A MIXED-MODE I/II FRACTURE CRITERION AND ITS APPLICATION IN CRACK GROWTH PREDICTIONS

Michael A. Sutton, Xiaomin Deng, and Fashang Ma
Department of Mechanical Engineering, University of South Carolina
Columbia, SC 29208, USA
Telephone: (803) 777-7158
E-Mail: sutton@engr.sc.edu

5/8 -39
037254

James S. Newman, Jr.
NASA Langley Research Center, Mechanics of Materials Branch
Hampton, VA 23681, USA

ABSTRACT

A crack tip opening displacement (CTOD)-based, mixed mode fracture criterion is developed for predicting the onset and direction of crack growth. The criterion postulates that crack growth occurs in either the Mode I or Mode II direction, depending on whether the maximum in either the opening or the shear component of CTOD, measured at a specified distance behind the crack tip, attains a critical value.

For crack growth direction prediction, the proposed CTOD criterion is shown to be equivalent to seven commonly used crack growth criteria under linearly elastic and asymptotic conditions. Under elastic-plastic conditions the CTOD criterion's prediction of the dependence of the crack growth direction on the crack-up mode mixity is in excellent agreement with the Arcan test results. Furthermore, the CTOD criterion correctly predicts the existence of a crack growth transition from mode I to mode II as the mode mixity approaches the mode II loading condition.

The proposed CTOD criterion has been implemented in finite element crack growth simulation codes Z1P2DL and FRANC2DL to predict the crack growth paths in (a) a modified Arcan test specimen and fixture made of AL 2024-T34 and (b) a double cantilever beam (DCB) specimen made of AL 7050. A series of crack growth simulations have been carried out for the crack growth tests in the Arcan and DCB specimens and the results further demonstrate the applicability of the mixed mode CTOD fracture criterion crack growth predictions and residual strength analyses for airframe materials.

1. INTRODUCTION

Among the various fracture parameters that have been proposed over the years, crack tip opening displacement (CTOD) has been shown to have potential in quantifying crack tip deformations during stable crack growth. Computational studies of stable crack growth under Mode I loading were performed by Newman et al.[1-3] to assess the viability of a CTOA-based fracture criterion for numerical simulation. In addition, Dawicke and Sutton[4] conducted a series of tests to obtain the critical CTOA value for an aluminum alloy (2024 T#). They found that the measured CTOA approached a constant value after an initial amount of crack growth approximately equal to the specimen thickness. Dawicke et al.[5] then used the CTOA criterion in two-dimensional finite element analyses and successfully predicted the crack-growth behavior of these test specimens.

Previous studies of CTOD-based fracture criterion were confined to Mode I crack extension along a fixed direction which closely approximated the initial crack line, which do not answer the important questions, "In what direction and at what loading will a stationary crack or a stable growing crack propagate under mixed mode loading?" Over the years, a variety of fracture criteria have been proposed to answer these questions, including among others the maximum circumferentical stress criterion, the maximum energy release rate criterion, the stationary strain energy density criterion, and the $K_{II} = 0$ criteria. In many cases, the theories proposed have adequately described the direction of crack growth for Mode I-type dominated fracture under mixed mode loading. However, the recent tests on Arcan specimens conducted by Amstutz et al.[6, 7] have shown that there is a sharp transition of crack growth behavior from predominantly Mode I type to Mode II type. In mode II, the crack grows under conditions that are locally shear-type in the crack tip region with the crack tip opening displacement (CTOD) dominated by the shear component parallel to the crack line.

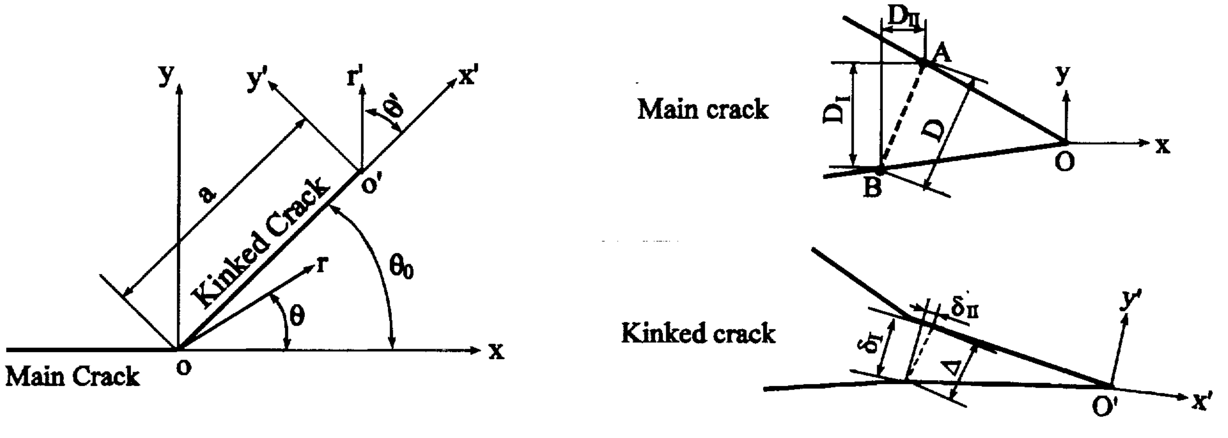
Preliminary studies[8] have shown that the CTOD-based fracture criterion can predict the load-crack extension response when crack propagation is forced to follow the experimentally observed crack growth paths. In order to develop a CTOD based fracture criterion for general mixed mode loading conditions, the authors[9] have studied the fundamental basis for the CTOD fracture criterion under mixed mode loading through analysis of *initial* crack kinking along arbitrary directions. The work has shown that, along the *initial* crack growth direction, in-plane crack tip deformations and stresses are either nearly Mode I or Model II type. Furthermore, either the opening or the shearing CTOD component of the kinked crack is a maximum during the *initial* increment of crack propagation. Using this data, *initial* crack growth is predicted to occur in either locally Mode I or locally Mode II direction, depending upon whether the opening or the shear component of CTOD measured at a specified distance behind the crack tip attains the critical value. Transition from Mode I-dominated initial crack growth to Mode II-dominated initial crack growth is predicted to occur when the shear component of CTOD reaches the critical CTOD value first.

In this work, a mixed mode, CTOD-based fracture criterion for the prediction of ***both initial kinking and stable crack propagation*** is outlined and verified through successful predictions of experimentally observed crack growth behavior in Arcan specimens[6, 7] made of AL 2024-T3 and in double-cantilever beam (DCB) specimens[10] made of AL 7050.

2. RATIONALE FOR THE CTOD-BASED MIXED MODE FRACTURE CRITERION

Figures 1a and 1b provide a graphical description of the CTOD components for the main crack and for the kinked crack. The initial idea behind the proposed CTOD criterion is that crack growth occurs along the direction where CTOD for the kinked crack (defined as Δ, at a fixed distance behind the current crack tip) is a maximum. Under linearly elastic and asymptotic conditions, several important conclusions have been obtained. For crack kinking along the mode I direction (say along $\theta_0 = \theta_c^I$), quantities $\Delta, \delta_I, \sigma_\theta, k_1$, and G take their respective maximum values, while quantities $\delta_{II}, \sigma_{r\theta}$ and k_2 become zero, where k_1 and k_2 are the stress intensity factors for the kinked crack, σ_θ and $\sigma_{r\theta}$ are the circumferential normal and shear stresses, and G is the strain energy release rate. Thus, using the maximum in δ_I or Δ as a fracture parameter for the prediction of crack growth is equivalent to using the maxima in commonly accepted fracture criteria (e.g, σ_θ, G, k_I). Similarly, for crack kinking along the mode II direction (say along $\theta_0 = \theta_c^{II}$), the quantities $\delta_{II}, \sigma_{r\theta}$ and k_2 are maxima and δ_I, σ_θ and k_1 approach zero. In this case, using the maximum in δ_{II} as a fracture parameter for prediction of the direction of crack growth is equivalent to using the maxima in commonly accepted fracture criteria (e.g,

$\sigma_{r\theta}, K_{II}$). These results suggest that the CTOD components, δ_I and δ_{II}, are viable fracture parameters for predicting crack growth under mixed mode loading conditions. Conceptually, the onset of crack growth can be assumed to occur in either the direction θ_I^c or θ_{II}^c, depending on whether $(\delta_I)_{\max} = \delta_I^c$ or $(\delta_{II})_{\max} = \delta_{II}^c$ is satisfied first, where δ_I^c and δ_{II}^c are the critical values for δ_I and δ_{II}, respectively, to be determined from Mode I and Mode II fracture experiments, respectively. It can be shown that $(\delta_I)_{\max}$ (if $\theta_0 = \theta_I^c$) and $(\delta_{II})_{\max}$ (if $\theta_0 = \theta_{II}^c$) are explicit functions of the crack tip opening displacement for the main crack, D, under linearly elastic and first-order asymptotic conditions. Thus, the attainment of a critical value for $(\delta_I)_{\max}$, $(\delta_{II})_{\max}$ corresponds to a critical value for D. Hence, it is possible to use only the CTOD quantities at the main crack tip to assess the direction and onset of crack propagation.



(a) (b)

Figure 1 A geometrical representation of the relation between a main crack and a short kinked crack: (a) coordinate systems for the main and kinked cracks, and (b) definitions of crack tip opening displacements for the main and kinked cracks.

For elastic-plastic crack growth, the finite element code ZIP2DL developed by Deng and Newman[11, 12] has been used with the mixed mode CTOD criterion and a small-scale yielding (SSY) model to predict the dependence of the crack growth direction in AL 20204-T3 on the crack-tip local mode mixity defined by $\alpha = \arctan(D_{II}/D_I)$. This is achieved by simulating initial crack growth in twelve possible directions in the interval of $-90 < \theta_0 < 90^0$. A finite element mesh with focused elements around the main crack tip is developed to allow crack growth along each of the twelve directions ahead of the main crack tip. The applied load is increased gradually until the computed CTOD value at a specified distance behind the initial crack tip attains a critical value of $D = D_c$ (D_c is obtained in the Arcan tests by Amstutz et al.[6, 7]). The crack is then extended along the radial line by one element length by allowing the appropriate node pair to separate. This provides the mechanism for determining the critical kinking direction $\theta_0 = \theta_c$ that has the maximum in δ_I or δ_{II}.

The SSY simulations were performed for three values of the T-stress (normalized and represented by B=-1, 0, 1). Figure 2 presents the predicted crack kinking direction (assuming $\delta_I^c = \delta_{II}^c$) as a function of the local mode mixity, α. The experimental data are for the Arcan specimens. It can be seen that the effect of the T-stress on the predicted initial crack direction is quite small, suggesting that the CTOD criterion is not sensitive to specimen size and geometry based on the SSY analyses. Also, the CTOD

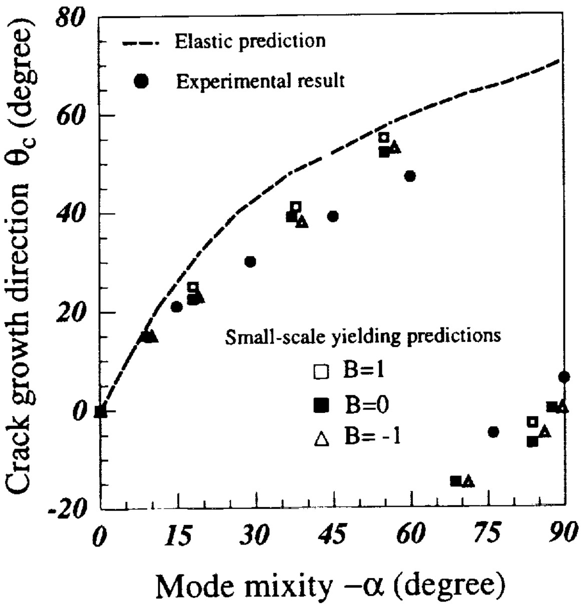


Figure 2 The crack growth direction θ_c as a function of the local mode mixity α, predicted under both linearly elastic and modified small-scale yielding conditions, and compared with the Arcan test results.

criterion predicts a transition of initial crack kinking mode from Mode I to Mode II as the mode mixity varies from Mode I values to Mode II values. This prediction is consistent with the recent experimental evidence of Amstutz et al for a 2024-T3 aluminum specimen, where a transition in fracture mode was observed during initial crack kinking for $\alpha \cong 70^0$. Finally, it is noted that the linear elastic prediction has the correct trends for predicting the initial kinking direction during Mode I crack growth, but it is not capable of predicting the transition to Mode II type crack growth.

The crack growth direction, θ_c, on the mode mixity, α, can be expressed in the following empirical expressions, which are to be used in the mixed mode CTOD criterion for the direct determination of crack growth directions under arbitrary mixed-mode loading conditions:

$$\left.\begin{aligned}\theta_c^I &= a_1 \arctan(b_1\alpha) && \text{for } \alpha<\alpha_c\\ \theta_c^{II} &= a_2\cos(b_2\alpha)\frac{\alpha}{|\alpha|} && \text{for } \alpha\ge\alpha_c\end{aligned}\right\} \quad (1)$$

where α_c is the critical local mode mixity for the transition of Mode I to Mode II type fracture and a_1, b_1, a_2 and b_2 are the fitting parameters. For 2024-T3 aluminum, the fitting parameters have been obtained from the Arcan tests and they are

$$\alpha_c = 70^0, a_1 = -36.5, b_1 = 2.2, a_2 = 57.3, b_2 = 1.0 \quad (2)$$

3. NUMERICAL SIMULATION OF CRACK GROWTH USING THE CTOD CRITERION

The mixed mode CTOD fracture criterion has been implemented in the finite element code FRANC2DL[13, 14], which uses a mapping algorithm to re-mesh the crack-tip region and to transfer the state variables. Upon satisfaction of the fracture criterion, nodal release and load relaxation techniques are employed to advance the crack. In this manner, crack propagation can be simulated in arbitrary directions, as specified by the CTOD fracture criterion.

The CTOD criterion as implemented in FRANC2DL has been used to predict the crack growth paths and other features of crack growth behavior observed in the Arcan tests[6, 7] and in the DCB tests[10]. Six-node triangular elements are used for both the Arcan and DCB specimens. The near-crack-tip element size is 0.5 mm. When the CTOD value at the second node behind the crack tip (which is at a distance of 1 mm) attains the critical value D_c, the fracture criterion predicts the crack growth direction θ_c, and the crack is extended by two elements (1mm). Re-meshing is performed and equilibrium re-established after crack advance and the process is repeated throughout the crack propagation process until the desired crack propagation length is achieved.

3.1 Predictions of Crack Growth Behavior of the Arcan Specimens

Test data for the Arcan tests include the load-crack extension curves and the crack growth paths. The critical CTOD value has been measured and the average, D_c=0.089 mm, is used in this study. Actual material properties for AL 2024-T3 (including the strain hardening curve) are used as input to the finite element models. The only input related to the crack growth behavior is the critical CTOD value. The loading fixture and specimen geometries have been discussed by Amstutz et al.[6, 7]

Simulations for the Arcan tests have been carried out for the full spectrum of mixed-mode loading cases. Due to page limitation, comparisons of the measured and predicted crack growth paths for the Arcan specimens are presented here only four loading cases (denoted by the loading angle Φ, where $\Phi=0^0$ is for mode I and $\Phi=90^0$ is for mode II), as shown in Fig. 3. The comparison demonstrates that the crack growth path predictions using the CTOD-based mixed mode fracture criterion are in good agreement with the experimental observations throughout the crack propagation process. Predictions of the load-crack extension curves (not shown here) are also in excellent agreement with test data.

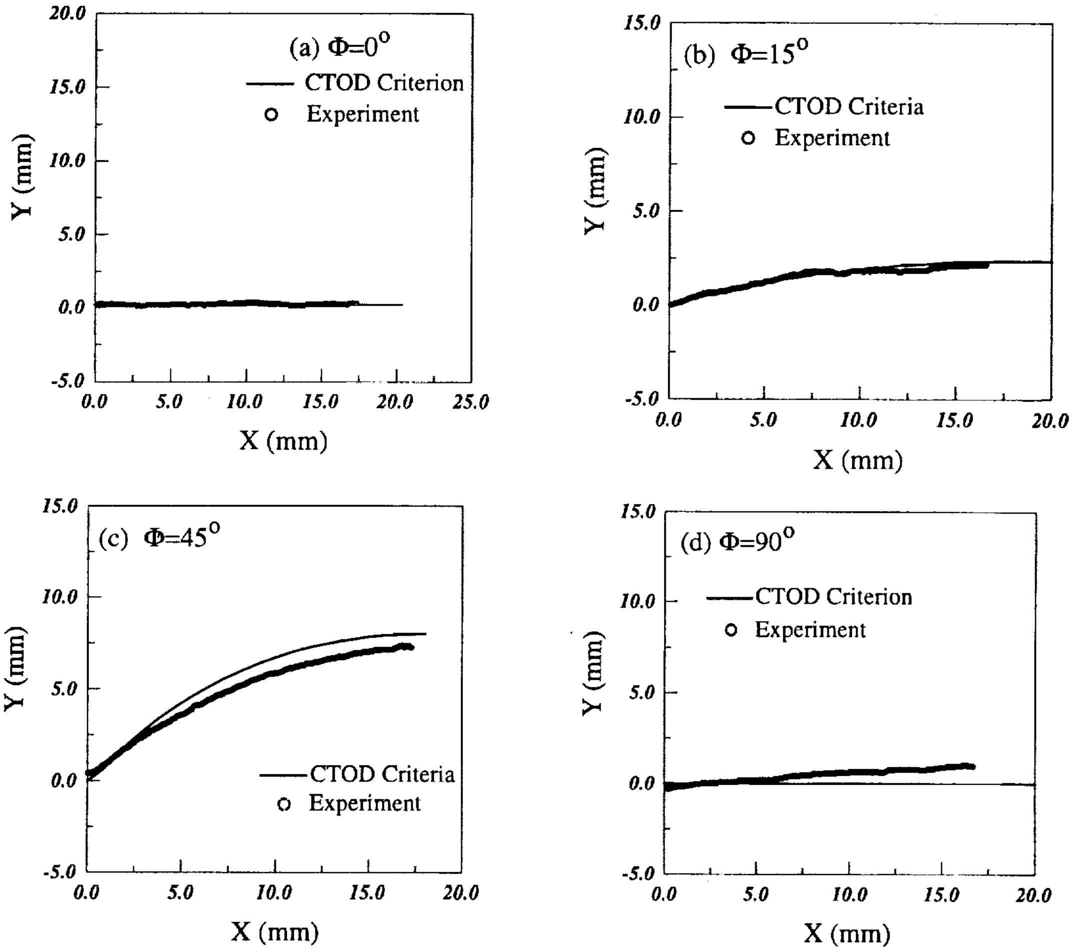


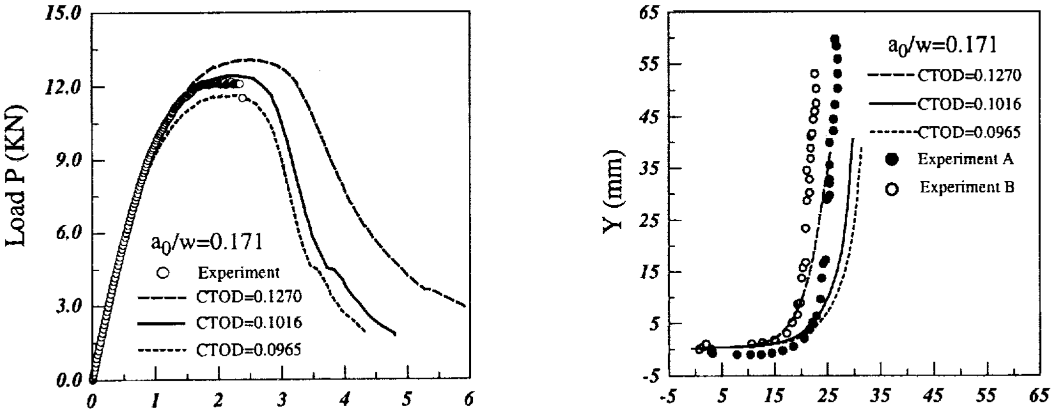
Figure 3 Predicted and measured crack growth paths for the Arcan tests for (a) $\Phi=0^0$, (b) $\Phi=15^0$, (c) $\Phi=45^0$, (d) $\Phi=90^0$.

3.2 Predictions of Crack Growth Behavior of the DCB Specimens

Mode I crack growth in DCB specimens is known to have an instability problem, in that the crack growth path is rarely along the expected straight line in the original crack direction. Depending on the particular built-in asymmetry in either the specimen geometry or the loading application, the crack growth path in a DCB test usually will curve away from the straight line either from one side or from the other. Without knowing the details of the built-in asymmetry, it is impossible to predict whether the crack will deviate from the straight line one way or another. In order for the CTOD criterion to predict the curved crack growth direction, details of the actual crack-surface geometry of the DCB specimens are modeled in the finite element meshes. These details provide the built-in geometrical asymmetry.

The database for the DCB specimens do not include the critical CTOD value nor the load-crack extension curves. As such, the measured load-load point displacement curve and the crack growth path for a particular DCB specimen are used in this study to estimate the critical CTOD value, D_c, for the material. Then the estimated critical CTOD value is used for all later predictions for this and other specimens made of the same material. The procedure used to estimate D_c is as follows.

Noting that for small α ($< 10^0$), the $\theta_c \sim \alpha$ relationship in Fig. 2 predicted by elastic and elastic-plastic analyses are nearly identical and that the elastic results are independent of material properties, it is expected that the $\theta_c \sim \alpha$ relation will depend only weakly on material properties for naturally growing cracks when the local mode mixity is dominantly mode I ($\alpha < 10^0$). Using the $\theta_c \sim \alpha$ relation for AL 2024-T3 ($a_1 = -36.5, b_1 = 2.2$) and assuming a value for D_c, the crack growth process in the DCB specimen with $a/w = 0.171$ is simulated, where a is the initial crack length and w is the width of the specimen in the crack direction. By comparing the measured and predicted load-load point displacement curves and the crack growth paths, a critical D_c value with the best overall fit can be chosen. As shown in Fig. 4a for the load-load point displacement (deflection) curve, and in Fig. 4b for the crack growth path, a best overall fit is found when $D_c \cong 0.1016$ mm. It is noted that experimental measurements A and B in Fig. 4b refer to the crack growth paths measured from the two surfaces of the same DCB specimen. It is



(a) (b)

Figure 4 Predictions compared with tests for a DCB specimen with $a/w = 0.171$: (a) load-load-line displacement curve (b) crack growth path.

also observed from Fig. 4b that the three predicted crack growth paths curve up on the same side of the original crack line and all can basically follow the measured crack growth trend.

To examine whether the chosen D_c value of 0.1016 mm can be used to produce reasonable predictions for other DCB specimens made of the same material, it is now used in finite element simulations of the crack growth tests in DCB specimens with $a/w = 0.404$ and 0.545. The results are presented in Figs. 5 and 6, where comparisons of the measured and predicted crack growth paths and load-load point displacement curves are shown, respectively. These comparisons shows that the CTOD criterion does a good job in predicting the rapid changes in the observed crack growth direction, with only slightly less curvature than what was measured, and that the predicted load-load point displacement curves are in excellent agreement with the measured response.

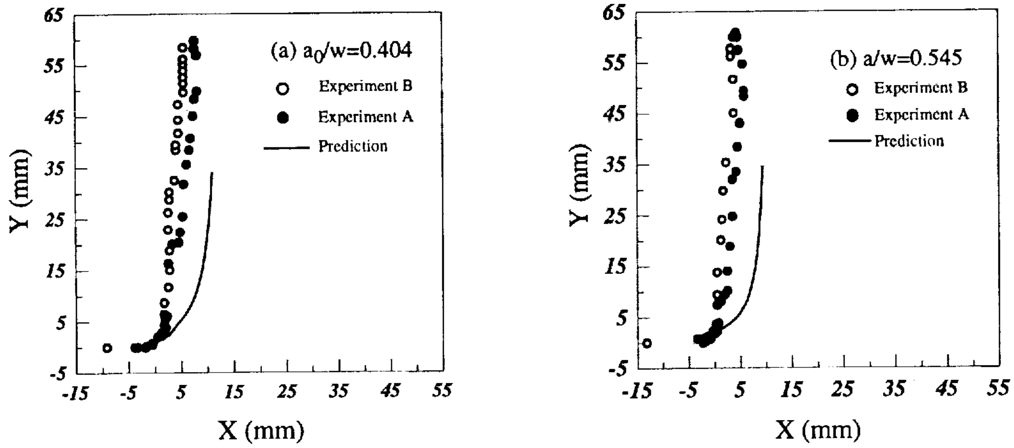


Figure 5 Predicted and measured crack growth paths for DCB specimens with (a) $a/w = 0.404$ and (b) $a/w = 0.545$, where the predictions are obtained using a critical CTOD of $0.1016mm$.

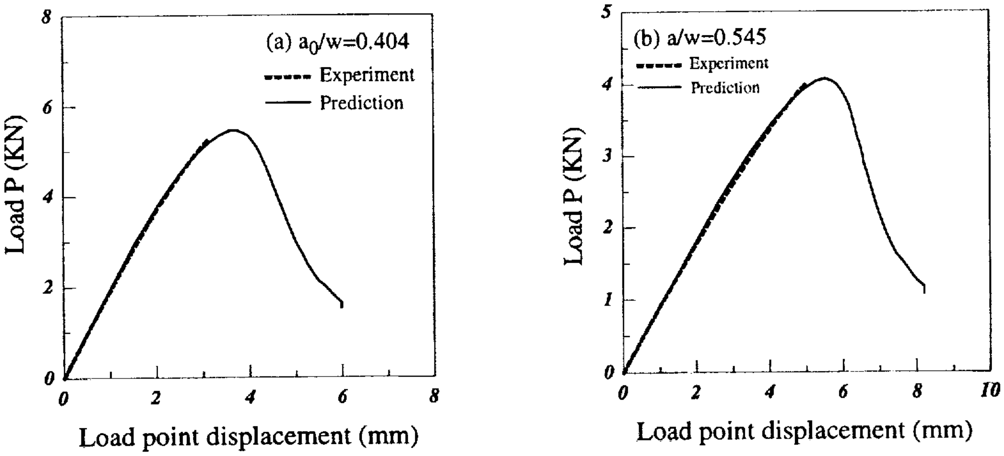


Figure 6 Predicted and measured load-load-line displacement curves for DCB specimens with (a) $a/w = 0.404$ and (b) $a/w = 0.545$, where the predictions are obtained using a critical CTOD of $0.1016mm$.

4. SUMMARY AND CONCLUSIONS

First, theoretical analyses and finite element simulations for crack growth in Arcan specimens indicate that crack growth occurs under either locally Mode I or locally Mode II conditions, which agree with experimental observations in the Arcan specimens. The mode of fracture which occurs depends on whether the tensile or shear component of CTOD first attains a critical value.

Second, the proposed mixed mode CTOD fracture criterion can adequately capture the mixed mode crack growth behavior (including the crack growth path, the load-crack extension curve and the load-load line displacement response) in ductile airframe materials. Results clearly show that predictions for both the Arcan-specimen and DCB are in good agreement with experimental data.

Third, since the fracture parameter is expected to be a material property, the CTOD criterion developed for 2024-T3 aluminum using the Arcan specimen data can be used for predicting crack growth for other specimen geometry. In fact, we have performed a range of crack growth simulations for both middle crack tension specimens and compact tension specimens machined from 2.3-mm thick, 2024-T3 aluminum. Results similar to those presented in this paper have been obtained for both specimens, with propagation occurring under locally Mode I conditions throughout the crack growth process.

Fourth, for the DCB specimen, due to the lack of a measured critical CTOD, the measured load-load point displacement curve and the crack growth path for a particular DCB specimen was used to estimate the critical CTOD value, D_c. By matching the measured and predicted load-load point displacement curves and also considering the prediction for the crack growth path, an estimate for D_c was determined and was used for later predictions for this and other DCB specimens made of the same material. The estimated critical value of CTOD for Al 7050 is approximately on the order of that obtained for Al-2024-T3, with both specimens nominally in the LT orientation (crack initially perpendicular to the rolling direction).

Finally, with regard to the crack growth paths shown in Figs. 4 and 5 for the DCB specimens, recent experimental observations[10] suggest that the fracture behavior of Al 7050 is highly anisotropic with $(K_{IC}^{TL}/ K_{IC}^{LT}) \approx 0.60$ for Al-7050, which would imply that $(\delta_{IC}^{TL}/ \delta_{IC}^{LT}) \approx 0.60$. Since our simulations used a constant CTOD value for all angles, the predicted crack paths would have less curvature than the actual crack growth paths, which is precisely what is shown in Figs. 4 and 5. It appears that the CTOD-based fracture criterion can be extended to include the effects of anisotropic material fracture behavior by including a functional form for critical CTOD as a function of direction. The functional form can be determined experimentally through a series of mixed mode tests. The specimens to be tested would have flaws at an initial angle to the material directions, and the effects of material direction on critical CTOD would be measured during crack growth.

ACKNOWLEDGMENTS

The authors wish to thank (a) Mr. Rick Pettit from Boeing, Inc. for both the use of his experimental data from several DCB crack growth tests and his timely discussions of key issues in the testing process, (b) Drs. Daniel Swenson and Mark James from the Kansas State University for their assistance in modifying and using FRANC2DL in our fracture simulations, and (c) Dr. David S. Dawicke at NASA Langley Research Center for his assistance in both fracture tests and fracture simulations. In addition, the financial support of NASA headquarters through an NASA EPSCoR grant is gratefully acknowledged.

REFERENCES

1. Newman, J. C., Jr., 1977. Finite Element Analysis of Crack Growth under Monotonic and Cyclic Loading. *ASTM STP* 637, 56-80.
2. Newman, J.C., Jr., 1984. An Elastic-Plastic Finite Element Analysis of Crack Initiation, Stable Crack Growth, and Instability. *Fracture Mechanics, ASTM STP* 833, 93-117.
3. Newman, J.C., Jr., Dawicke, D. S., and Bigelow, C. A., 1992. Finite Element Analyses and Fracture Simulation in Thin Sheet Aluminum Alloy. *NASA TM*-107662, NASA Langley Research Center, Hampton, Virginia.
4. Dawicke, D. S., Sutton, M. A., 1994. CTOA and Crack-Tunneling Measurements in Thin 2024-T3 Aluminum Alloy. *Experimental Mechanics* 34, 357-368.
5. Dawicke, D. S., Sutton, M. A., Newman, J. C., Jr., Bigelow, C. A., 1995. Measurement and Analysis of Critical CTOA for Aluminum Alloy Sheet. *Fracture Mechanics, ASTM STP* 1220, 358-379.
6. Amstutz, B. E., Sutton, M. A., Dawicke, D. S. and Newman, J. C., Jr., 1995. An Experimental Study of CTOD for Mode I/ II Stable Crack Growth in Thin 2024-T3 Aluminum Specimens. *Fracture Mechanics, ASTM STP* 1256, 256-271.
7. Amstutz, B. E., Sutton, M. A., Dawicke, D. S. and Boone, M. L., 1995. Effects of Mixed Mode I/II Loading and Grain Orientation on Crack Initiation and Stable Tearing in 2024-T3 aluminum. *Fatigue and Fracture, ASTM STP* 1296, 105-125.
8. Sutton, M. A., Zhao, W., Deng, X., Dawicke, D. S., and Newman, J. C., Jr., 1997. Numerical Investigations Into the Viability of CTOD as a Fracture Parameter for Mixed-Mode I/II Tearing of Thin Aluminum Sheets. *Proceedings of the FAA-NASA Symposium on Continued Airworthiness of Aircraft Structures (Atlanta, Georgia, August 28-30, 1996), DOT/FAA/AR*-97/2, Vol. II, 461-472.
9. Ma, F., Deng, X., Sutton, M. A. and Newman, J.C., Jr., 1998. A CTOD Based Mixed Mode Fracture Criterion. *Mixed Mode Crack Behavior, ASTM STP* 1359 (in press).
10. Pettit, R., 1998, Private Communication with M. A. Sutton.
11. Deng, X., Newman, J. C., Jr., 1997. Implementation and Application of a Large-Rotation Finite Element Formulation in NASA Code ZIP2DL. *FAA-NASA Symposium on Continued Airworthiness of Aircraft Structures (Atlanta, Georgia, August 28-30, 1996), DOT/FAA/AR*-97/2, Vol. II, 377-390.
12. Deng, X., Newman, J. C., Jr. 1997. ZIP2DL – An Elastic-Plastic, Large-Rotation Finite-Element Stress Analysis and Crack-Growth Simulation Program. *NASA Technical Memorandum* 110332, NASA Langley Research Center, Hampton, Virginia.
13. Swenson, D., James, J., 1997. FRANC2D/L: A Crack Propagation Simulator for Plane Layered Structures. User's Guide, Version 1.4.
14. James, M. A., Swenson, D., 1998. A Software Framework for Two-Dimensional Mixed Mode I/II Elastic-Plastic Fracture. *Mixed Mode Crack Behavior, ASTM STP* 1359 (in press).