AAS 98-334

S33-35
169331

A TRANSITION MATRIX APPROACH TO THE DAVENPORT GYRO CALIBRATION SCHEME*

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The in-flight gyro calibration scheme commonly used by NASA Goddard Space Flight Center (GSFC) attitude ground support teams closely follows an original version of the Davenport algorithm developed in the late seventies. Its basic idea is to minimize the least-squares differences between attitudes gyro-propagated over the course of a maneuver and those determined using post-manuever sensor measurements. The paper represents the scheme in a recursive form by combining necessary partials into a rectangular matrix, which is propagated in exactly the same way as a Kalman filter's square transition matrix. The nontrivial structure of the propagation matrix arises from the fact that attitude errors are not included in the state vector, and therefore their derivatives with respect to estimated gyro parameters do not appear in the transition matrix defined in the conventional way.

In cases when the required accuracy can be achieved by a single iteration, representation of the Davenport gyro calibration scheme in a recursive form allows one to discard each gyro measurement immediately after it was used to propagate the attitude and state transition matrix. Another advantage of the new approach is that it utilizes the same expression for the error sensitivity matrix as that used by the Kalman filter. As a result the suggested modification of the Davenport algorithm made it possible to reuse software modules implemented in the Kalman filter estimator, where both attitude errors and gyro calibration parameters are included in the state vector.

The new approach has been implemented in the ground calibration utilities used to support the Tropical Rainfall Measuring Mission (TRMM). The paper analyzes some preliminary results of gyro calibration performed by the TRMM ground attitude support team. It is demonstrated that an effect of the second iteration on estimated values of calibration parameters is negligibly small, and therefore there is no need to store processed gyro data. This opens a promising opportunity for onboard implementation of the suggested recursive procedure by combining it with the Kalman filter used to obtain necessary attitude solutions at the beginning and end of each maneuver.

* This work was supported by the National Aeronautics and Space Administration (NASA) / Goddard Space Flight Center (GSFC), Greenbelt, Maryland, Contract GS-35F-4381G, Task Order No. S-03365-Y.

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I. INTRODUCTION

Propagation of a state vector from one measurement time to another is usually done[1] by introducing a transition matrix formed by partial derivatives of the current state with respect to the state at an epoch time. A well-known technique[2,3] has been developed to propagate the transition matrix between sequential measurements using gyro data. The paper extends this propagation technique to the Davenport gyro calibration scheme.[4-7] The main obstacle to such an extension comes from the fact that the cited gyro calibration scheme treats an a priori given change in the spacecraft attitude within a specified time interval as a pseudo-measurement, and therefore attitude errors are not included in the state vector anymore; as a result, their derivatives with respect to estimated gyro parameters (such as misalignments, biases, and scale factors) do not appear in the transition matrix defined in the conventional way. To overcome this complication, the new approach combines necessary partials into a *rectangular* matrix, which can be propagated in exactly the same way as the conventional (*square* by definition) transition matrix.[2,3]

Assuming that the first iteration eliminates bulk errors, representation of the least-squares gyro calibration scheme in a recursive form allows one to discard each gyro measurement immediately after it is used to propagate the attitude and state transition matrix. Due to a significant decrease in required storage size, this feature of the new approach seems especially promising for onboard applications.

The next Section presents a simplified derivation of the original version of the Davenport algorithm.[4,5] Its final result is an explicit expression of the vector attitude residual in terms of the error sensitivity matrices ψ_k utilized by Kalman filter estimator.[3] It is shown that the derived expression turns into the conventional one[7] if only linear terms are kept in the expansion of each matrix ψ_k as a Taylor series in the duration Δt_k of the k^{th} propagation interval.

Section III introduces a *rectangular* matrix which gyro-governed evolution is performed via the same recurrence relations as those used for gyro propagation of the conventional state transition matrix. The derivation is accomplished in Section IV, which outlines main steps of the suggested recursive procedure.

The new algorithm has been implemented[8] and successfully used to calibrate gyros[9] for the Tropical Rainfall Measuring Mission (TRMM). One of the advantages of the suggested modification of the Davenport algorithm is that it allowed a reuse of software modules implemented in the Kalman filter estimator,[3] where both attitude errors and gyro calibration parameters are included in the solve-for state vector. Section V discusses some preliminary results of the TRMM gyro calibration. It is shown that the required accuracy of gyro calibration can be indeed achieved by a single iteration, and therefore each gyro measurement can be indeed discarded immediately after it was used. The paper also studies a possibility to reduce a volume of processed gyro data without jeopardizing the accuracy of calibration.

II. MATHEMATICAL GROUNDS OF DAVENPORT ALGORITHM

The Davenport method[4] is a two-step procedure. The first step is to determine attitudes at the ends of specially selected calibration intervals. For successful calibration, the selected time intervals usually cover a series of maneuvers associated with significant changes in body rates. It is essential that, regardless of maneuver specifics, each calibration interval must both start and end in a constant-rate mode. To determine the spacecraft attitude at the ends of each interval, sensor measurements are then collected only during time periods within constant-rate modes, when unknown errors in gyro misalignments and scale factors are compensated by additional gyro biases estimated simultaneously with the spacecraft attitude. As a result, one can assume that gyro propagation from one sensor measurement time to another is done accurately enough, despite the fact that gyros have not been properly calibrated yet.

The second step is quaternion propagation starting from the predetermined attitude quaternion at the beginning of each calibration interval and stopping at its end. The resultant propagated quaternion is then compared with the second of two attitude quaternions predetermined for this calibration interval. The comparison is done by multiplying one of the two quaternions at the ending time by the inverse of other. The vector part of the product is then treated as a vector residual, with the total number of these attitude quaternion vector residuals (AQVRs) always equal to the number of the calibration intervals.

Davenport's principal result is an approximate expression for the AQVRs in terms of vector deviations of the observed angular velocity vectors from the true rates. The outline of the Davenport method presented here mainly follows Keat's[5] interpretation of Davenport's original work.[4] To simplify the notation, the discussion will be limited only to a single maneuver so that the index labeling different maneuvers can be omitted. An extension of the final expression for an AQVR to a series of sequential maneuvers is performed in a trivial way by attaching an additional index to both residual and all angular velocity vectors.

Let $\vec{\omega}_k^{adj}$ be an observed angular velocity vector obtained by adjusting measured rates with some estimated parameters, where subscript k refers to the k-th available gyro measurement within the maneuver in question. The observed vector differs from a true vector $\vec{\omega}_k$ by a rate error $\delta\vec{\omega}_k$, that is,

$$\vec{\omega}_k = \vec{\omega}_k^{adj} - \delta\vec{\omega}_k . \tag{1}$$

Both vectors $\vec{\omega}_k^{adj}$ and $\vec{\omega}_k$ are assumed to remain constant during a time interval Δt_k so that the quaternion propagated over n intervals (starting from the known quaternion $\bar{q}_{init}$) can be represented as

$$\bar{q}_{prop} = \bar{q}_{init} \prod_{k=1}^{n} \bar{q}(\vec{\omega}_k^{adj} \Delta t_k) , \tag{2}$$

where[10] $\bar{\mathbf{q}}(\vec{\omega}\,\Delta t) = \left[\hat{\omega}\,\sin(\phi/2),\, \cos(\phi/2)\right]$, with $\hat{\omega} \equiv \vec{\omega}/|\vec{\omega}|$ and $\phi \equiv |\vec{\omega}|\Delta t$. The AQVR $\vec{\mathbf{Z}}$ is defined via the relation:*

$$\left[\,\vec{\mathbf{Z}},\ \sqrt{1-|\vec{\mathbf{Z}}|^2}\,\right] = \delta\overline{\mathbf{Q}} \equiv \bar{\mathbf{q}}_{\mathrm{fin}}^{-1}\,\bar{\mathbf{q}}_{\mathrm{prop}}\ , \tag{3}$$

where $\bar{\mathbf{q}}_{\mathrm{fin}}$ is the given attitude quaternion in the end of the maneuver.

It is assumed that the attitude quaternion $\bar{\mathbf{q}}_{\mathrm{fin}}$ can be obtained from $\bar{\mathbf{q}}_{\mathrm{init}}$ by propagating the latter with the true constant angular velocity vectors $\vec{\omega}_k$ over the time intervals Δt_k, so that

$$\bar{\mathbf{q}}_{\mathrm{fin}} = \bar{\mathbf{q}}_{\mathrm{init}} \prod_{k=1}^{n} \bar{\mathbf{q}}(\vec{\omega}_k \Delta t_k)\,. \tag{4}$$

To express the AQVR $\vec{\mathbf{Z}}$ in terms of errors in gyro parameters, one first needs to linearize the quaternion product $\bar{\mathbf{q}}_{\mathrm{fin}}^{-1}\,\bar{\mathbf{q}}_{\mathrm{prop}}$ in $\delta\vec{\omega}_k$. At this point one has to deal with unnormalized quaternions, which form the so-called 'associative algebra'. Note that both attitude and propagation quaternions discussed above are normalized quaternions, which cannot be either summed up or multiplied by a scalar, in contrast with unnormalized quaternions. On the other hand, the inverse operation $\bar{\mathbf{q}}^{-1}$ is well defined just for normalized quaternions. Only the multiplication law given by Eq. (D-8) in Ref. 10 is common for both normalized and unnormalized quaternions. It is essential that, by analogy with orthogonal matrices, the multiplication law is associative, i. e., $\bar{\mathbf{q}}(\bar{\mathbf{q}}'\bar{\mathbf{q}}'') = (\bar{\mathbf{q}}\bar{\mathbf{q}}')\bar{\mathbf{q}}''$ for any three unnormalized quaternions, $\bar{\mathbf{q}}, \bar{\mathbf{q}}'$, and $\bar{\mathbf{q}}''$. Another important features of the multiplication law are that $\bar{\mathbf{q}}(\bar{\mathbf{q}}'+\bar{\mathbf{q}}'') = \bar{\mathbf{q}}\bar{\mathbf{q}}'+\bar{\mathbf{q}}\bar{\mathbf{q}}''$ and that $\bar{\mathbf{q}}(k\bar{\mathbf{q}}') = (k\bar{\mathbf{q}})\bar{\mathbf{q}}'$ for a scalar multiplication. After the mentioned features of the multiplication law are established, unnormalized quaternions can be formally treated in the same way as square matrices, with the norm of quaternion given by Eq. (D-9) in Ref. 10 used instead of the matrix determinant. In particular, all Taylor expansions look very similarly, except that each product should be computed based on the quaternion multiplication law.

Substituting Eqs. (2) and (4) into Eq. (3) and keeping only terms linear in $\delta\vec{\omega}_k$, one can represent the latter expression as

$$\delta\overline{\mathbf{Q}} \approx (1-n)\left[\vec{0},\, 1\right] + \sum_{k=1}^{n} \bar{\mathbf{q}}_{k\to n}^{-1}\, \delta\bar{\mathbf{q}}_k\, \bar{\mathbf{q}}_{k\to n}\,, \tag{5}$$

where

$$\bar{\mathbf{q}}_{k\to k+\Delta k} \equiv \prod_{k'=k+1}^{k+\Delta k} \bar{\mathbf{q}}(\vec{\omega}_{k'}^{\,\mathrm{adj}}\,\Delta t_{k'}) \qquad \text{for } \Delta k = 1,\ldots, n\text{-}k > 0, \tag{6a}$$

* Note that our definition of the AQVR differs by the factor (-1) from that used by Keat.[5]

$$\bar{\mathbf{q}}_{n\to n} \equiv \left[\vec{0},\, 1\right], \tag{6b}$$

and

$$\left[\vec{\mathbf{z}}_k,\, \sqrt{1-|\vec{\mathbf{z}}_k|^2}\,\right] = \delta\bar{\mathbf{q}}_k \equiv \bar{\mathbf{q}}(-\vec{\omega}_k \Delta t_k)\,\bar{\mathbf{q}}(\vec{\omega}_k^{\,adj}\,\Delta t_k)\ . \tag{7}$$

Note that the sum in the right-hand side of Eq. (5) is formed by the products of normalized quaternions, and hence, to simplify each product, one can take advantage of the existent isomorphism between normalized quaternions and 3×3 orthogonal matrices. Making use of Eq. (12-7b) in Ref. 10, one can easily verify that

$$\mathbf{R}\,\mathbf{A}(\vec{\mathbf{g}})\mathbf{R}^{T} = \mathbf{A}(\mathbf{R}\vec{\mathbf{g}}), \tag{8}$$

where $\mathbf{R}$ is an arbitrary 3×3 orthogonal matrix, whereas the rotation matrix $\mathbf{A}(\vec{\mathbf{g}})$ associated with the Gibbs vector $\vec{\mathbf{g}}$ is given by Eq. (12-7b) in Ref. 10, with $\vec{\mathbf{g}} \equiv \hat{e}\tan(\Phi/2)$. Representing Eq. (8) in the quaternion form and substituting the resultant expression in the sum in the right-hand side of Eq. (5), one can finally represent the AQVR $\vec{\mathbf{Z}}$ as

$$\vec{\mathbf{Z}} = \sum_{k=1}^{n} \left[\, \mathbf{R}_{k\to n}\,\vec{\mathbf{z}}_k + O\!\left(\left|\delta\vec{\omega}_k\right|^2\right) \right], \tag{9}$$

where $\mathbf{R}_{k\to k'}$ is the rotation matrix associated with the propagation quaternion $\bar{\mathbf{q}}_{k\to k'}$.

As discussed in detail in Ref. 11, an explicitly expression of the vector $\vec{\mathbf{z}}_k$ in terms of the rate error $\delta\vec{\omega}_k$ has the form:

$$\vec{\mathbf{z}}_k = \frac{1}{2}\psi_k\,\delta\vec{\omega}_k + O\!\left(\left|\delta\vec{\omega}_k\right|^2\right), \tag{10}$$

where ψ_k is the error sensitivity matrix used by the Kalman filter estimator,[3] that is,

$$\psi_k \equiv \mathbf{I}_3\,\Delta t_k - \frac{1}{2}[\vec{\omega}_k^{\,adj}\times]\Delta t_k^2\, \nu^2\!\left(\phi_k^{adj}/2\right) + 2\,[\vec{\omega}_k^{\,adj}\times]^2\,\eta\!\left(\phi_k^{adj}\right)\Delta t_k^3, \tag{11}$$

with $\nu(\varphi) \equiv \sin\varphi/\varphi$, $\eta(\varphi) \equiv 0.5\,(1-\sin\varphi)/\varphi^2$, and $\phi_k^{adj} \equiv |\vec{\omega}_k^{\,adj}|\Delta t_k$. Note that a slightly different representation for error sensitivity matrix (11), compared with Ref. 3, makes it possible to compute this matrix using expansions explicitly stable at the limit $|\vec{\omega}_k^{\,adj}| \to 0$.

Finally, AQVRs (9) are represented as linear combinations of errors Δx_i ($i=1,\ldots,p$) in gyro parameters:

$$\vec{\mathbf{Z}} \approx \frac{1}{2}\sum_{i=1}^{p} \Delta x_i \sum_{k=1}^{n} \mathbf{R}_{k\to n}\psi_k \vec{\mathbf{f}}_i\!\left(\vec{\omega}_k^{\,adj}\right). \tag{12}$$

by substituting the rate errors

$$\delta\vec{\omega}_k = \sum_{i=1}^{p} \vec{\mathbf{f}}_i\left(\vec{\omega}_k^{adj}\right)\Delta x_i \,. \tag{13}$$

into Eq. (10). Computation of necessary partials is then performed in a trivial way.

Note that Keat's formula[5] for the AQVR (utilized in the conventional version[7] of the Davenport algorithm[4]) is obtained from Eq. (13) by keeping only the first term in the right-hand side of Eq. (11), which seems to be a sufficiently accurate approximation in most cases (see comments made in the end of Section V). Another minor modification comes from a slightly different choice of the state vector $\Delta\bar{x}$, which is formed by three bias errors Δb_i (i=1,2,3), three scale factor errors Δk_i (i=1,2,3), and two misalignment angle errors $\varepsilon_{k,i}$ (k=1,2) for each of three gyros (i=1,2,3). Such a choice of gyro calibration parameters[8] makes it possible to calibrate each gyro separately, which is convenient in case of a spacecraft having only one gyro, such as Solar and Heliospheric Observatory (SOHO).[12]

III. PROPAGATION OF ATTITUDE MATRIX VECTOR RESIDUALS

The main purpose of this Section is to show that an attitude residual can be represented in the general form:

$$\vec{\theta} = \widetilde{\mathbf{H}}\widetilde{\Phi}_{state}\Delta\bar{\mathbf{x}}, \tag{14}$$

where $\vec{\theta}$, $\Delta\bar{\mathbf{x}}$, $\widetilde{\mathbf{H}}$, and $\widetilde{\Phi}_{state}$ are usually referred to as a measurement residual, a state error vector, a sensitivity matrix, and a state transition matrix, respectively. The crucial point is that the transition matrix $\widetilde{\Phi}_{state}$ can be computed as the last term $\widetilde{\Phi}_{state} = \widetilde{\Phi}_n$ in a sequence of recurrence relations:

$$\widetilde{\Phi}_k = \Phi_{k-1\to k}\,\widetilde{\Phi}_{k-1}, \tag{15}$$

where the (p +3) × (p+3) matrix $\Phi_{k-1\to k}$ is an incremental transition matrix conventionally used in Kalman filter applications to propagate the attitude state vector (see, for example. Eq. (F8-26) in Ref. 3). A certain complication, however, comes from the fact that the calibration scheme in question estimates only gyro calibration parameters, and therefore attitude errors are not included into the state vector $\Delta\bar{\mathbf{x}}$. As a result $\widetilde{\Phi}_k$ turn out to be rectangular matrices having p+3 rows but only p columns.

As mentioned above, the initial and final attitudes in the Davenport method are determined using sensor measurements in constant-rate modes, when solved-for biases compensate for errors in gyro misalignments and scale factors. The attitude matrix vector residual (AMVR), $\vec{\theta}$, is then defined via the relation:

$$\exp[\vec{\theta}\times] \equiv \mathbf{A}_{\text{obs}}\mathbf{A}_{\text{prop}}^{\text{T}} \quad , \tag{16}$$

where $\mathbf{A}_{\text{obs}}$ and $\mathbf{A}_{\text{prop}}$ are the attitude matrices associated with the attitude quaternions $\overline{\mathbf{q}}_{\text{fin}}$ and $\overline{\mathbf{q}}_{\text{prop}}$ in the previous Section.* One thus finds

$$\vec{\theta}\ \sin(|\vec{\theta}|/2)/|\vec{\theta}| = \vec{\mathbf{Z}}\ . \tag{17}$$

Since we are interested only in terms linear in gyro errors, one can simply put

$$\vec{\theta} \approx 2\vec{\mathbf{Z}} \tag{18}$$

and make use of Eq. (12) to represent the AMVR $\vec{\theta}$ as the last term $\vec{\theta} \approx \vec{\theta}_n$ in the sequence

$$\vec{\theta}_k = \sum_{i=1}^{p} \Delta x_i\ \psi_k \vec{\mathbf{f}}_i\left(\vec{\omega}_k^{\text{adj}}\right) + \mathbf{R}_{k\text{-}1\rightarrow k}\vec{\theta}_{k-1}, \tag{19}$$

with $\vec{\theta}_0 = \vec{0}$.

Eq. (19) immediately leads to the conventional Kalman filter expression[3] for propagation of the combined attitude error / gyro calibration parameter state vector:

$$\begin{bmatrix} \vec{\theta}_k \\ \\ \Delta\overline{\mathbf{x}} \end{bmatrix} = \Phi_{k-1\rightarrow k} \begin{bmatrix} \vec{\theta}_{k-1} \\ \\ \Delta\overline{\mathbf{x}} \end{bmatrix}, \tag{20}$$

where

$$\Phi_{k-1\rightarrow k} \equiv \begin{bmatrix} \mathbf{R}_{k\text{-}1\rightarrow k} & \psi_k \mathbf{F}_k \\ \\ \mathbf{0}_{p\times 3} & \mathbf{I}_p \end{bmatrix}, \tag{21}$$

with

$$\mathbf{F}_k \equiv \left[\vec{\mathbf{f}}_1\left(\vec{\omega}_k^{\text{adj}}\right), \ldots, \vec{\mathbf{f}}_p\left(\vec{\omega}_k^{\text{adj}}\right)\right]. \tag{22}$$

By initializing sequence (15) via the relation

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* The author is thankful to J. Sedlak for pointing to a misprint in the definition of the AMVR $\vec{\theta}$ in Ref. 11 leading to a sign error in Eqs. (III-5), (III-6), (III-8), (III-14), and (III-17) there.

$$\tilde{\Phi}_0 \equiv \begin{bmatrix} \mathbf{0}_{3\times p} \\ \mathbf{I}_p \end{bmatrix} \tag{23}$$

one finds that

$$\tilde{\Phi}_1 \equiv \begin{bmatrix} \psi_1 \mathbf{F}_1 \\ \mathbf{I}_p \end{bmatrix} \tag{24}$$

and hence, making use of Eq. (20) at k=1,

$$\vec{\theta}_1 = \tilde{\mathbf{H}}\tilde{\Phi}_1 \Delta\bar{\mathbf{x}}, \tag{25}$$

with

$$\tilde{\mathbf{H}} \equiv \left[\mathbf{I}_3 \;\; \mathbf{0}_{3\times p}\right]. \tag{26}$$

By applying mathematical induction to Eq. (20) and making use of the fact that the first three rows of the matrix $\tilde{\mathbf{H}}\tilde{\Phi}_k$ coincide with the first three rows of the transition matrix $\tilde{\Phi}_k$ for any k, one can easily verify that

$$\vec{\theta}_k = \tilde{\mathbf{H}}\tilde{\Phi}_k \Delta\bar{\mathbf{x}}. \tag{27}$$

Substituting the state transition matrix $\tilde{\Phi}_{\text{state}}$ for $\tilde{\Phi}_n$ then immediately leads to Eq. (14), which constitutes the main result of this work.

IV. REPRESENTATION OF THE DAVENPORT GYRO CALIBRATION SCHEME IN A RECURSIVE FORM

The suggested recursive procedure has been implemented in the following way.[8]

Estimation starts by setting elements of the so-called 'measurement accumulation' vector $\Delta\bar{\mathbf{u}}$ to zero. One also initializes elements of the covariance matrix $\mathbf{P}$ with some a priori values. The attitude matrix is then propagated from one gyro measurement to another:

$$\mathbf{A}_k = \mathbf{R}_{k\text{-}1\to k}\mathbf{A}_{k\text{-}1} \tag{28}$$

starting from the given observed attitude $\mathbf{A}_0$ associated with the quaternion $\bar{\mathbf{q}}_{\text{init}}$ at the beginning of the first maneuver. At the same step one also computes error sensitivity matrix (11) and rate-dependent partials in the right-hand side of Eq. (22) which are then substituted, together with the rotation matrix $\mathbf{R}_{k-1\to k}$, into Eq. (21) for the matrix $\Phi_{k-1\to k}$ used to propagate the state transition matrix via recurrence sequence (15). At the end of the maneuver the AMVR $\bar{\theta}$ is computed by linearizing Eqs. (16):

$$\bar{\theta} \approx \frac{1}{2}\left[\delta A_{23} - \delta A_{32},\ \delta A_{31} - \delta A_{13},\ \delta A_{12} - \delta A_{21}\right]^T \tag{29}$$

where δA_{ij} are elements of the orthogonal matrix

$$\delta \mathbf{A} \equiv \mathbf{A}_{obs} \mathbf{A}_{prop}^T \ . \tag{30}$$

After the AMVR $\bar{\theta}$ is computed, one updates the measurement accumulation vector and the inverse covariance matrix $\mathbf{W} \equiv \mathbf{P}^{-1}$ according to the standard equations:

$$\Delta\bar{\mathbf{u}} \leftarrow \Delta\bar{\mathbf{u}} + \tilde{\Phi}_{state}^T \tilde{\mathbf{H}}^T \mathbf{C}^{-1} \bar{\theta} \tag{31}$$

and

$$\mathbf{W} \leftarrow \mathbf{W} + \tilde{\Phi}_{state}^T \tilde{\mathbf{H}}^T \mathbf{C}^{-1} \tilde{\mathbf{H}} \tilde{\Phi}_{state} , \tag{32}$$

where $\mathbf{C}$ is a 3×3 measurement covariance matrix. The state transition matrix is then reset to $\tilde{\Phi}_0$ and the processing continues starting from the beginning of the next maneuver.

After the last maneuver is processed, one obtains the covariance matrix $\mathbf{P}$ by inverting the resultant $\mathbf{W}$ matrix and computing the state error vector $\Delta\bar{\mathbf{x}}$ from the measurement accumulation vector $\Delta\bar{\mathbf{u}}$:

$$\Delta\bar{\mathbf{x}} = \mathbf{P}\,\Delta\bar{\mathbf{u}} \, . \tag{33}$$

The magnitude of the state error vector $\Delta\bar{\mathbf{x}}$ is then compared with the given tolerance to proceed with iterations if necessary.

V. TRMM IN-FLIGHT GYRO CALIBRATION

The TRMM is an Earth-pointing three-axis stabilized spacecraft. Its body z-axis is nominally pointed along the geodetic nadir.[13] It can be in '+x forward' or '-x forward' nominal mode, with its body x-axis being approximately either parallel or anti-parallel to the spacecraft velocity. For power and thermal protection of science instruments from direct exposure to the Sun, yaw maneuvers from one nominal mode to another (similar to those depicted in Figs. 1a and 1b) are periodically performed. Since the body y-axis is parallel (anti-parallel) to the orbit normal in the '-x forward' ('+x forward') mode, the only nonzero component of the spacecraft angular velocity vector is the pitch rate, nominally equal to +1 revolution per orbit (RPO) in the '-x forward' mode and -1 RPO in '+x forward' mode. This is illustrated by the flat portions of the dashed curves in Figures 1a and 1b. It is essential that rates remain nearly constant in both nominal modes, and therefore the spacecraft attitude can be determined with a sufficient accuracy without a complete gyro calibration, provided that gyro biases are included in the state vector to be solved for. (Unknown errors in scale factors and gyro misalignments manifest themselves as some additional biases, which differ for different modes.)

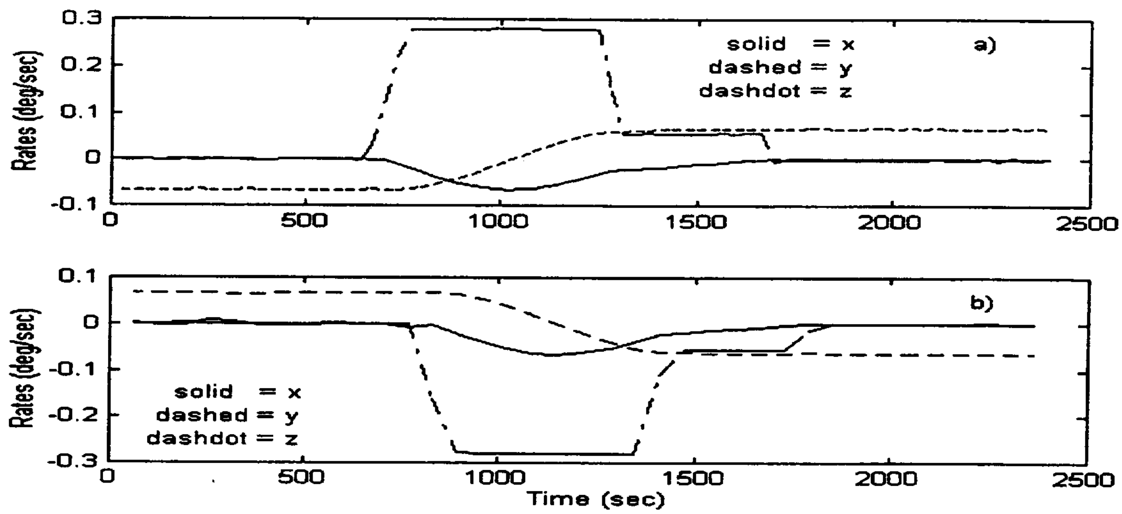

Figure 1. TRMM body rates during +X to –X (upper) and –X to +X (lower) yaw maneuvers.

On December 14, 1997 the TRMM was also placed in the '-y forward' mode to calibrate scientific instruments, with the y body axis being anti-parallel to the spacecraft velocity vector. As seen from gyro rate profiles depicted in Fig. 2, the spacecraft was rotating for about one hour around its body x-axis with the rate of +1 RPO, before coming back to the '-x forward' mode.

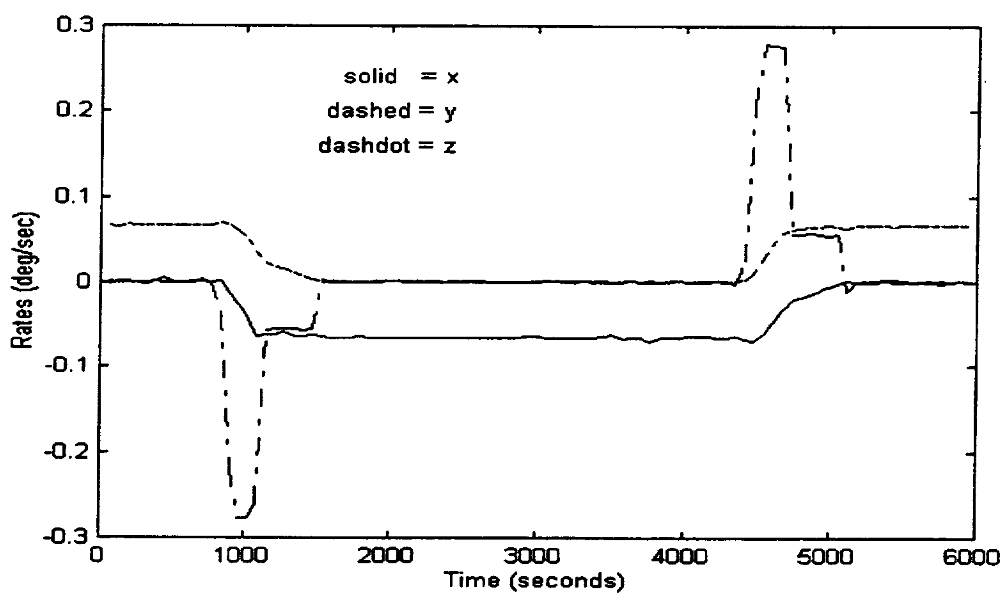

Figure 2. TRMM body rates in the –Y forward mode.

In addition to maneuvers between the three Earth-pointing modes mentioned above, the TRMM was commanded on January 7, 1998 to stay for one orbit in the inertial hold mode used to calibrate a science instrument by pointing it toward cold space. Fig. 3 presents the corresponding x and y body rates. (The z body rate is omitted since its deviations from zero would be practically invisible at the figure scale.)

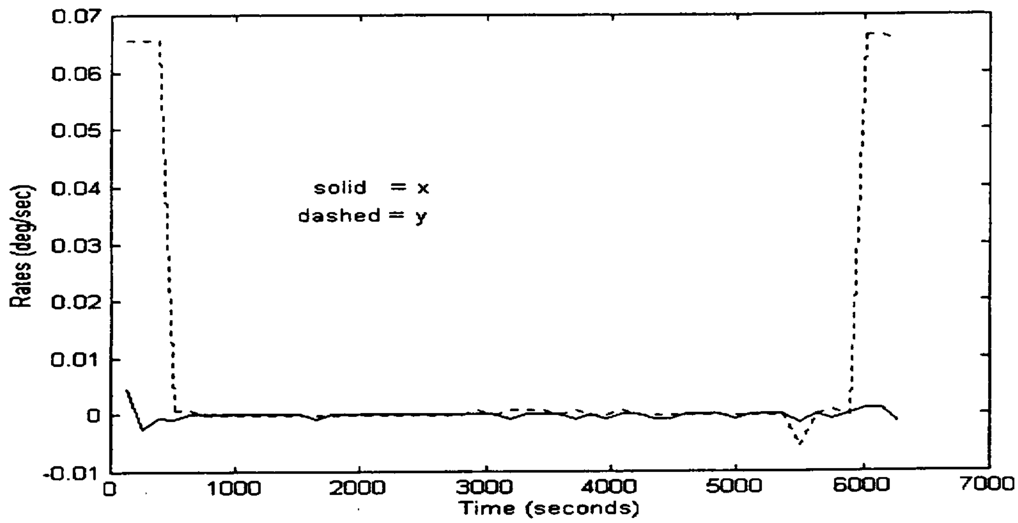

Figure 3. TRMM body rates during inertial hold.

Table 1 lists time intervals selected for calibration of the TRMM inertial reference unit (IRU).[9] The reference attitudes were determined at the beginning and at the end of each maneuver using Digital Sun Sensor (DSS) and Barnes Static Earth Sensor Assembly (SESA) measurements.[13]

Table 1 - Intervals of Gyro Data Used for IRU Calibration

Maneuvers	Time Interval	Spacecraft Activity
1	9:45:00 – 11:25:00, Dec. 13, 1997	+x forward
2	13:11:00 – 13:29:00, Dec. 13, 1997	+x to –x yaw maneuver
3	15:05:00 - 16:40:00, Dec. 13, 1997	–x forward
4	11:43:30 - 12:58:00, Dec. 14, 1997	–y forward mode
5	13:07:00 - 14:39:15, Jan. 7, 1998	Inertial hold
6	20:47:30 - 21:06:00, Jan. 14, 1998	–x to +x yaw maneuver

Three residuals per maneuver were then obtained by propagating the spacecraft attitude with gyro rates from the beginning of each maneuver and comparing the result with the predetermined reference attitude at the end of the maneuver. Table 2 presents roll, pitch, and yaw attitude residuals obtained by gyro propagation with pre-launch and calibrated gyro biases, scale factors, and misalignment matrix. The corresponding values of calibration parameters for each gyro (i. e., a bias, a scale factor, and misalignment angles* relative to body axes) are listed in Table 3.

* A deviation of the gyro axis from its nominal direction can be derived from two other misalignments; however, it is included in Table 2 just to simplify the notation, with zeros standing for some rather small numbers completely irrelevant to our discussion.

Table 2- Roll, Pitch, and Yaw Attitude Residuals (deg) Before and After Calibration

Maneuvers	Pre-launch			1st iteration		
	$\Delta t = 0.5$ s	$\Delta t = 1$ s	$\Delta t = 2$ s	$\Delta t = 0.5$ s	$\Delta t = 1$ s	$\Delta t = 2$ s
1	0.012	0.022	-0.008	0.005	0.024	0.001
	-0.044	-0.072	-0.105	-0.015	-0.004	-0.008
	0.045	0.030	0.053	0.002	0.002	0.012
2	0.195	0.212	0.202	-0.006	0.009	-0.009
	0.002	-0.001	-0.003	-0.004	-0.003	-0.001
	0.024	0.031	0.031	0.003	-0.004	0.002
3	0.018	0.015	-0.021	0.006	0.001	-0.040
	-0.433	-0.412	-0.441	-0.007	-0.002	-0.003
	-0.011	0.003	0.034	0.003	0.008	-0.042
4	0.235	0.233	0.265	-0.0002	0.001	-0.001
	-0.243	-0.233	-0.243	-0.0001	-0.001	-0.001
	-0.108	-0.116	-0.131	0.003	-0.006	0.007
5	0.073	0.094	0.099	-0.003	-0.001	-0.001
	-0.194	-0.223	-0.248	0.028	0.008	0.011
	0.066	0.071	0.075	0.024	0.008	0.006
6	0.273	0.266	0.271	0.002	-0.004	0.006
	0.105	0.086	0.078	0.010	0.005	0.003
	0.047	0.045	0.074	0.006	-0.005	0.004

Table 3 - Calibration parameters

	Pre-launch	1st iteration		
		$\Delta t = 0.5$ s	$\Delta t = 1$ s	$\Delta t = 2$ s
Biases (deg/sec)	-1.41×10^{-4}	-1.55×10^{-4}	-1.58×10^{-4}	-1.59×10^{-4}
	1.58×10^{-4}	1.98×10^{-4}	2.00×10^{-4}	2.04×10^{-4}
	0.86×10^{-4}	0.78×10^{-4}	0.75×10^{-4}	0.73×10^{-4}
Scale factors	1.00000	1.00044	1.00035	1.00031
	1.00000	1.00053	1.00046	1.00046
	1.00000	1.00088	1.00092	1.00096
Misalignment angles (deg) relative body axis x	0.000	0.000	0.000	0.000
	0.059	0.044	0.051	0.044
	0.005	0.054	0.067	0.071
Misalignment angles (deg) relative body axis y	-0.036	-0.058	-0.068	-0.086
	0.000	0.000	0.000	0.000
	0.030	0.096	0.063	0.079
Misalignment angles (deg) relative body axis z	0.006	0.039	0.034	0.033
	-0.033	-0.124	-0.119	-0.118
	0.000	0.000	0.000	0.000

To investigate the possibility of reducing the amount of processed gyro data (about 60 000 points for all six maneuvers), propagation was performed using several time steps: Δt = 0.5 s, 1 s, 2s. Inspection of Table 2 and Table 4 shows that every other gyro measurement can be successfully skipped without any noticeable effect on the accuracy of estimation. Skipping three of every four gyro measurements still gives reasonably good results, though some degradation in the accuracy can be clearly seen.

Table 4 - Mean and RMS Residuals (deg)

	Pre-launch			1st iteration		
	$\Delta t = 0.5$ s	$\Delta t = 1$ s	$\Delta t = 2$ s	$\Delta t = 0.5$ s	$\Delta t = 1$ s	$\Delta t = 2$ s
Mean Residuals	0.1343 -0.1343 0.0103	0.1404 -0.1426 0.0105	0.1343 -0.1603 0.0226	0.0008 0.0016 0.0068	0.0050 0.0004 0.0004	-0.0073 0.0001 0.0123
RMS Residuals	0.0694 0.0908 0.0242	0.0707 0.0892 0.0249	0.0738 0.0960 0.0303	0.0015 0.0058 0.0043	0.0043 0.0019 0.0024	0.0070 0.0024 0.0075

It is essential that the calibration be accomplished by a single iteration. This can be easily seen by comparing corrections to gyro biases and elements of the G-matrix due to the first and second iterations, presented in Table 5. Contributions to attitude residuals from the second iteration are so small that they would have no effect on the values t in Tables 2 and 4 (to the precision shown).

Table 5- Bias and G-Matrix Corrections

	1st iteration			2nd iteration		
	x-gyro	y-gyro	z-gyro	x-gyro	y-gyro	z-gyro
Bias corrections (deg/sec)	-1.74×10^{-5}	4.15×10^{-5}	-1.15×10^{-5}	-3.89×10^{-8}	4.18×10^{-8}	0.11×10^{-8}
Corrections to G-matrix elements	-0.44×10^{-3} 0.27×10^{-3} -0.86×10^{-3}	0.39×10^{-3} -0.54×10^{-3} -1.16×10^{-3}	-0.57×10^{-3} 1.59×10^{-3} -0.88×10^{-3}	-2.69×10^{-6} -2.86×10^{-6} -0.85×10^{-6}	-1.90×10^{-6} -0.58×10^{-6} -4.08×10^{-6}	-0.72×10^{-6} 0.81×10^{-6} -1.43×10^{-6}

To study the significance of higher-order powers of Δt_k in the error sensitivity, calibration was repeated using only the linear terms in Eq. (11), which is equivalent to the use of the Davenport method in its conventional implementation.[7] It was found that neglecting higher- order terms does not practically affect the calibration results, so that the linear approximation seems to be sufficiently accurate for calibration purposes.

VI. CONCLUSIONS

A new approach has been implemented in the ground calibration utilities[8] and successfully used for the TRMM IRU calibration. It has been shown that the required accuracy can be achieved by a single iteration. As a result the new approach seems to be especially useful for onboard applications by allowing one to discard each gyro measurement immediately after it is used to update the state vector and covariance matrix.

Recently the *Rossi X-Ray Timing Explorer (RXTE)* ground launch support team has reported[14] some problems in Kalman filter estimation of gyro scale factors and gyro misalignments, and the new least-squares approach to gyro calibration makes it possible to extend advantages of the Davenport algorithm to onboard applications. To avoid memory-consuming batch attitude determination, one can use the Kalman filter to determine the spacecraft attitudes before and after each of the selected maneuvers. The Kalman filter state vector is composed only of attitude and gyro bias errors, so that gyro biases will change with each new gyro measurement, in contrast with those used in the Davenport method. For this reason one has to propagate in parallel two separate transition matrices, namely, propagation rates for the spacecraft attitude and the transition matrix used by the suggested recursive algorithm are obtained by adjusting raw measurements with a priori biases which remain the same for all the selected maneuvers. On the other hand, propagation rates for attitude errors and the transition matrix utilized by the Kalman filter are obtained by adjusting raw measurements with the solved-for biases (and the same scale factors and misalignments as in the former case). Feasibility of this approach is currently investigated using Submillimeter Wave Astronomy Satellite (SWAS) simulated data.

ACKNOWLEDGEMENTS

The author would like to thank J. Sedlak for very thorough editing of the manuscript and numerous critical remarks, J. Hashmall for providing preliminary calibration results, as well as the necessary attitude history file and gyro measurements, and J. Glickman for a comprehensive overview of TRMM mission requirements and some useful comments on the paper.

References

1. L. Fallon, III, and P. V. Rigterink, "Introduction to Estimation Theory" in *Spacecraft Attitude Determination and Control*, J. Wertz, editor, D. Reidel, Dordrecht, Holland, 1978

2. M. Nicholson, F. Markley, and E. Seidewitz, "Attitude Determination Error Analysis System (ADEAS) Mathematical Specifications Document," CSC/TM-88/6001, prepared by Computer Sciences Corporation, October 1988

3. J. Landis et al, "Multimission Three-Axis Stabilized Spacecraft (MTASS) Flight Dynamics Support System," Section 2.4, CSC/TR-91/6071R1UD0, prepared by Computer Sciences Corporation, Sept. 1995

4. P. Davenport, "In-flight Calibration of Gyros," Goddard Space Flight Center, Spring 1976

5. J. Keat, "Gyro Calibration Analysis for the High Energy Astronomy observatory-A (HEAO-A)," CSC/TM-77/6082, prepared by Computer Sciences Corporation, June 1977

6. P. Davenport and G. Welter, "Algorithm for In-flight Gyroscope Calibration," Flight Mechanics/Estimation Theory Symposium, May 10–11, 1988

7. J. Landis et al, "Multimission Three-Axis Stabilized Spacecraft (MTASS) Flight Dynamics Support System," Section 4.2, CSC/TR-91/6071R1UD0, prepared by Computer Sciences Corporation, Sept. 1995

8. G. Klitsch, M. Lambertson, G. Natanson, R. Strang, et al., "Flight Dynamics Distributed Systems (FDDS) Generalized Support Software (GSS) Functional Specification," Revision 1, Update 4, CSC/TR-92/6023R1UD4, prepared by Computer Sciences Corporation, September 1996

9. J. Hashmall, "TRMM Inertial Reference Unit Calibration," unpublished memo, Febr. 1998.

10. F. Markley, "Three-Axis Attitude Determination Methods" in *Spacecraft Attitude Determination and Control*, J. Wertz, editor, D. Reidel, Dordrecht, Holland, 1978

11. G. Natanson, "A Transition Matrix Approach to the Davenport Gyro Calibration Scheme ", Memo CSC-27434-62, Nov. 1997.

12. T. Becher et al., "International Solar-Terrestial Physics (ISTP) / Collaborative Solar-Terrestial Research (COSTR) Initiative: Solar and Heliospheric Observatory (SOHO) Mission, Flight Dynamics Support System (FDSS) Functional Specifications," CSC/TR-92/6102R0UD0, prepared by Computer Sciences Corporation, February 1993

13. J. Glickman et al., "Tropical Rainfall Measuring Mission (TRMM) Flight Dynamics Support System (FDSS) Functional Specifications, " CSC/TR-94/6045R0UD0, prepared by Computer Sciences Corporation, October 1994

14. D. Fink, W. Davis et al., "Rossi X-Ray Timing Explorer (RXTE) Postlaunch Report," CSC 10032526, prepared by Computer Sciences Corporation, June 1996