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X33 REUSABLE LAUNCH VEHICLE CONTROL ON SLIDING MODES:
CONCEPTS FOR A CONTROL SYSTEM DEVELOPMENT

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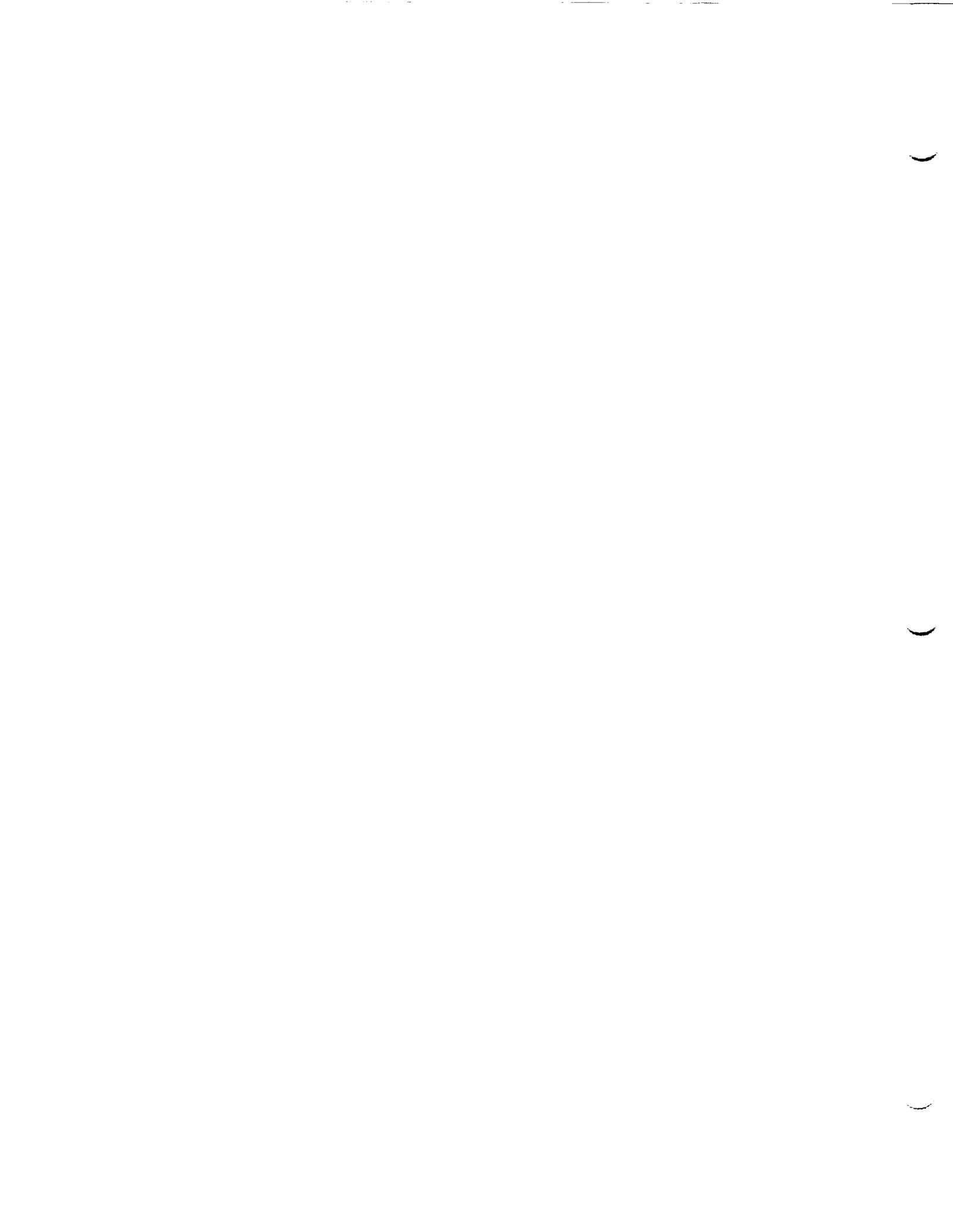
Introduction

Control of the X33 reusable launch vehicle is considered. The launch control problem consists of automatic tracking the launch trajectory which as assumed to be optimally precalculated. It requires development of reliable, robust control algorithm that can automatically adjust to some changes in mission specifications (mass of payload, target orbit) and the operating environment (atmospheric perturbations, interconnection perturbations from the other subsystems of the vehicle, thrust deficiencies, failure scenarios). One of the effective control strategies successfully applied in nonlinear systems is the *Sliding Mode Control*. The main advantage of the Sliding Mode Control is that the system's state response in the sliding surface remains insensitive to certain parameter variations, nonlinearities and disturbances.

Concepts for a Control System Development

The three-time-scale controller is designed for the X33 vehicle launch (ascending) mode. The outer loop (guidance) controller is designed using dynamic inversion or sliding mode control technique. The vector of angular rates command is formed in the outer loop to provide an asymptotic tracking of the Euler angles' reference profiles. This vector is tracked in the inner loop via the smoothed sliding mode controllers. Roll, pitch and yaw torque signals are considered as control inputs. The inner loop transient response must be much faster then the outer loop one. A control allocation algorithm is employed to allocate torque commands into end-effector deflection commands, which are executed by actuators. The desired transient response of execution of end-effector deflection commands can be achieved via the "very-inner loop" controller. A structure of the X33 vehicle control system is developed and shown in fig.1.
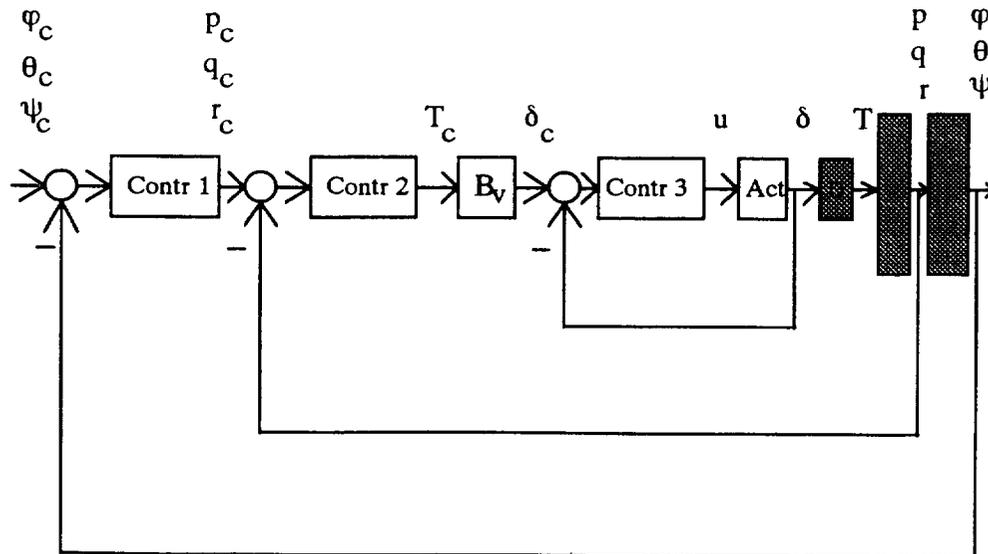

Fig. 1 A structure of the X-33 vehicle control system

Mathematical Model

Orientation equations are given in terms of Euler angles ψ, θ, φ and body rates p, q, r.

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = R(\varphi, \theta, \psi) \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \qquad R(\varphi, \theta, \psi) = \begin{bmatrix} 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \dfrac{\sin\varphi}{\cos\theta} & \dfrac{\cos\varphi}{\cos\theta} \end{bmatrix} \qquad (1)$$

The equations of the X33 vehicle rotational motion are

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_1(p,q,r,t,J) \\ f_2(p,q,r,t,J) \\ f_3(p,q,r,t,J) \end{bmatrix} + B_1(J) \begin{bmatrix} L \\ M \\ N \end{bmatrix} \qquad (2)$$

$$\begin{cases} f_1(p,q,r,t,J) = \dfrac{1}{J_{xx}}\left[J_{xy}\dot{r} - qr\left(J_{zz} - J_{yy}\right) + J_{xz}pq \right] + M_{1d}(t) \\ f_2(p,q,r,t,J) = \dfrac{1}{J_{yy}}\left[-pr\left(J_{xx} - J_{zz}\right) - J_{xz}\left(p^2 - r^2\right) \right] + M_{2d}(t), \\ f_3(p,q,r,t,J) = \dfrac{1}{J_{zz}}\left[J_{xz}\dot{p} - pq\left(J_{yy} - J_{xx}\right) - J_{xz}qr \right] + M_{3d}(t) \end{cases} \quad B_1(J) = \begin{bmatrix} \dfrac{1}{J_{xx}} & 0 & 0 \\ 0 & \dfrac{1}{J_{yy}} & 0 \\ 0 & 0 & \dfrac{1}{J_{zz}} \end{bmatrix}$$

The vector of control moments $T = \begin{bmatrix} L, M, N \end{bmatrix}^T$ is generated by five aerodynamic surfaces and an engine, and is related to the deflection vector $\delta \in \Re^8$ as follows: $T = D(.)\delta$, where $D(.) \in \Re^{3\times 8}$ is a nonlinear matrix calculated on the basis of a table-look-up data. The mathematical models of actuators are taken as follows:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = -\omega_i^2 x_{i,1} - 2\xi_i \omega_i x_{i,2} + \omega_i^2 u_i \\ \delta_i = x_{i,1} \end{cases} \qquad (3)$$

where u_i is a control input, $\omega_i = 26.4\,rad/s$, and $\xi_i = 0.7 \ \forall i = \overline{1,3}$.

The problem is to design the control laws $u_i \ \forall i = \overline{1,8}$ to provide the robust de-coupled tracking of Euler angle reference profiles to the X33 vehicle in a launch mode.

The Outer Loop (Guidance) Smoothed Sliding Mode Controller Design

The sliding surfaces are introduced as follows:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix} = \begin{bmatrix} \tilde{\varphi} \\ \tilde{\theta} \\ \tilde{\psi} \end{bmatrix} + K \begin{bmatrix} \int \tilde{\varphi} dt \\ \int \tilde{\theta} dt \\ \int \tilde{\psi} dt \end{bmatrix}, \quad K = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \tag{4}$$

where $\tilde{\varphi} = \varphi_c - \varphi$, $\tilde{\theta} = \theta_c - \theta$, $\tilde{\psi} = \psi_c - \psi$ are Euler angles' tracking errors. A smoothed sliding mode controller is designed to provide attractivity of the ε_i - vicinities of the sliding surfaces

$$\begin{bmatrix} p_c \\ q_c \\ r_c \end{bmatrix} = R^{-1}(\varphi, \theta, \psi) \left\{ \begin{bmatrix} \dot{\varphi}_c \\ \dot{\theta}_c \\ \dot{\psi}_c \end{bmatrix} + K \begin{bmatrix} \tilde{\varphi} \\ \tilde{\theta} \\ \tilde{\psi} \end{bmatrix} + \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} \begin{bmatrix} sat \dfrac{\sigma_1}{\varepsilon_1} \\ sat \dfrac{\sigma_2}{\varepsilon_2} \\ sat \dfrac{\sigma_3}{\varepsilon_3} \end{bmatrix} \right\}, \quad \varepsilon_i > 0, \rho_i > 0 \ \forall i = \overline{1,3} \tag{5}$$

Inner Loop Smoothed Sliding Mode Controller Design

The corresponding sliding surfaces are designed as follows:

$$\begin{bmatrix} \tilde{\sigma}_1 \\ \tilde{\sigma}_2 \\ \tilde{\sigma}_3 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + C^1 \begin{bmatrix} \int e_1 d\tau \\ \int e_2 d\tau \\ \int e_3 d\tau \end{bmatrix}, \quad C^1 = \begin{bmatrix} c_1^1 & 0 & 0 \\ 0 & c_2^1 & 0 \\ 0 & 0 & c_3^1 \end{bmatrix}, \tag{6}$$

where $e_1 = p_c - p$, $e_2 = q_c - q$, $e_3 = r_c - r$ are the angular rates tracking errors. The inner loop smoothed sliding mode controller is designed to provide attractivity of the ε_i - vicinities of the sliding surfaces

$$T_c = \begin{bmatrix} L_c \\ M_c \\ N_c \end{bmatrix} = \begin{bmatrix} \hat{L}_{eq} \\ \hat{M}_{eq} \\ \hat{N}_{eq} \end{bmatrix} + \begin{bmatrix} \tilde{\rho}_1 & 0 & 0 \\ 0 & \tilde{\rho}_2 & 0 \\ 0 & 0 & \tilde{\rho}_3 \end{bmatrix} \begin{bmatrix} sat \dfrac{\tilde{\sigma}_1}{\varepsilon_1} \\ sat \dfrac{\tilde{\sigma}_2}{\varepsilon_2} \\ sat \dfrac{\tilde{\sigma}_3}{\varepsilon_3} \end{bmatrix}, \quad \tilde{\rho}_i > a_i \ \forall i = \overline{1,3}. \tag{7}$$

$$\left| \Delta_{1eq} \right| = \left| L_{eq} - \hat{L}_{eq} \right| \leq a_1, \quad \left| \Delta_{2eq} \right| = \left| M_{eq} - \hat{M}_{eq} \right| \leq a_2, \quad \left| \Delta_{3eq} \right| = \left| N_{eq} - \hat{N}_{eq} \right| \leq a_3. \tag{8}$$

$$\begin{bmatrix} L_{eq} \\ M_{eq} \\ N_{eq} \end{bmatrix} = B_1^{-1} \left\{ \begin{bmatrix} \dot{p}_c \\ \dot{q}_c \\ \dot{r}_c \end{bmatrix} + C^1 \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} - \begin{bmatrix} f_1(p,q,r,t,J) \\ f_2(p,q,r,t,J) \\ f_3(p,q,r,t,J) \end{bmatrix} + \frac{1}{2} B_1^{-1} \frac{dB_1}{dt} \tilde{\sigma} \right\} \tag{9}$$

Control Allocation

The command vector of control moments T_c is executed by corresponding actuators through deflections of five aerodynamic surfaces and an orientation of the thrust vector of the rocket engine. An optimal allocation matrix $B_v(.)$ must be identified such that $\delta_c = B_v(.)T_c$.

"Very" Inner Loop Controller Design

The control inputs $u_i \ \forall i = \overline{1,8}$ of the actuators must be designed to make the compensated dynamics of the actuators much faster then the inner loop sliding mode dynamics.

X33 Launch Vehicle Smoothed Sliding Mode Controller Design in Launch Mode

The elements of the inertia matrix of the X33 vehicle in the launch mode are given

$$\begin{cases} J_{xx} = 806555.0 - 1261.0t & J_{yy} = 2371654.0 - 6175.0t \\ J_{zz} = 2781705.0 - 7026.0t & J_{xz} = 16494 - 198.0t \qquad J_{xy} = J_{yz} = 0 \end{cases} \tag{10}$$

The outer loop smoothed sliding mode controller is designed for the X33 vehicle as follows:

$$\begin{cases} p_c = a - b\sin\varphi \\ q_c = b\cos\varphi + c\sin\varphi\cos\theta \ , \\ r_c = -b\sin\varphi + c\cos\varphi\cos\theta \end{cases} \quad \begin{cases} a = \dot{\varphi}_c + 0.4\tilde{\varphi} + \rho_1(\tilde{\varphi} + 0.4\int\tilde{\varphi}dt) \\ b = \dot{\theta}_c + 0.4\tilde{\theta} + \rho_2(\tilde{\theta} + 0.4\int\tilde{\theta}dt) \\ c = \dot{\psi}_c + 0.4\tilde{\psi} + \rho_3(\tilde{\psi} + 0.4\int\tilde{\psi}dt) \end{cases} \tag{11}$$

The values of parameters $\rho_1 = 4.0$, $\rho_2 = 3.5$, $\rho_3 = 4.0$ are chosen experimentally. The inner loop smoothed sliding mode controller for the X33 vehicle is designed as well. This is

$$\begin{cases} L_c = \hat{L}_{eq} + \tilde{\rho}_1(e_1 + 1.884\int e_1 dt) \\ M_c = \tilde{M}_{eq} + \tilde{\rho}_2(e_2 + 1.884\int e_2 dt) \\ N_c = \tilde{N}_{eq} + \tilde{\rho}_3(e_3 + 1.884\int e_3 dt) \end{cases} \quad \begin{cases} \tilde{L}_{eq} = J_{xx}(-f_1(.) + 1.884e_1) \\ \tilde{M}_{eq} = J_{yy}(-f_2(.) + 1.884e_2) \\ \tilde{N}_{eq} = J_{zz}(-f_3(.) + 1.884e_3) \end{cases} \tag{12}$$

$$
\begin{cases}
f_1(.) = \dfrac{1}{J_{xx}}\left[-qr\left(J_{zz}-J_{yy}\right)+J_{xz}pq\right] \\[2mm]
f_2(.) = \dfrac{1}{J_{yy}}\left[-pr\left(J_{xx}-J_{zz}\right)-J_{xz}\left(p^2-r^2\right)\right] \\[2mm]
f_3(.) = \dfrac{1}{J_{zz}}\left[J_{xz}\dot{p}-pq\left(J_{yy}-J_{xx}\right)-J_{xz}qr\right]
\end{cases}
\tag{13}
$$

The values of parameters $\tilde{\rho}_1 = 1.0\cdot 10^7$, $\tilde{\rho}_2 = 1.5\cdot 10^7$, $\tilde{\rho}_3 = 1.5\cdot 10^7$ are chosen experimentally. Smoothed sliding mode controllers (11) - (13) are implemented as follows:

$$
\begin{aligned}
L_c &= \tilde{\rho}_1[\rho_1\tilde{\varphi} - \rho_2\tilde{\theta}\sin\varphi + 0.4\rho_1\int\tilde{\varphi}d\tau - 0.4\rho_2\sin\varphi\int\tilde{\theta}d\tau - p] + 1.884\tilde{\rho}_1[\rho_1\int\tilde{\varphi}d\tau \\
&\quad - \rho_2\int\tilde{\theta}\sin\varphi d\tau + 0.4\rho_1\iint\tilde{\varphi}d\tau dt - 0.4\rho_2\int\sin\varphi\int\tilde{\theta}d\tau dt - \int p\,dt]
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
M_c &= \tilde{\rho}_2[\rho_2\tilde{\theta}\cos\varphi + \rho_3\tilde{\psi}\sin\varphi\cos\theta + 0.4\rho_2\cos\varphi\int\tilde{\theta}d\tau + 0.4\rho_3\sin\varphi\cos\theta\int\tilde{\psi}d\tau - q] \\
&\quad + 1.884\tilde{\rho}_2[\rho_2\int\tilde{\theta}\cos\varphi dt + \rho_3\int\tilde{\psi}\sin\varphi\cos\theta dt + 0.4\rho_2\int\cos\varphi\int\tilde{\theta}d\tau dt \\
&\quad + 0.4\rho_3\int\sin\varphi\cos\theta\int\tilde{\psi}d\tau dt - \int q\,dt]
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
N_c &= \tilde{\rho}_3[-\rho_2\tilde{\theta}\sin\varphi + \rho_3\tilde{\psi}\cos\varphi\cos\theta - 0.4\rho_2\sin\varphi\int\tilde{\theta}d\tau + 0.4\rho_3\cos\varphi\cos\theta\int\tilde{\psi}d\tau - r] \\
&\quad + 1.884\rho_3[-\rho_2\int\tilde{\theta}\sin\varphi dt + \rho_3\int\tilde{\psi}\cos\varphi\cos\theta dt - 0.4\rho_2\int\sin\varphi\int\tilde{\theta}d\tau dt \\
&\quad + 0.4\rho_3\int\cos\varphi\cos\theta\int\tilde{\psi}d\tau dt - \int r\,dt]
\end{aligned}
\tag{16}
$$

Simulations showed that the designed smoothed sliding mode controllers provide a very accurate, highly robust tracking of Euler angle profiles during the X33 vehicle launch mode.

Conclusions

Employing time scaling concept a new two (three)-loop structure of the control system for the X33 launch vehicle was developed. Smoothed sliding mode controllers were designed to robustly enforce the given close-loop dynamics. Simulations of the 3-DOF model of the X33 launch vehicle with the table-look-up models for Euler angle reference profiles and disturbance torque profiles showed a very accurate, robust tracking performance.

Acknowledgment