} = \underline{b}_i^T A \underline{s}_j + n_{ij} + w_{ij} \tag{3}$$

where $\Delta\tilde{\phi}_{ij}$ denotes the phase difference measurement for the i^{th} baseline and j^{th} sightline, and w_{ij} represents a zero-mean Gaussian measurement error with standard deviation ϖ_{ij} which is $0.5\,\text{cm}/\lambda = 0.026$ wavelengths for typical phase noise.[9]

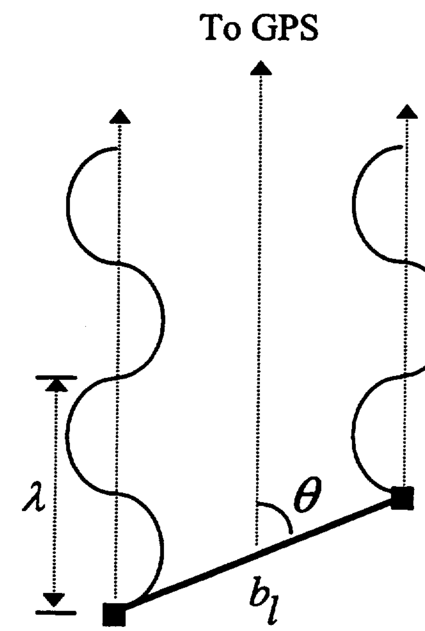

Figure 1 GPS Wavelength and Wavefront Angle

QUASI-STATIC APPROACH

Cohen's quasi-static method[9] is a motion-based technique that begins by taking measurements for $k = 1$ to L ("measurement epochs") to which a single attitude solution will be determined. At each epoch it is assumed that M baselines exist and N sightlines. The measurement model is linearized by assuming a small perturbation about a nominal attitude A_0 and an assumed set of integer phases $(n_0)_{ij}$, so that

$$A = A_0\left(I_{3\times3} + [\underline{\delta\theta}\times]\right) \tag{4}$$

where $I_{3\times3}$ is a 3×3 identity matrix, $\underline{\delta\theta}$ is assumed to be a small angle rotation, and $[\underline{\delta\theta}\times]$ is a cross product matrix with

$$[\underline{a}\times] \equiv \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{5}$$

Substituting Equation (4) into Equation (3) for all available measurements yields

$$\begin{aligned} \underline{\delta\phi} = \left[\underline{\Delta\phi} - \underline{\Delta\phi}_0\right] &= \begin{bmatrix} \underline{s}_1^T A_0^T [\underline{b}_1 \times] & \bar{I} \\ \vdots & \\ \underline{s}_N^T A_0^T [\underline{b}_M \times] & \end{bmatrix} \begin{bmatrix} \underline{\delta\theta} \\ \underline{\delta n} \end{bmatrix} \\ &\equiv \begin{bmatrix} H & \bar{I} \end{bmatrix} \begin{bmatrix} \underline{\delta\theta} \\ \underline{\delta k} \end{bmatrix} \end{aligned} \tag{6}$$

where $\bar{I}$ is a quasi-identity matrix with possible zeros along the diagonal where states have been removed at various measurement epochs, and

$$\underline{\Delta\phi} \equiv \begin{bmatrix} \Delta\phi_{11} \\ \vdots \\ \Delta\phi_{MN} \end{bmatrix}, \quad \underline{\delta n} \equiv \begin{bmatrix} (n-n_0)_{11} \\ \vdots \\ (n-n_0)_{MN} \end{bmatrix}, \quad (\Delta\phi_0)_{ij} = \underline{b}_i^T A_0 \underline{s}_j + (n_0)_{ij} \tag{7}$$

Equation (6) is a set of MN equations for $(3+MN)$ states. Allowing perturbations at all epochs leads to

$$\begin{bmatrix} \underline{\delta\phi}(1) \\ \underline{\delta\phi}(2) \\ \vdots \\ \underline{\delta\phi}(L) \end{bmatrix} = \begin{bmatrix} H(1) & 0 & \cdots & 0 & \bar{I}(1) \\ 0 & H(2) & \ddots & \vdots & \bar{I}(2) \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & H(L) & \bar{I}(L) \end{bmatrix} \begin{bmatrix} \underline{\delta\theta}(1) \\ \vdots \\ \underline{\delta\theta}(L) \\ \underline{\delta n} \end{bmatrix} \tag{8}$$

This compact representation has LMN rows and $(3L+MN)$ states. In principle, the integers and the attitude of the vehicle of the vehicle at each measurement epoch may be found by applying an iterative linearized least-squares approach using Equation (8) and updating the nominal attitude using Equation (4).

The quasi-static method has been successfully implemented to resolve the integer ambiguities on an actual system (Ref. 1), and works extremely well when a fairly accurate a-priori attitude is known, and significant vehicle motion is present. However, this approach has a number of disadvantages. First, if the a-priori attitude estimate is poorly known, then the solution may never converge (even if the integers are known exactly). Second, not only does this algorithm require a good a-priori guess, but requires fairly accurate attitude estimates at all measurement times. The reason for this is that as time increases, the perturbations to the a-priori attitude guess may become too large for the solution to converge. This may be overcome by augmenting the state equations to include a constant, but unknown, body rate that is also estimated. Finally, a $(3L+MN)$ matrix inverse is required, which may cause computational problems. Many of these problems may be overcome by developing an algorithm that is independent of any attitude information.

NEW ALGORITHM

In this section a new algorithm to resolve the integer ambiguities is shown. The main advantage of this algorithm is that it is attitude independent. First, a conversion of the sightline vectors into the body frame is shown. This converts the problem into the familiar magnetometer-bias problem. Then, a batch solution for this problem is shown, followed by a sequential approach.

The new algorithm begins by determining the sightline vector in the body frame, denoted by $\breve{\underline{s}} = A\underline{s}$. This is accomplished by minimizing the following loss function[11]

$$J(\breve{\underline{s}}_j) = \frac{1}{2}\sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2}\left(\Delta\tilde{\phi}_{ij} - n_{ij} - \underline{b}_i^T \breve{\underline{s}}_j\right)^2 \quad \text{for } j = 1,2,\ldots,N \tag{9}$$

If at least three non-coplanar baselines exist, the minimization of Equation (9) is straightforward and leads to

$$\breve{\underline{s}}_j = \hat{\underline{s}}_j - \underline{c}_j \tag{10a}$$

$$\hat{\underline{s}}_j = B_j^{-1}\left[\sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2}\Delta\tilde{\phi}_{ij}\underline{b}_i\right] \tag{10b}$$

$$\underline{c}_j = B_j^{-1}\left[\sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2} n_{ij}\,\underline{b}_i\right] \tag{10c}$$

$$B_j = \sum_{i=1}^{M} \frac{1}{\varpi_{ij}^2}\underline{b}_i \underline{b}_i^T \tag{10d}$$

The computed sightline in the body frame is related to the sightline vector in the reference frame by

$$\hat{\underline{s}}_j = A\underline{s}_j + \underline{c}_j + \underline{\varepsilon}_j \tag{11}$$

where $\underline{c}_j$ is a constant bias since the baselines are assumed constant, and $\underline{\varepsilon}_j$ is a zero-mean Gaussian process with covariance $R_j = B_j^{-1}$. Again, the inverse in Equation (10) exists only if three non-coplanar baseline vectors exist.

The next step is to use an attitude-independent method to find the phase-bias vector $\underline{c}_j$. Doing this for each sightline gives us all the sightlines in both the body frame and the reference frame. The explicit integer phases are not needed for this solution, but it is important to check that they are close to integer values, as mentioned in the Introduction. In the general case, the explicit integer phases can be found from the attitude solution.

The three-baseline case ($M = 3$) is simpler, for in this case Equation (10c) can be inverted to give

$$n_{ij} = \underline{b}_i^T \underline{c}_j \tag{12}$$

With more than three baselines, however, Equation (10c) does not have a unique solution for $\underline{c}_j$, so the M integer phases for sightline $\underline{s}_j$ cannot be found from $\underline{c}_j$ alone. We will consider the three-baseline case, which is the most common in practice. If more baselines are available, we are always free to select a three-baseline subset. Then, after the integer phases have been determined, a refined attitude estimate can be computed using all baselines (i.e., three baselines are sufficient to determine an attitude, which may then be used to resolve the integers corresponding to the other baselines).

To eliminate the dependence on the attitude, the square of Equation (11) is computed, so that

$$\begin{aligned} \|\underline{s}_j\|^2 = \|A\underline{s}_j\|^2 &= \|\hat{\underline{s}}_j - \underline{c}_j - \underline{\varepsilon}_j\|^2 \\ &= \|\hat{\underline{s}}_j\|^2 - 2\hat{\underline{s}}_j \cdot \underline{c}_j + \|\underline{c}_j\|^2 - 2(\hat{\underline{s}}_j - \underline{c}_j) \cdot \underline{\varepsilon}_j + \|\underline{\varepsilon}_j\|^2 \end{aligned} \tag{13}$$

Next, the following effective measurement and noise are defined

$$z_j \equiv \|\hat{\underline{s}}_j\|^2 - \|\underline{s}_j\|^2 \tag{14a}$$

$$v_j \equiv 2(\hat{\underline{s}}_j - \underline{c}_j) \cdot \underline{\varepsilon}_j - \|\underline{\varepsilon}_j\|^2 \tag{14b}$$

Then, the effective measurement can be written as

$$z_j = 2\hat{\underline{s}}_j \cdot \underline{c}_j - \|\underline{c}_j\|^2 + v_j \tag{15}$$

Alonso and Shuster (Ref. 12) showed that v_j is approximately Gaussian for small $\underline{\varepsilon}_j$ with mean given by

$$\mu_j \equiv E\{v_j\} = -\text{trace}\{R_j\} \tag{16}$$

and variance given by

$$\sigma_j^2 \equiv E\{v_j^2\} - \mu_j^2 = 4(\hat{\underline{s}}_j - \underline{c}_j)^T R_j (\hat{\underline{s}}_j - \underline{c}_j) \tag{17}$$

The negative-log-likelihood function for the bias is given by

$$J(\underline{c}_j) = \frac{1}{2}\sum_{k=1}^{L}\left\{\frac{1}{\sigma_j^2(k)}\left[z_j(k) - 2\hat{\underline{s}}_j(k)\cdot\underline{c}_j + \|\underline{c}_j\|^2 - \mu_j(k)\right]^2 + \log\sigma_j^2(k) + \log 2\pi\right\} \tag{18}$$

The symbol k denotes the variable at time t_k. The maximum-likelihood estimate for $\underline{c}_j$, denoted by $\underline{c}_j^*$, minimizes the negative-log-likelihood function, and satisfies

$$\left.\frac{\partial J(\underline{c}_j)}{\partial \underline{c}_j}\right|_{\underline{c}_j^*} = \underline{0} \tag{19}$$

The minimization of Equation (18) is not straightforward since the likelihood function is quartic in $\underline{c}_j$. A number of algorithms have been proposed for estimating the bias (see Ref. 12 for a survey). The simplest solution is obtained by scoring, which involves a Newton-Raphson iterative approach. Another approach avoids the minimization of a quartic loss function by using a "centered" estimate. A statistically correct centered estimate is also derived in Ref. 12. Furthermore, Alonso and Shuster show a complete solution of the statistically correct centered estimate that determines the exact maximum likelihood estimate $\underline{c}_j^*$. This involves using the statistically correct centered estimate as an initial estimate, and iterating on a correction term using a Gauss-Newton method. Although this extension to the statistically correct centered estimate can provide some improvements, this part is not deemed necessary for the GPS problem since the estimated quantity for n_{ij} is rounded to the nearest integer.

Batch Solution

In this section the statistically correct centered estimate algorithm (see Ref. 12 for details) and its application to the integer ambiguity problem are shown. First, the following weighted averages are defined

$$\begin{aligned} \bar{z}_j &\equiv \bar{\sigma}_j^2 \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} z_j(k), & \bar{\underline{s}}_j &\equiv \bar{\sigma}_j^2 \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} \hat{\underline{s}}_j(k), \\ \bar{v}_j &\equiv \bar{\sigma}_j^2 \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} v_j(k), & \bar{\mu}_j &\equiv \bar{\sigma}_j^2 \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} \mu_j(k) \end{aligned} \tag{20}$$

where

$$\frac{1}{\bar{\sigma}_j^2} \equiv \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} \tag{21}$$

Next, the following variables are defined

$$\tilde{z}_j(k) \equiv z_j(k) - \bar{z}_j, \quad \tilde{\underline{s}}_j(k) \equiv \hat{\underline{s}}_j(k) - \bar{\underline{s}}_j, \quad \tilde{v}_j(k) \equiv v_j(k) - \bar{v}_j, \quad \tilde{\mu}_j(k) \equiv \mu_j(k) - \bar{\mu}_j \tag{22}$$

The statistically correct centered estimate now minimizes the following loss function

$$\tilde{J}(\underline{c}_j) = \frac{1}{2} \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} \left[\tilde{z}_j(k) - 2\tilde{\underline{s}}_j(k) \cdot \underline{c}_j - \mu_j(k)\right]^2 \tag{23}$$

which is now a quadratic function in $\underline{c}_j$. The minimization leads directly to

$$\underline{c}_j^* = P_j \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} \left[\tilde{z}_j(k) - \tilde{\mu}_j(k)\right] 2\,\tilde{\underline{s}}_j(k) \tag{24}$$

where the estimate error covariance is given by

$$P_j = \left[\sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} 4\,\tilde{\underline{s}}_j(k)\tilde{\underline{s}}_j^T(k)\right]^{-1} \tag{25}$$

The ambiguity for the i^{th} baseline and j^{th} sightline can be resolved by rounding the following to the nearest integer

$$n_{ij} = \underline{b}_i^T \underline{c}_j^* \tag{26}$$

The integer error covariance, denoted by Q_{ij}, can be shown to be given by

$$Q_{ij} = \underline{b}_i^T P_j \underline{b}_i \tag{27}$$

Equation (27) can be used to develop an integrity check for the algorithm. For example, a suitable criterion can be developed from a three-sigma bound using $3\sqrt{Q_{ij}}$.

Sequential Formulation

This section expands upon the batch solution so that a sequential estimate of the integers can be found. The main advantage of a sequential formulation is that the convergence (integrity) check can be made on-the-fly (i.e., in real-time). The covariance in Equation (25) to be expanded to the $L+1$ time point, so that

$$\begin{aligned} P_j^{-1}(L+1) &= \sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} 4\,\tilde{\underline{s}}_j(k)\tilde{\underline{s}}_j^T(k) + \frac{1}{\sigma_j^2(L+1)} 4\,\tilde{\underline{s}}_j(L+1)\tilde{\underline{s}}_j^T(L+1) \\ &= P_j^{-1}(L) + \frac{1}{\sigma_j^2(L+1)} 4\,\tilde{\underline{s}}_j(L+1)\tilde{\underline{s}}_j^T(L+1) \end{aligned} \tag{28}$$

From the matrix inversion lemma,[13] the following sequential formation for the covariance is developed

$$P_j(k+1) = K_j(k) P_j(k) \tag{29}$$

where

$$K_j(k) \equiv I - P_j(k)\tilde{\underline{s}}_j(k+1)\left[\tilde{\underline{s}}_j^T(k+1)P_j(k)\tilde{\underline{s}}_j(k+1) + \frac{1}{4}\sigma_j^2(k+1)\right]\tilde{\underline{s}}_j^T(k+1) \tag{30}$$

In order to derive sequential formulas for the quantities in Equation (20), first consider the following identity

$$\sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)} z_j(k) = \frac{1}{\bar{\sigma}_j^2(L)} \bar{z}_j(L) = \left[\sum_{k=1}^{L} \frac{1}{\sigma_j^2(k)}\right] \bar{z}_j(L) \tag{31}$$

Expanding out this expression using $L-1$ points in the summation yields

$$\frac{1}{\bar{\sigma}_j^2(L-1)}\bar{z}_j(L-1)+\frac{1}{\sigma_j^2(L)}z_j(L)=\left[\frac{1}{\bar{\sigma}_j^2(L-1)}+\frac{1}{\sigma_j^2(L)}\right]\bar{z}_j(L) \tag{32}$$

and so

$$\bar{z}_j(L)=\frac{\sigma_j^2(L)\bar{z}_j(L-1)+\bar{\sigma}_j^2(L-1)z_j(L)}{\sigma_j^2(L)+\bar{\sigma}_j^2(L-1)} \tag{33}$$

Therefore, the following sequential expressions for the quantities in Equation (20) are given

$$\bar{z}_j(k+1)=\frac{1}{\sigma_j^2(k+1)+\bar{\sigma}_j^2(k)}\left[\sigma_j^2(k+1)\bar{z}_j(k)+\bar{\sigma}_j^2(k)z_j(k+1)\right] \tag{34a}$$

$$\bar{\underline{s}}_j(k+1)=\frac{1}{\sigma_j^2(k+1)+\bar{\sigma}_j^2(k)}\left[\sigma_j^2(k+1)\bar{\underline{s}}_j(k)+\bar{\sigma}_j^2(k)\hat{\underline{s}}_j(k+1)\right] \tag{34b}$$

$$\bar{\mu}_j(k+1)=\frac{1}{\sigma_j^2(k+1)+\bar{\sigma}_j^2(k)}\left[\sigma_j^2(k+1)\bar{\mu}_j(k)+\bar{\sigma}_j^2(k)\mu_j(k+1)\right] \tag{34c}$$

where

$$\frac{1}{\bar{\sigma}_j^2(k+1)}=\frac{1}{\bar{\sigma}_j^2(k)}+\frac{1}{\sigma_j^2(k+1)} \tag{35}$$

The estimated bias in Equation (24) can also be found in a similar manner, so that

$$\underline{c}_j^*(k+1)=K_j(k)\underline{c}_j^*(k)+\frac{1}{\sigma_j^2(k+1)}\left[\tilde{z}_j(k+1)-\tilde{\mu}_j(k+1)\right]2\,P_j(k+1)\bar{\underline{s}}_j(k+1) \tag{36}$$

Since the baselines are constant, Equations (26) and (27) can be used directly to determine the sequential integer value and error covariance, given by

$$n_{ij}(k)=\underline{b}_i^T\underline{c}_j^*(k) \tag{37a}$$

$$Q_{ij}(k)=\underline{b}_i^T P_j(k)\underline{b}_i \tag{37b}$$

The complete solution proceeds as follows. First, use Equations (10b) and (10d) to convert the sightline vectors into the body frame. Then, perform an initial batch solution using Equations (20)-(25) in order to initialize the sequential routine (an accurate initial estimate is not required as will be seen in the results section). Then, perform a sequential estimate for the integers using Equations (29), (30), and (34)-(37). Finally, continue until the covariance in Equation (37b) is below a pre-specified value.

There are many advantages of the new algorithm. First, the algorithm is fully autonomous (i.e., it requires no a-priori information such as an a-priori attitude guess). Second, the largest matrix inverse is of a 3×3 matrix, which makes the algorithm computationally efficient and stable. Third, it is non-iterative, which makes it suitable for a sequential formulation. This has a significant advantage since the convergence can be checked during the actual motion in the vehicle. Finally, the integers for other sightlines can be easily resolved by calling the same subroutine. Therefore, the algorithm can easily be implemented using all available sightlines, and attitude determination can begin once the integers corresponding to two sightlines have been resolved. For these reasons, the new algorithm provides an attractive approach to resolve the integers.

HARDWARE SIMULATION AND RESULTS

A hardware simulation of a typical spacecraft attitude determination application was undertaken to demonstrate the performance of the new algorithm. For this simulation, a Northern Telecom 40 channel, 4 RF output STR 2760 unit was used to generate the GPS signals that would be received at a user specified location and velocity. The signals are then provided directly (i.e., they are not actually radiated) to a GPS receiver that has been equipped with software tracking algorithms that allow it operate in space (see Figure 2).

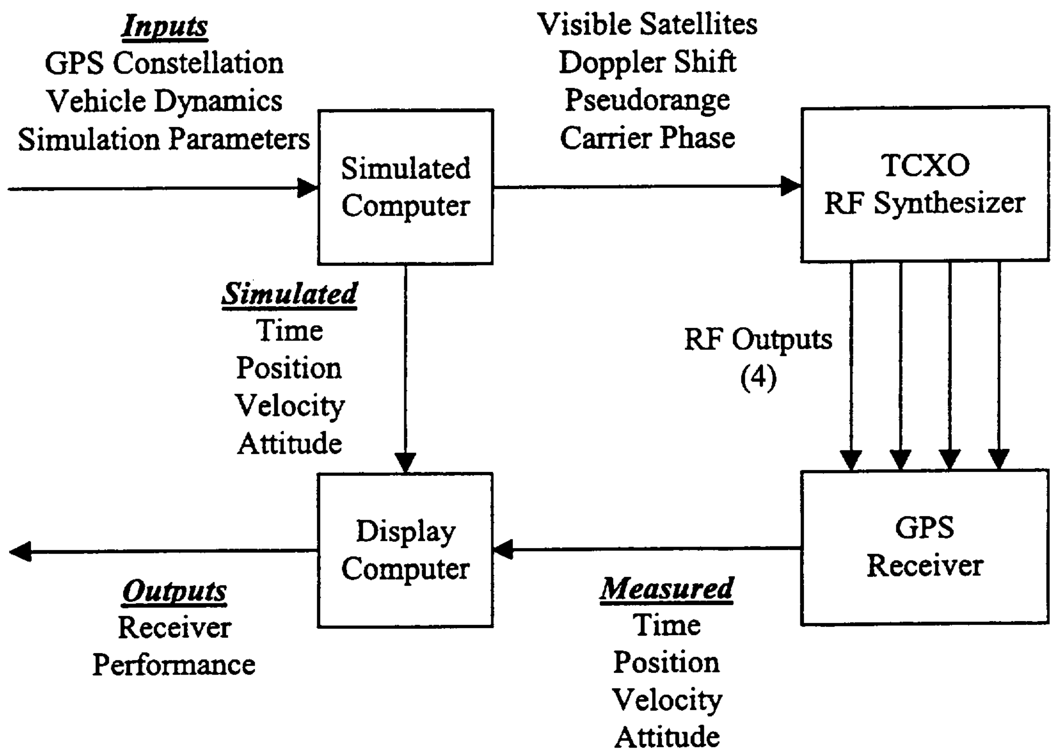

Figure 2 Hardware Simulation Block Diagram

The receiver that was used was a Trimble TANS Vector; which is a 6 channel, 4 RF input multiplexing receiver that performs 3-axis attitude determination using GPS carrier phase and line of sight measurements. This receiver software was modified at Stanford University and NASA-Goddard to allow it to operate in space. This receiver model has been flown and operated successfully on several spacecraft, including: REX-II, OAST-Flyer, GANE, Orbcomm, Microlab, and others.

The simulated motion profile was for an actual spacecraft, the Small Satellite Technology Initiative (SSTI) Lewis satellite, which carried an experiment to assess the performance of GPS attitude determination on-orbit. Although the spacecraft was lost due to a malfunction not related to the GPS experiment shortly after launch, this motion profile is nonetheless very representative of the types of attitude determination applications. The orbit parameters and pointing profile used for the simulation are given in Table 1.

Table 1 SSTI Lewis Orbit parameters

Semimajor axis (a)	6901.137 km
Inclination (i)	97.45 deg
Right Ascension of Ascending Node (RAAN)	-157.1 deg
Eccentricity (e)	0.0001
Pointing profile	Earth pointed
Launch date	August 22, 1997

The simulated SSTI Lewis spacecraft has four GPS antennas that form three baselines. The antenna separation distances are 0.61 m, 1.12 m, and 1.07 m, respectively. One antenna (in baseline 3) is located 0.23 m out of plane (below) the other three antennas. On the spacecraft, the antennas are mounted on pedestals with ground planes to minimize signal reflections and multipath. For the simulation, the signal was provided to the GPS receiver without multipath noise. The baseline vectors in wavelengths are given by

$$\underline{b}_1 = \begin{bmatrix} 2.75 \\ 1.64 \\ -0.12 \end{bmatrix}, \quad \underline{b}_2 = \begin{bmatrix} 0.00 \\ 6.28 \\ -0.17 \end{bmatrix} \quad \underline{b}_3 = \begin{bmatrix} -3.93 \\ 3.93 \\ -1.23 \end{bmatrix} \tag{38}$$

Line biases are first determined before the new algorithm is tested to resolve the integer ambiguities. The GPS raw measurements are processed at 1 Hz over a forty minute simulation. During the simulated run, a minimum of three visible GPS are given at all times. Also, there are a number of eight minute spans when two of the same (in time) sightlines are available for the ambiguity resolution algorithm. Again, in practice, all available sightlines should be processed simultaneously, but with three baseline vectors only two simultaneously available sightlines are required to determine the attitude of the vehicle.

As mentioned previously, the first step in the algorithm involves using the baselines and phase difference measurements to convert the sightline vector into the body-frame, using Equations (10b) and (10d). Then, a small batch run is used to initialize the sequential routine. For this case, only 5 seconds of data was required to perform the initialization. Again, only two sightlines are required to determine the attitude. Sequential error results (i.e., actual integer minus the computed values without rounding) for the first sightline are shown in Figure 3. The integer error can be found be rounding the values in Figure 3 to the nearest integer. Clearly, for this case, the integer ambiguities have been resolved even before the sequential process begins (i.e., within 5 seconds). A plot of the $3\sqrt{Q_{ij}}$ values is shown in Figure 4 for the first sightline (a suitable integrity check is given when $3\sqrt{Q_{ij}}$ is below 0.5). Clearly, the integrity check shows that the ambiguities are resolved within 5 minutes. Note, that this is a sufficiency test (i.e., the integers may be resolved well before 5 minutes, which is seen in this case). A plot of the errors for the second sightline is shown in Figure 5. For this case, all of the ambiguities have been resolved within 30 seconds (the error value corresponding to the second baseline goes below -0.5 before 30 seconds). A plot of integrity check for the second sightline is shown in Figure 6. The integrity check shows that the ambiguities are resolved within 7 minutes. This hardware simulation of a spacecraft clearly demonstrates that the new algorithm presented in this paper provides an accurate method to resolve the integer ambiguities with even slight vehicle motion.

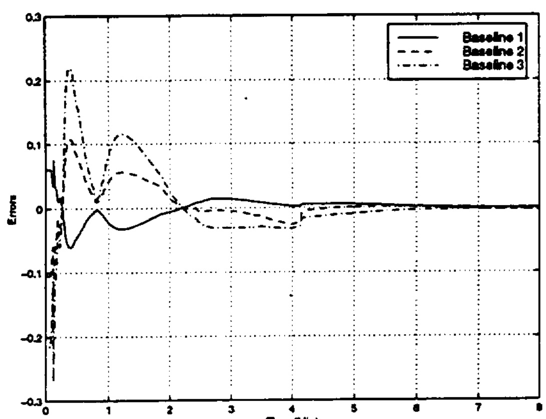

Figure 3 Errors for First Sightline

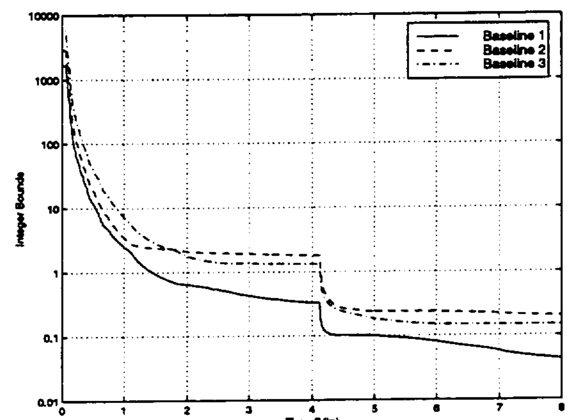

Figure 4 Integrity Check for First Sightline

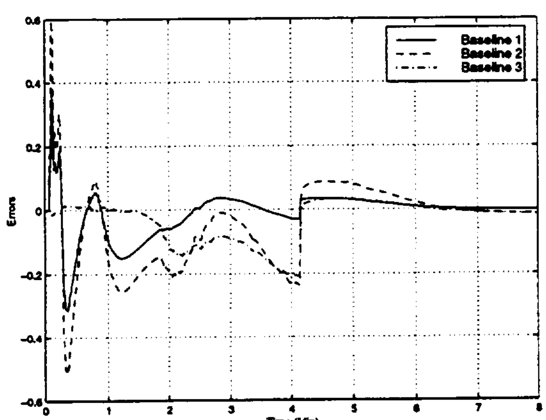

Figure 5 Errors for Second Sightline

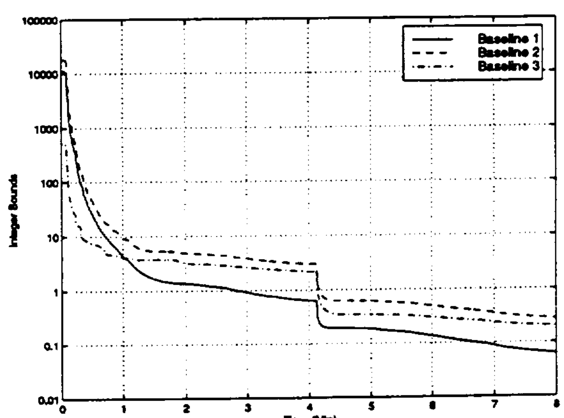

Figure 6 Integrity Check for Second Sightline

CONCLUSIONS

In this paper, a new algorithm was developed for GPS integer ambiguity resolution. The new algorithm has several advantages over previously existing algorithms. First, the algorithm is attitude independent so that no a-priori attitude estimate (or assumed vehicle motion) is required. Second, the algorithm is sequential so that it may be implemented in real-time. Also, a suitable integrity check can be used to determine when the determined values have converged to the correct values. Finally, the algorithm is computationally efficient since only a 3×3 matrix inverse is required, and the same subroutine can be used on different sightlines. The only disadvantage of the new algorithm is that it requires at least three non-coplanar baselines. The algorithm was tested using a GPS hardware simulator to simulate the motions of a typical low-altitude Earth-orbiting spacecraft. Results indicated that the new algorithm provides a viable and attractive means to effectively resolve the integer ambiguities.

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