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MODELING OF A TWO-PHASE JET PUMP WITH PHASE CHANGE, SHOCKS  
AND TEMPERATURE-DEPENDENT PROPERTIES

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## **Introduction**

One of the primary motivations behind this work is the attempt to understand the physics of a two-phase jet pump which constitutes part of a flow boiling test facility at NASA-Marshall. The flow boiling apparatus is intended to provide data necessary to design highly efficient two-phase thermal control systems for aerospace applications. The facility will also be capable of testing alternative refrigerants and evaluate their performance using various heat exchangers with enhanced surfaces. The test facility is also intended for use in evaluating single-phase performance of systems currently using CFC refrigerants.

Literature dealing with jet pumps is abundant and covers a very wide array of application areas. Example application areas includes vacuum pumps which are used in the food industry, power station work, and the chemical industry; ejector systems which have applications in the aircraft industry as cabin ventilators and for purposes of jet thrust augmentation; jet pumps which are used in the oil industry for oil well pumping; and steam-jet ejector refrigeration, to just name a few. Examples of work relevant to this investigation includes those of Fairuzov and Bredikhin (1995).

While past researchers have been able to model the two-phase flow jet pump using the one-dimensional assumption with no shock waves and no phase change, there is no research known to the author apart from that of Anand (1992) who was able to account for condensation shocks. Thus, one of the objectives of this work is to model the dynamics of fluid interaction between a two-phase primary fluid and a subcooled liquid secondary fluid which is being injected employing atomizing spray injectors. The model developed accounts for phase transformations due to expansion, compression, and mixing. It also accounts for shock waves developing in the different parts of the jet pump as well as temperature and pressure dependencies of the fluid properties for both the primary two-phase mixture and the secondary subcooled liquid.

## **Analysis**

The objective of this research effort is to develop an analytical model capable of predicting the performance of a two-phase jet pump. The model developed will account for phase transformations which are likely to take place in the nozzle, mixing chamber and diffuser (see Figure 1). Complete details of this model can be obtained from the NASA colleague.

**Primary Nozzle.** The approach employed in the analysis described in this report is an extension of the isentropic homogeneous expansion (IHE) model described in the literature. In addition to the primary nozzle, the extension to the model covers the rest of the jet pump components and is thus labeled as an isentropic homogeneous expansion/compression (IHE/C) method. This method assumes that the velocities of the two phases are equal and that thermal equilibrium between the phases exists. It further assumes that both the expansion in the nozzle and compression in the diffuser are isentropic and that the working fluid property data correspond to those of a static, equilibrium, two-phase system with plane interfaces.

In order to be able to deal with the flow in the jet pump as a homogeneous two-phase mixture, equivalent properties are derived for the mixture in terms of corresponding properties for its vapor and liquid phases. In the nozzle part of the jet pump, the two-phase primary fluid will be treated as predominantly gaseous with liquid particles suspended in it. It is hypothesized that the presence of the liquid particles in the mixture is such that the latter can still behave as a gas, but with modified properties.

The mass flow rate of the two-phase fluid flowing through the primary nozzle can be expressed according to the following equation (Shapiro, 1953):

$$m_n = \frac{A_n P_{ni}}{\sqrt{T_{ni}}} \sqrt{\frac{\gamma_n}{R_n}} \frac{M_n}{\left[1 + \frac{(\gamma_n - 1)}{2} M_n^2\right]^{(\gamma_n + 1)/[2(\gamma_n - 1)]}} \quad (1)$$

where the quantities with the subscript "n" are those of the two-phase primary fluid while flowing through the nozzle. To obtain an expression for the specific heat ratio of the primary fluid,  $\gamma_n$ , the mixture quality in the primary nozzle,  $x_n$ , is used in conjunction with the definition of  $\gamma_n$ :

$$\gamma_n = \frac{C_{p,n}}{C_{v,n}} = \frac{x_n C_{p,g,n} + (1 - x_n) C_{L,n}}{x_n C_{v,g,n} + (1 - x_n) C_{L,n}} \quad (2)$$

Utilizing the following relationships

$$R_{g,n} = C_{p,g,n} - C_{v,g,n} \quad (3)$$

$$C_{p,g,n} = \gamma_{g,n} C_{v,g,n} \quad (4)$$

Equation (2) can be rewritten as

$$\gamma_n = \frac{x_n \gamma_{g,n} R_{g,n} + (1 - x_n)(\gamma_{g,n} - 1) C_{L,n}}{x_n R_{g,n} + (1 - x_n)(\gamma_{g,n} - 1) C_{L,n}} \quad (5)$$

To obtain an expression for the two-phase gas constant of the primary fluid,  $R_n$ , we utilize the expressions for  $C_{p,n}$ ,  $C_{v,n}$  and the definition of  $R_n$  to get

$$R_n = x_n R_{g,n} \quad (6)$$

With the exception of the specific heat at constant pressure of the vapor phase, the properties of interest are fairly independent of temperature and pressure. For the two-phase primary fluid, the properties  $C_{p,n}$ ,  $C_{v,n}$ ,  $\gamma_n$ , and  $R_n$  are therefore going to be predominantly functions of the primary fluid vapor-liquid composition or the quality,  $x_n$ , and will be functions of temperature to the extent that  $C_{p,g,n}$  depends on the temperature. Having established this fact, we can now proceed to perform two-phase calculations

throughout the nozzle to establish the conditions of the two-phase primary fluid at the nozzle exit. This will be done by first computing the mixture specific entropy at nozzle inlet,  $s_{ni}$ , as follows:

$$s_{ni} = s_{ni,L} + x_{ni} (s_{ni,g} - s_{ni,L}) \quad (7)$$

where  $s_{ni,g}$  and  $s_{ni,L}$  represent the saturated vapor and saturated liquid specific entropies evaluated at the nozzle inlet pressure,  $p_{ni}$ .

Next, the nozzle exit pressure is assumed and the saturated vapor and saturated liquid specific entropies,  $s_{ne,g}$  and  $s_{ne,L}$ , respectively, are computed. Since the flow is assumed isentropic throughout the nozzle, the specific entropy at the nozzle exit will be equal to that at the nozzle inlet. This allows the quality of the primary fluid at the exit,  $x_{ne}$ , to be computed

$$x_{ne} = \frac{(s_{ne} - s_{ne,L})}{(s_{ne,g} - s_{ne,L})} \quad (8)$$

Since the exit pressure has been assumed, the exit temperature will have also been specified (since the fluid is in the mixture region). This allows computing the vapor specific heat at constant pressure,  $C_{p,g,ne}$ , the gas constant of the vapor phase,  $R_{g,ne}$ , and the vapor phase ratio of specific heats,  $\gamma_{g,ne}$  all evaluated at the nozzle exit. These, in addition to the primary fluid quality computed from Equation (8), can then be used to compute the primary fluid specific heat ratio from Equation (5).

Now that the exit pressure and mixture specific heat ratio are known, the exit Mach number can be computed employing the following equation (Shapiro, 1953):

$$M_{ne} = \left\{ \frac{2}{(\gamma_{ne} - 1)} \left[ \left( \frac{p_{ni}}{p_{ne}} \right)^{(\gamma_{ne}-1)/\gamma_{ne}} - 1 \right] \right\}^{1/2} \quad (9)$$

Using the exit Mach number,  $M_{ne}$ , along with the value of the primary fluid specific heat ratio,  $\gamma_{ne}$ , the area ratio between the exit and throat can be computed using the following equation (Shapiro, 1953):

$$\frac{A_{ne}}{A_{nt}} = \frac{1}{M_{ne}} \left[ \frac{2}{(\gamma_{ne} + 1)} \left\{ 1 + \frac{(\gamma_{ne} - 1)}{2} M_{ne}^2 \right\} \right]^{(\gamma_{ne}+1)/(2(\gamma_{ne}-1))} \quad (10)$$

Since  $A_{ne}/A_{nt}$  is a known quantity for a specified nozzle geometry, the area ratio computed from Equation (10) can now be compared to that assigned by the nozzle configuration. If the two values did not agree, then Equation (10) can be used to compute the Mach number at the nozzle exit corresponding to the actual area ratio, but employing the computed value of  $\gamma_{ne}$  from the previous calculation step. The new value of the Mach number can now be used to compute a new value for the nozzle exit pressure,  $p_{ne}$ , using the following equation (Shapiro, 1953):

$$\frac{p_{ni}}{p_{ne}} = \left[ 1 + \frac{(\gamma_{ne} - 1)}{2} M_{ne}^2 \right]^{\gamma_{ne}/(\gamma_{ne}-1)} \quad (11)$$

Again, the value of  $\gamma_{ne}$  used in Equation (11) is the same as the one employed in the previous calculation step. Once the nozzle exit pressure has been found, the exit temperature,  $T_{ne}$  can also be found since the primary fluid is a two-phase mixture. The new exit temperature is then used to correct the value of the vapor phase specific heat at constant pressure,  $C_{p,g,ne}$ , and the vapor phase gas constant,  $R_{g,ne}$ , both evaluated at the nozzle exit. As mentioned earlier, variations in the gas phase ratio of specific heats as well as the specific heat of the liquid phase are relatively insensitive to temperature variations in the temperature range of interest to this study. While updating the values of these parameters to account for the newly calculated temperature value at the nozzle exit is relatively straightforward, it will not be performed here since its effect on the accuracy of the results should be fairly minimal.

With the newly calculated exit pressure, the specific entropies of the saturated vapor and saturated liquid evaluated at the nozzle exit pressure can be found and used to calculate the updated value of the primary fluid quality at the nozzle exit using Equation (8) and the isentropic flow assumption. This quality can, in turn, be used to calculate the primary fluid ratio of specific heats at the exit,  $\gamma_{ne}$ , employing Equation (5). Knowing,  $\gamma_{ne}$ , a new value for the Mach number at the exit can be calculated using Equation (9) and subsequently a new area ratio can be calculated using Equation (10). The calculated area ratio should then be compared to the actual area ratio, and in case they do not agree, the whole procedure is repeated until convergence.

After convergence has been reached, the values of all the relevant parameters of the primary fluid at the nozzle exit should be updated. This should include the ratio of specific heats  $\gamma_{ne}$ , gas constant  $R_{ne}$ , specific heats  $C_{p,ne}$  and  $C_{v,ne}$ , temperature  $T_{ne}$ , pressure  $p_{ne}$ , quality  $x_{ne}$ , entropy  $s_{ne}$ , enthalpy  $h_{ne}$ , density  $\rho_{ne}$ , and Mach number  $M_{ne}$ . The enthalpy can be computed using the following equation:

$$h_{ne} = h_{ne,L} + x_{ne} (h_{ne,g} - h_{ne,L}) \quad (12)$$

where  $h_{ne,L}$  and  $h_{ne,g}$  represent the saturated liquid and saturated vapor enthalpies evaluated at the exit pressure,  $p_{ne}$ . The density,  $\rho_{ne}$ , on the other hand, can be computed from the following equation (Shapiro, 1953):

$$\frac{\rho_{ni}}{\rho_{ne}} = \left[ 1 + \frac{(\gamma_{ne} - 1)}{2} M_{ne}^2 \right]^{1/(\gamma_{ne}-1)} \quad (13)$$

Of course the specific volume at the nozzle exit,  $v_{ne}$ , is the reciprocal of the density and can easily be determined. The mass flow rate of the primary fluid can now be computed from Equation (1). The area employed in the equation should correspond to the exit section since the Mach number and fluid properties used correspond to the exit.

With the knowledge of the mass flow rate, as well as the exit area and density, the nozzle exit velocity can be computed using the continuity equation

$$V_{ne} = \dot{m}_n / (\rho_{ne} A_{ne}) \quad (14)$$

where the quantities  $v_{ni,g}$  and  $v_{ni,L}$  represent the specific volume of the saturated vapor and saturated liquid of the primary fluid evaluated at the nozzle inlet pressure,  $p_{ni}$ .

**Liquid Sprayers.** As was reported earlier, atomizing sprayers are used to inject subcooled liquid into the mixing chamber of the jet pump. In order to relate the pressure and velocity of the secondary liquid at the inlet of the mixing chamber to those at the inlet of the sprayers, Bernoulli's equation is used

$$(p_{si} - p_{mi}) = \frac{1}{2} \rho_s (V_{s,mi}^2 - V_{si}^2) \quad (15)$$

Additional details can be found with the NASA colleague.

**Mixing Chamber.** The governing conservation equations for a control volume representing the mixing chamber are those of the continuity, momentum, and energy (in integrated forms):

$$\dot{m}_m = \dot{m}_n + \dot{m}_s \quad (17)$$

$$A_{ne} p_{ne} + A_s p_{mi} \cos \alpha - A_{me} p_{me} = \dot{m}_m V_{me} - \dot{m}_n V_{p,mi} - \dot{m}_s V_{s,mi} \cos \alpha \quad (18)$$

$$\dot{m}_m \left( h_{me} + \frac{V_{me}^2}{2} \right) = \dot{m}_n \left( h_{ni} + \frac{V_{ni}^2}{2} \right) + \dot{m}_s \left( h_{si} + \frac{V_{si}^2}{2} \right) \quad (19)$$

Additional details about the mixing chamber analysis can be found with the NASA colleague.

**Throat and Diffuser.** Because it is highly likely that a condensation shock would take place some where in either the throat or the diffuser, the latter will be divided into three segments; the part of the diffuser upstream of the shock, the shock layer itself, and the part of the diffuser downstream of the shock. For both parts upstream and downstream of the shock, control volume analyses will be performed. For the shock layer, the standard normal shock relations for a two-phase substance will be implemented.

The problem on hand can proceed in two different but equivalent routes. The first route is to assume the location of the condensation shock and let the solution predict the diffuser exit conditions. This route leads to predicting the jet pump compression ratio with a prescribed shock location. The other route is one in which the exit pressure of the diffuser is controlled (such as the work of Fairuzov and Bredikhin, 1995) and the location of the shock is determined as part of the solution. In this report, the latter approach is chosen. Complete details on the governing equations of the throat and diffuser can be found with the NASA colleague.

## Conclusions

The research effort on which this document partly reports described a relatively simple model capable of describing the performance of a two-phase flow jet pump. The model is based on the isentropic homogeneous expansion/compression hypothesis and is capable of fully incorporating the effects of shocks in both the mixing chamber and the throat/diffuser parts of the pump. The physical system chosen is identical to that experimentally tested by Fairuzov and Bredikhin (1995) and should therefore relatively easy to validate.

## References

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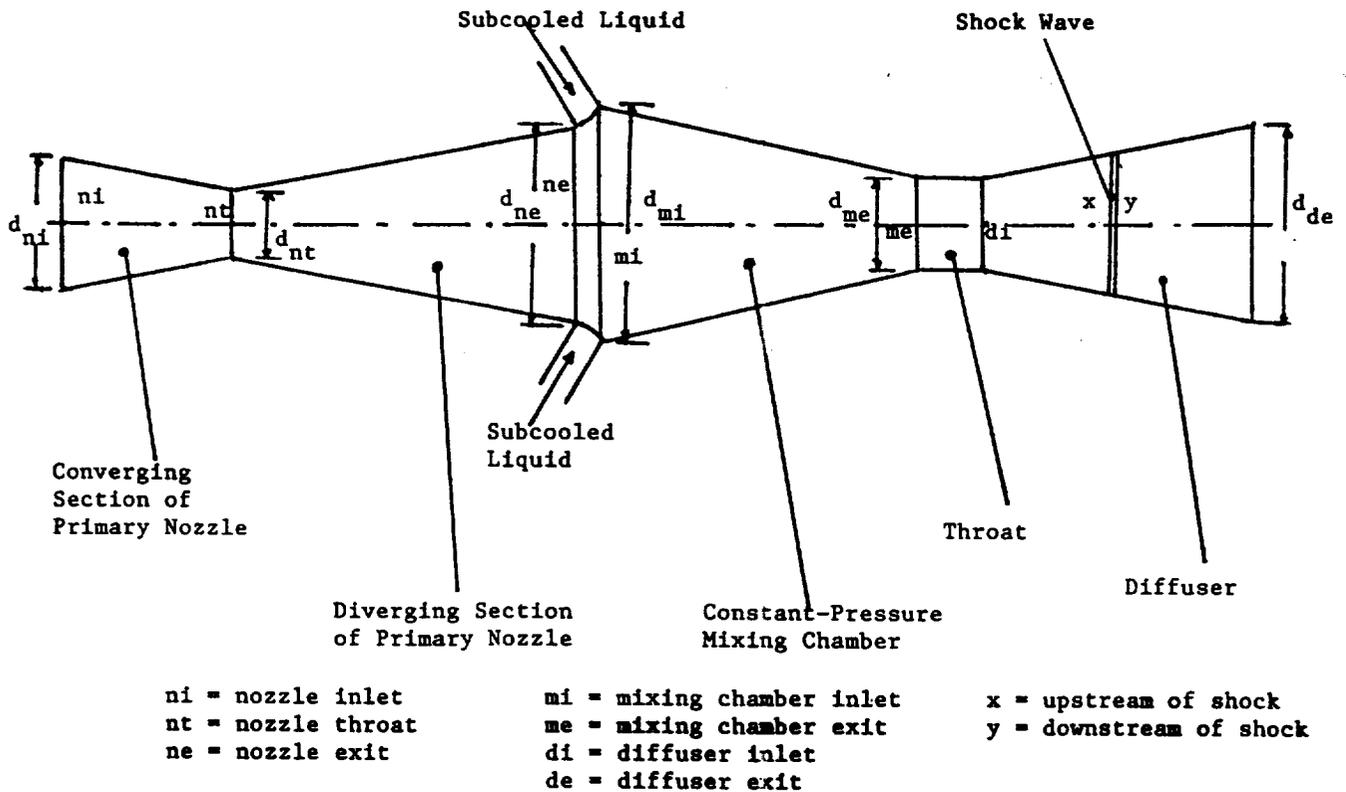


Figure 1. Physical model of the jet pump