Can a "Hyperspace" really Exist?

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Abstract

The idea of "hyperspace" is suggested as a possible approach to faster-than-light (FTL) motion. A brief summary of a 1986 study on the Euclidean representation of space-time by the author is presented. Some new calculations on the relativistic momentum and energy of a free particle in Euclidean "hyperspace" are now added and discussed. The superimposed Energy-Momentum curves for subluminal particles, tachyons, and particles in Euclidean "hyperspace" are presented. It is shown that in Euclidean "hyperspace", instead of a relativistic time dilation there is a time "compression" effect. Some fundamental questions are presented.

1. INTRODUCTION

In the George Lucas and 20th Century Fox Production of STAR WARS, Han Solo engages a drive mechanism on the Millennium Falcon to attain the "boost-to-light-speed". The star field visible from the bridge streaks backward and out of sight as the ship accelerates to light speed. Science fiction writers have fantasized time and time again the existence of a special space into which a space ship makes a transition. In this special space, objects can exceed the speed of light and then make a transition from the special space back to "normal space". Can a special space (which I will refer to as hyperspace) exist? Our first inclination is to emphatically say no! However, what we have learned from areas of research such as Kaluza-Klein theory[1] , Grand Unified supersymmetry theory [2], and Heterotic string theory [3] is that there indeed can be many types of spaces. Quantum Geometrodynamics indicates (as a theory) that space-time geometry may be fluctuating at distances within the Planck length [4]. Moreover, not only can space have different geometries but it can have enormous virtual energy content as a result of the zero-point quantum fluctuations of the electromagnetic field of the vacuum. [5] ***Thus, it is not unreasonable to at least explore the possibility that nature may provide a four-dimensional or higher dimensional space-time which exists throughout the universe and is physically accessible for the motion of particles (such as electrons, protons, photons, or neutrinos) at speeds beyond the speed of light.*** This subject would tantamount to an investigation of enormous scope, but here the author will present a brief discussion of one of the simplest space-time geometries in which he is particularly interested.

2. THE EUCLIDEAN-FOUR SPACE

Consider a Euclidean four-space geometry [6] expressed by the line element:

$$(ds)^2 = (dt)^2 + \sum_{\mu=1}^{3} (dx^\mu)^2 \quad . \tag{1}$$

The coordinates of a space-time point in this space , (x^0,x^1,x^2,x^3), are defined as cartesian coordinates (t, x,y,z) . Here, the temporal coordinate x^0 , is really ct, where t is time as measured on a clock by an observer in an inertial reference frame S, and c (the speed of light) has been set equal to one. We can ask the question: Is there a set of coordinate transformations S→ S' (where S' is moving at a speed v relative to S) that leaves $(ds)^2$ invariant? The answer is yes. Such a set of transformations is given by:

$$dt' = \gamma\,(dt + v\,dx) \quad (2)$$
$$dx' = \gamma\,(dx - v\,dt) \quad (3)$$
$$dy' = dy \quad (4)$$
$$dz' = dz \quad (5)$$
$$\gamma = (1 + v^2)^{-1/2} \quad (6)$$

Here v is really the relative speed of S' with respect to S expressed as a fraction of the speed of light. Although the above transformation equations look very similar to the Lorentz transformations they are not. Equation (2) has a plus sign where the corresponding Lorentz transformation has a minus sign. The factor γ has a plus sign , whereas Lorentz transformations would have a minus sign. Note that the set of transformations (2) through (5) do ***not*** go undefined as $v \to c$.

3. ELECTROMAGNETIC FIELD THEORY IN EUCLIDEAN FOUR-SPACE

Now [6] has found that a complete set of covariant electromagnetic field equations can be formulated for this Euclidean four-space representation of Minkowski space-time. It appears superficially on the surface as though there is a parallel space with Maxwell-like fields. But we can write the "new" field equations in their 3-space plus time representation and get the following:

$$\nabla \bullet \mathbf{E} = 4\pi\rho \quad (7)$$
$$\nabla \bullet \mathbf{B} = 0 \quad (8)$$
$$\nabla \times \mathbf{E} = \partial B/\partial t \quad (9)$$
$$\nabla \times \mathbf{B} = 4\pi \mathbf{J} + \partial E/\partial t \quad (10)$$

The obvious change is in (9) where there is a sign modification on the differential equation that expresses "Faraday's Law". This sign modification is all-important. It changes all of electrodynamics as we know it. Firstly, there is a reverse of Lenz's Law. The magnetic field produced by an induced current is in a direction such as to ***reinforce*** the original change in magnetic flux that produces it. This would increase the total magnetic flux through a loop, which, in turn, would increase the emf. This would lead to a "run-away" magnetic flux and emf. Secondly, it is shown in [6] that there will be a self-damping of mono-chromatic electromagnetic waves in the vacuum. Thirdly, the electromagnetic wave-equations in the presence of sources (charge density or current density vector) have a general solution that looks like an "advanced" and "retarded" time solution. However, this is a Euclidean-four space "analog" of classical electrodynamics since it contains imaginary terms. Specifically, the two solutions are:

$$\Psi(\mathbf{x},t) = \alpha \int d^3x' \, \frac{f(\mathbf{x}', t + i\,|\mathbf{x} - \mathbf{x}'|)}{|\mathbf{x}' - \mathbf{x}|} \quad (11)$$

$$\Psi(\mathbf{x},t) = \alpha \int d^3x' \, \frac{f(\mathbf{x}', t - i\,|\mathbf{x} - \mathbf{x}'|)}{|\mathbf{x} - \mathbf{x}'|} \quad (12)$$

where $\Psi(x,t)$ is either the scalar potential or a component of the magnetic vector potential. The " f " function represents the source density. (either charge density or the current density vector respectively) The factor α is a constant.

Question 1: The **E** and **B** fields do not appear to be the electric and magnetic fields we know. Could we "fix-up" the Maxwell-like equations (7) through (10) so that the difficulties are surmounted but the Euclidean four-space retained? What type of electrodynamics would emerge? This is a formidable problem and is yet to be solved.

Question 2: What equations of relativistic dynamics would hold for particles moving within Euclidean four-space? Some progress can be made on question (2).

4. EFFECT ON TIME MEASUREMENTS IN EUCLIDEAN FOUR SPACE

Consider a pair of events such as the emission and detection of an atomic particle. Suppose that an observer in a reference frame S' moves along with the particle at the same velocity so that the particle is observed to be "at rest" with respect to the observer in S'. We can re-introduce "c" in the line-element (1), so that dx^0 is given by cdt. For the observer co-moving with the frame S', v'=0, and the observer may define a proper time so that c dτ =ds. This leads directly to the result that ,

$$dt = d\tau \, (1+ v^2/c^2)^{-1/2} \quad . \tag{13}$$

This result is strange indeed since there is now a ***time compression*** effect from relative motion rather than a time dilation effect as in Special Relativity. Equation (13) means the following:

The apparent time interval dt (the interval between the two events as measured by an observer in the S reference frame) appears to be ***shortened*** *with respect to the time interval (dτ) as measured in the S' frame that is co-moving with the particle. Another way to interpret this result is to say that clocks appear to speed up when they are in relative motion within Euclidean hyperspace.*

5. MOMENTUM AND ENERGY

We can derive the formulas for total Relativistic Momentum and Energy of a free particle in "Hyperspace" by calculating the components of the covariant four-momentum vector. [7] If m is the rest mass,

$$\begin{aligned} P^\mu &= m\, dx^\mu/d\tau \\ &= m(dx^\mu/dt)(dt/d\tau) \\ &= mv^\mu (1+ v^2/c^2)^{-1/2} \end{aligned} \tag{14}$$

$$\begin{aligned} E &= cP^0 = mc\, (dx^0/dt)(dt/d\tau) \\ &= mc^2 (1+ v^2/c^2)^{-1/2} \end{aligned} \tag{15}$$

Resulting in:

$$E^2 + c^2p^2 = m^2c^4 \tag{16}$$

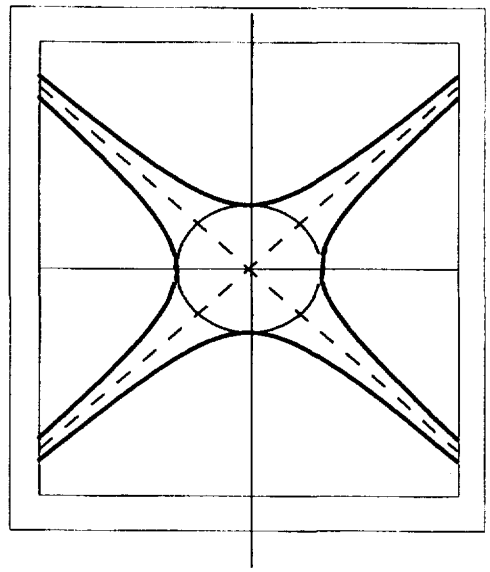
The speed of light has been explicitly represented as c to show similarity to the formulas of Special Relativity. Note that the momentum and energy analogs in Euclidean hyperspace do not go unbounded at the point v=c. Moreover, E and P remain real and bounded as $v \to \infty$, specifically, $P \to mc$, and $E \to 0$. No hypothesis of imaginary rest mass (as in tachyon theory [8]) is required for P and E to remain real as $v \to \infty$.

At this point the reader may ask what is the graph of equation (16) ? For the purpose of easy comparisons and graphing ease, once again,we set c equal to one. Equation (16) becomes:

$$E^2 + p^2 = m^2 \tag{17}$$

which is the equation of a circle. Where is this circle found?

Graphs of :	$E^2 - p^2 = m^2$	Minkowski space-time version (massive particle)	(18)
	$E^2 - p^2 = -m^2$ ($m \rightarrow im$)	Version of above relation for Tachyons	(19)
	$E^2 + p^2 = m^2$	Euclidean Representation of space-time ("Hyperspace")	(20)
	$E^2 - p^2 = 0$	Minkowski space-time version (for photons)	(21)



The diagonal asymptotes (dotted lines [9]) are the graphs of the energy-momentum equation for photons, since for photons, the "rest mass" is zero, or m=0, leaving us with the equations of lines.

For photons,

$$E^2 - p^2 = 0 \tag{22}$$

Which implies,

$$E = +p \tag{23}$$

and,

$$E = -p\,. \tag{24}$$

The graph for E in the two equations intersects the origin at a 45 degree angle. There are four hyperbolas. The upper hyperbola represents particles of positive rest mass and positive relativistic energy moving at subluminal speeds. The hyperbola to the right represents tachyons of either positive or negative relativistic energy , whereas the lower hyperbola (on the bottom) represents particles of negative relativistic energy at subluminal speeds. The hyperbola to the left represents tachyons with either positive or negative relativistic energy. The tachyons are a class of particles that must always move at a speed greater than the speed of light in order to exist and have imaginary rest mass. However, at the center of the graph is the circle which is the plot of equation 17. It touches all of the E vs. P hyperbolas at exactly one point. Imagine sub-atomic particles moving in Euclidean "hyperspace". Such particles will have strictly bounded relativistic momentum and energy for each value of proper mass m. (rest mass) However, particle speed is free to assume values anywhere within the interval $[0, \infty)$.

The particle speed can exist anywhere within $[0, \infty)$ without creating infinities in the coordinate transformations, relativistic momentum or relativistic energy. Moreover, when $v > c$, momentum and energy remain real valued. There is, however, a notable exception to the rule, namely the case of photons.

Note the interesting property of Euclidean four space when $m=0$. This leads to the solutions for E given by:

$$E = -i p \tag{17}$$

and

$$E = +i p . \tag{18}$$

Photons (particles with zero rest mass) would have imaginary total relativistic energy in Euclidean four-space.

6. FINAL REMARKS

More study is needed in "superluminal physics" to understand it at a level deeper level. Profound questions await an answer. Can we re-formulate the theories of electromagnetism and gravitation to be consistent with the Euclidean representation of space-time? Would major paradoxes remain or could they be eliminated? What physical principle would have to be operable in nature so that particles could make transitions into and out-of hyperspace? The following hypothesis is put forth by the author:

It is believed that there exists a "unified theory" of superluminal physics in flat space-time which can contain the theory of Tachyons, complex speed [10], and Euclidean four-space. But what is it?

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