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EVAULATION OF LRC SPECTRAL ANALYSES OF SIMULATED GUST VELOCITY DATA

by

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Prepared For

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SUMMARY

The NASA Langley Research Center has been computing power spectra of simulated atmospheric turbulence data by various techniques, in preparation for the B-57 gust velocity data analysis program. The results of these studies have confirmed earlier suspicions that conventional first-difference pre-whitening of gust velocity data can produce serious distortions in the spectral densities at very low frequencies (below the frequency of the gust velocity spectral "knee"). The results also indicate that the frequency averaging procedures have certain merits over ensemble averaging procedures in the computation of spectra by direct Fourier transform operations. Finally, the results do not reveal any significant difference in the spectral estimates obtained using Hann versus Parzen smoothing procedures.

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I. INTRODUCTION

The NASA Langley Research Center (LRC) is proceeding with a program to collect and analyze atmospheric turbulence data using an instrumented B-57 aircraft. In preparation for this program, the LRC gust velocity data acquisition system and the proposed analysis techniques were evaluated by Bolt Beranek and Newman (BBN) in June 1973.[1/] These evaluations lead to several important conclusions, as follows :

a) Although pre-whitening has been traditionally used in the calculation of gust velocity spectra, there is reason to believe it may not be desirable from the viewpoint of accurately defining a "knee" in such spectra.

b) Assuming that gust velocity spectra are calculated using direct Fourier transform operations with an FFT algorithm, there is reason to believe that frequency averaging will provide better detail at the lowest frequencies than ensemble averaging.

c) The smoothing window used to supress leakage through the side-lobs of the analysis spectral window is not critical compared to other factors in the calculations, such as the pre-whitening and averaging techniques.

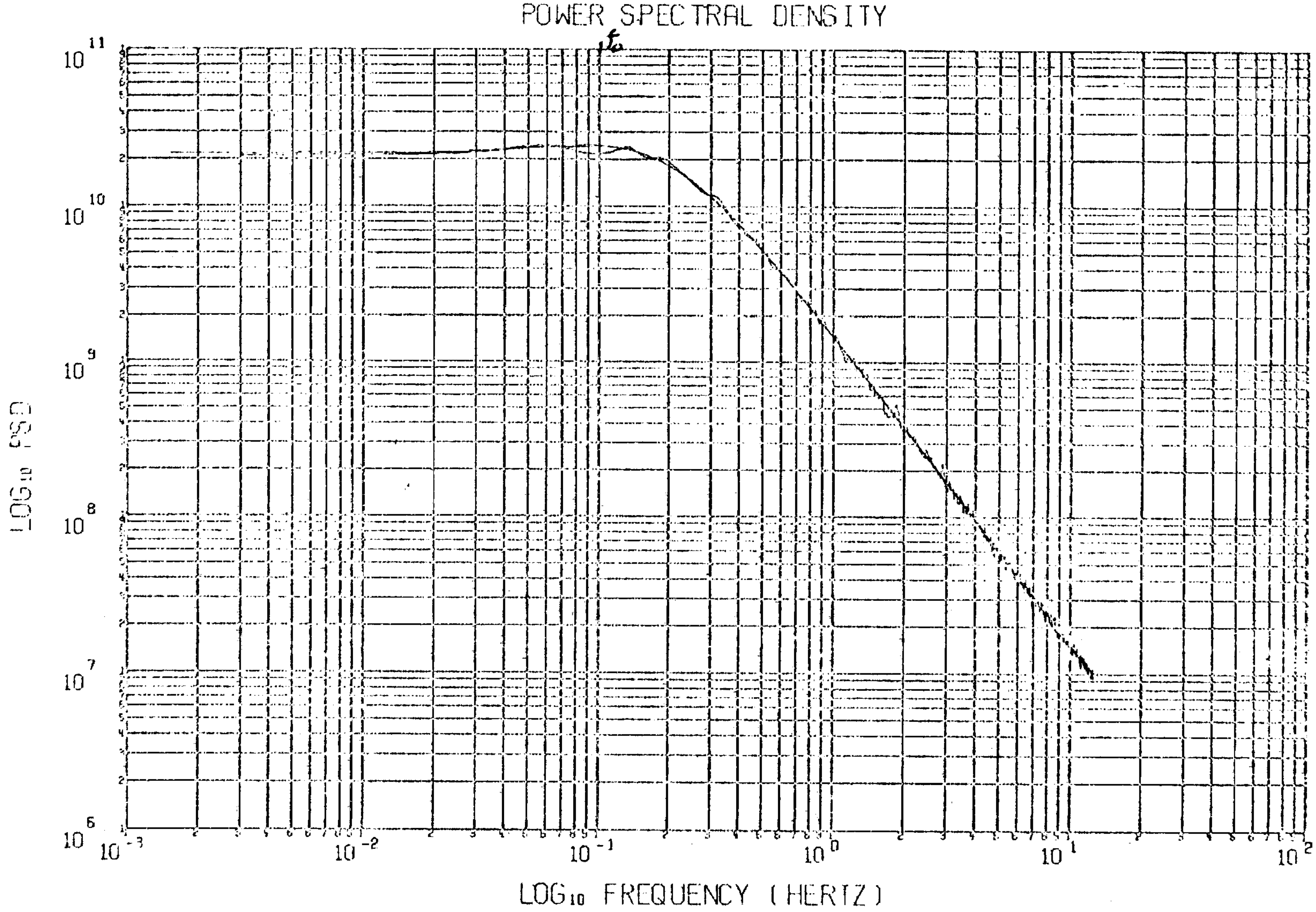
Based upon these conclusions, LRC performed a series of studies where a Von Karman spectrum was generated on a digital computer and analyzed by various techniques. The simulated Von Karman spectrum had a "knee" peaking at about 0.1 Hz, broadly representative of what might be expected in practice. The original signal was generated in two different ways, designated the "Otnes" signal and the "Keisler" signal. The actual signal generation techniques are of no direct importance, however, since they appear to produce equivalent results. All of the calculations were performed using a large number of degrees-of-freedom to suppress the influence of random errors on the spectral data.

2. PRE-WHITENING

When power spectra of typical gust velocity data are computed without special signal conditioning, there is no question that a bias error results which becomes increasingly severe as the resolution bandwidth of the analysis is increased. This fact is clearly illustrated in Figs. 1 through 4, which present spectra computed using resolution bandwidths varying from about $f_o/4$ to $4f_o$, where f_o is the frequency of the "knee" in the Von Karman spectrum. Note that the four spectra in these figures were computed by otherwise identical procedures (Blackman-Tukey technique with Parzen smoothing).

Referring to Fig. 1, the computed spectrum matches the actual spectrum with no significant error for the resolution of $B_e=f_o/4$. When the resolution bandwidth is doubled to $B_e=f_o/2$, a slight bias error appears, as seen in Fig. 2. Note that the error involves an over prediction at frequencies below the "knee" where the spectrum has a large positive 2nd derivative, and an under prediction at frequencies around the "knee" where the spectrum has a large negative second derivative. This is exactly as would be expected from theory.[2/] When the resolution bandwidth is increased further, this error is increased to a very significant magnitude, as illustrated in Figs. 3 and 4. Note that the nature of the bias error causes the "knee" to be smoothed and appear to be at a higher frequency.

If LRC is lucky enough to obtain sample records with sufficient length to permit an analysis with a resolution bandwidth of $B_e<f_o/4$, then the issue of pre-whitening does not arise since the spectral computations will not involve a significant bias error anyhow. It is quite possible, however, that LRC will be analyzing data with a resolution of perhaps $B_e>f_o$, due either to limited record lengths or the existance of a very low f_o.

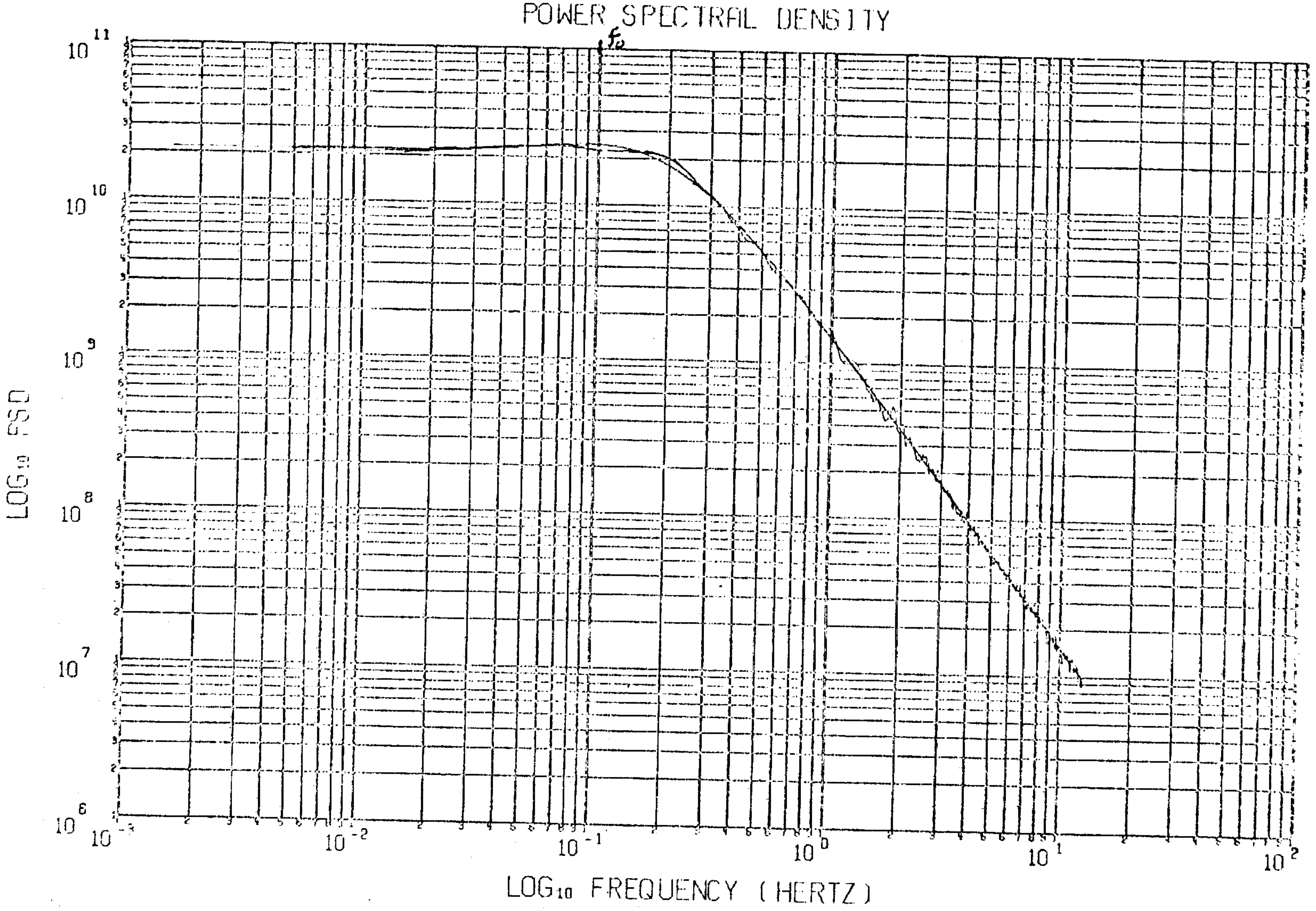


SERIAL NUMBER = 099099
DELTA T = .04000
DELTA F = 12207 x 10^{-6}
158710 INPUT DATA POINTS
1025 POWER POINTS PLOTTED

ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY
WITH PARZEN AUTOCORRELATION WINDOW OVER 1025LAGS
RESULTS ENSEMBLE AVERAGED OVER 10 TIME SEGMENTS
$B_e = 0.0244$ Hz. $\approx f_0/4$; $n \approx 300$ dof

FIGURE 1. SIMULATED SPECTRUM COMPUTED WITHOUT PRE-WHITENING ($B_e = 0.0244$ Hz)



SERIAL NUMBER = 09999

DELTA T = .04000

DELTA F = 24414 x 10^{-6}

63484 INPUT DATA POINTS

513 POWER POINTS PLOTTED

ALGORITHM:

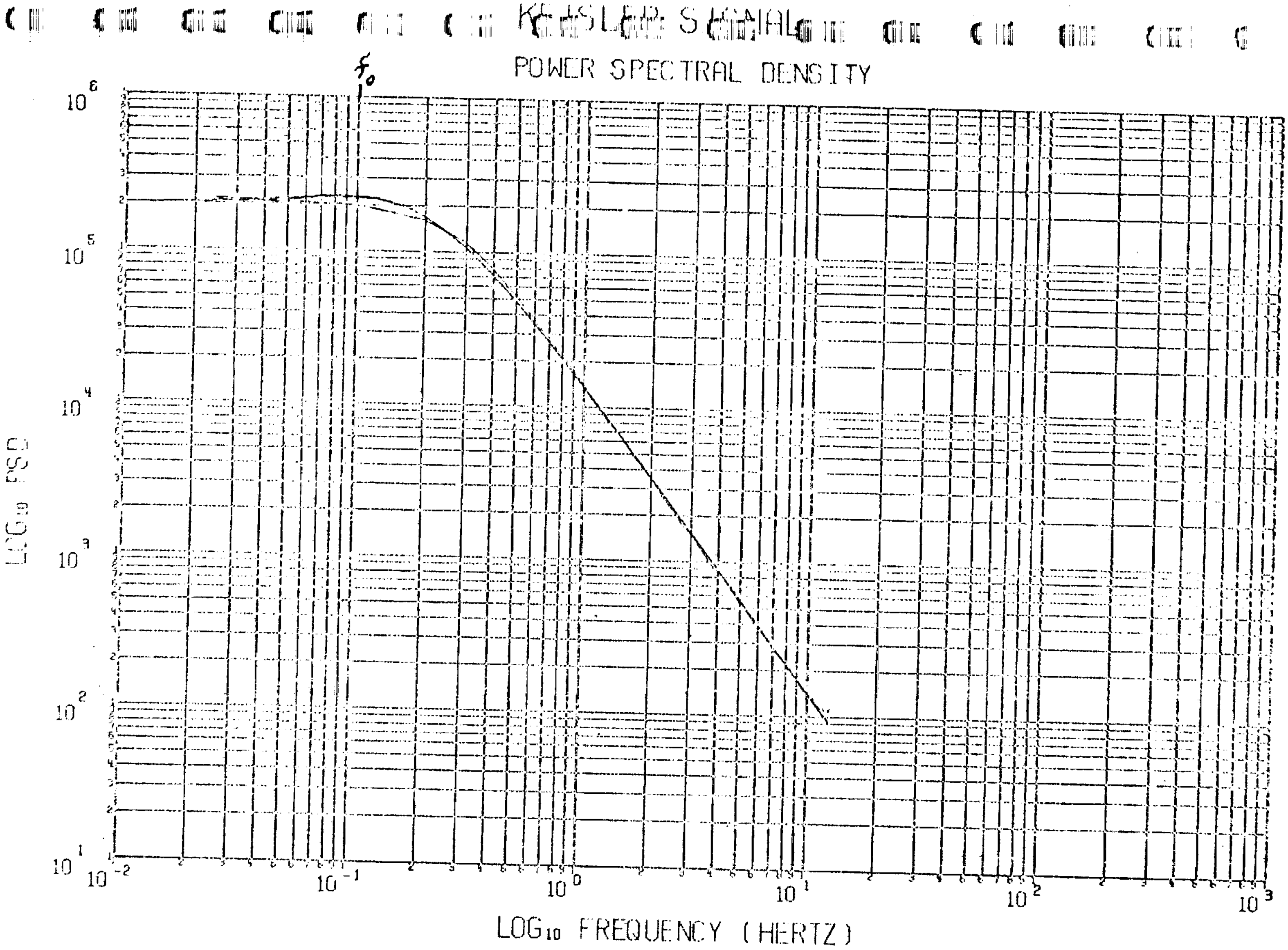
FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH PARZEN AUTOCORRELATION WINDOW OVER 513 LAGS

RESULTS ENSEMBLE AVERAGED OVER 4 TIME SEGMENTS

$B_e = 0.0488$ Hz. $\approx f_o/2$; $n \approx 250$ dof

FIGURE 2. SIMULATED SPECTRUM COMPUTED WITHOUT PRE-WHITENING ($B_e = 0.0488$ Hz)



SERIAL NUMBER = 099999

DELTA T = .04000

DELTA F = 97656 x 10^{-6}

15871 INPUT DATA POINTS

129 POWER POINTS PLOTTED

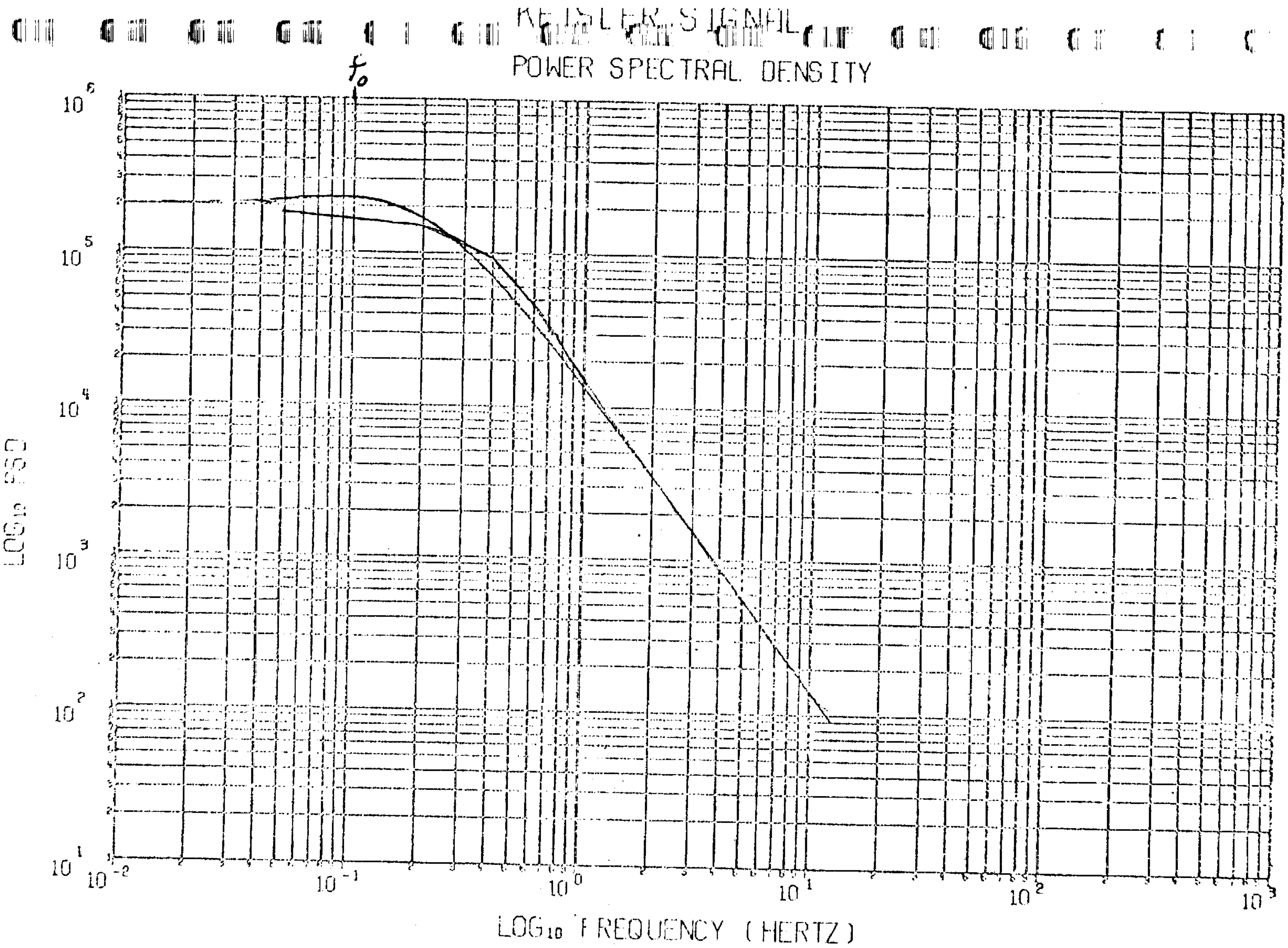
ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH PARZEN AUTOCORRELATION WINDOW OVER 129 LAGS.

$B_e = 0.195$ Hz. $\approx 2f_0$; $n \approx 250$ dof

FIGURE 3. SIMULATED SPECTRUM COMPUTED WITHOUT PRE-WHITENING ($B_e = 0.195$ Hz)



SERIAL NUMBER = 003999

DELTA T = .04000

DELTA F = 19531 x 10^{-5}

15571 INPUT DATA POINTS

65 POWER POINTS PLOTTED

ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH PARZEN AUTOCORRELATION WINDOW OVER 65 LAGS

$B_e = 0.390$ Hz. $\approx 4 f_o$; $n \approx 500$ d.o.f.

FIGURE 4. SIMULATED SPECTRUM COMPUTED WITHOUT PRE-WHITENING (B_e - 0.390 Hz).

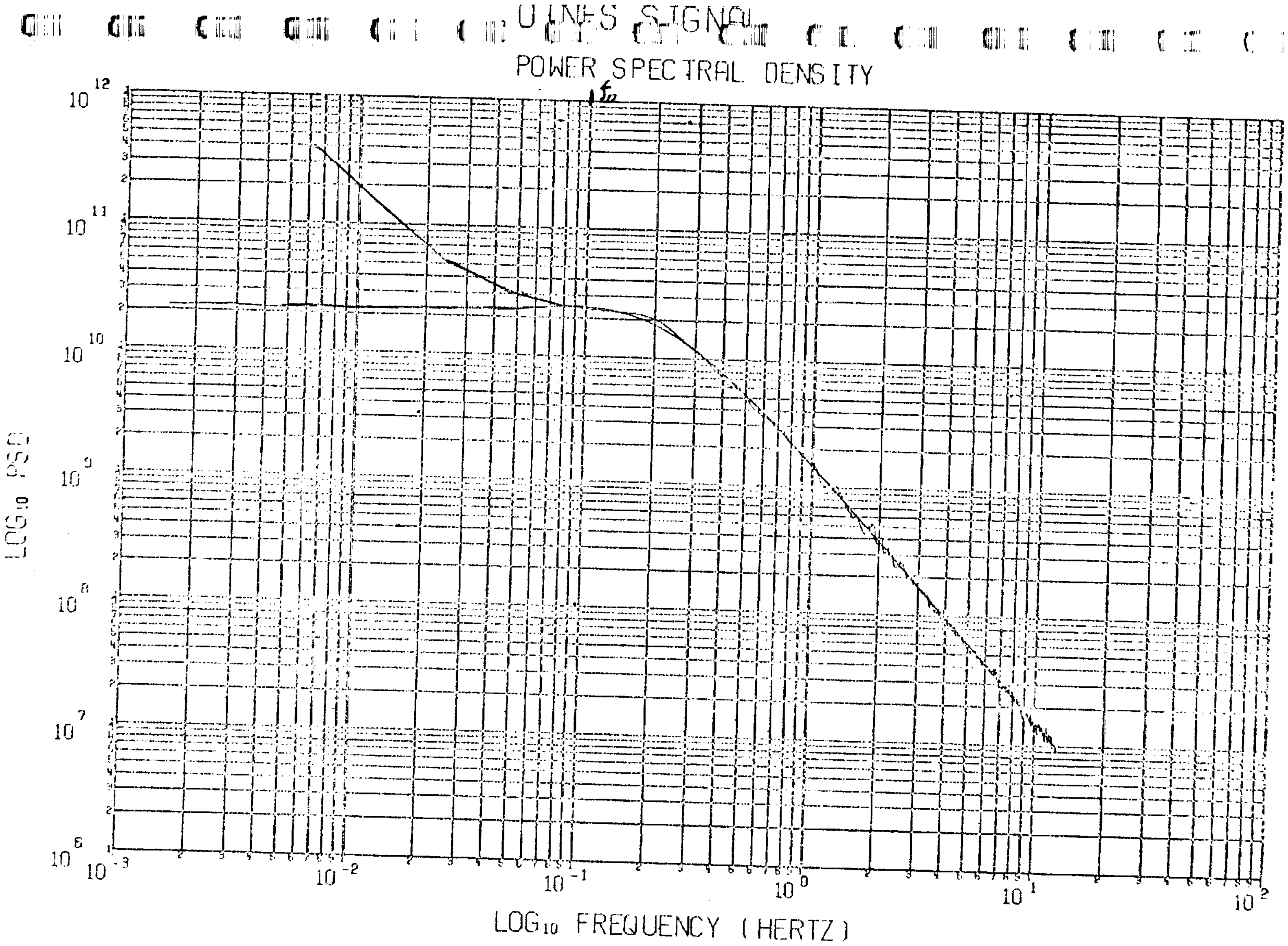
Hence, these poorly resolved cases are of great practical importance. These, of course, are the cases where pre-whitening is usually recommended.

The most common form of pre-whitening used for the spectral analysis of gust velocity data is a first-difference filter where the filtered data points y_i (i=1, 2, 3, ..., n) are given by

$$y_i = x_i - x_{i-1} \tag{1}$$

As noted in earlier evaluations[1/], there is no question that this operation will suppress bias errors in the frequency range above the spectral "knee", but there is reason to believe it will aggrevate the bias errors in the vicinity of the "knee". To investigate this possibility, the simulated Von Karman data were reanalyzed at the resolution bandwidths of $B_e = f_o/2$, $2f_o$, and $4f_o$ using first-difference pre-whitening, while holding all other factors constant. The results are presented in Figs. 5 through 7.

By comparing the results in Figs. 5 through 7 with the corresponding spectra in Figs. 2 through 4, it is clear that the pre-whitening does suppress bias errors above the "knee", but only at the expense of far more serious errors in the vicinity of the "knee". It should be mentioned that the spectral component at zero Hz has been plotted in Figs. 1 through 7 at $B_e/4$ to accomodate the logarithmic presentation. This tends to distort the spectral shapes, particularly for the pre-whitened data. Nevertheless, it is clear that the net effect of pre-whitening is to blur the "knee" in the spectrum. Similar results were obtained when the spectra were computed using the direct Fourier transform procedure, rather than the Blackman-Tukey procedure, and a Hann smoothing window, rather than the Parzen window.



SERIAL NUMBER = 099999

DELTA T = .04000

DELTA F = .24414 x 10^{-6}

63484 INPUT DATA POINTS

513 POWER POINTS PLOTTED

ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY

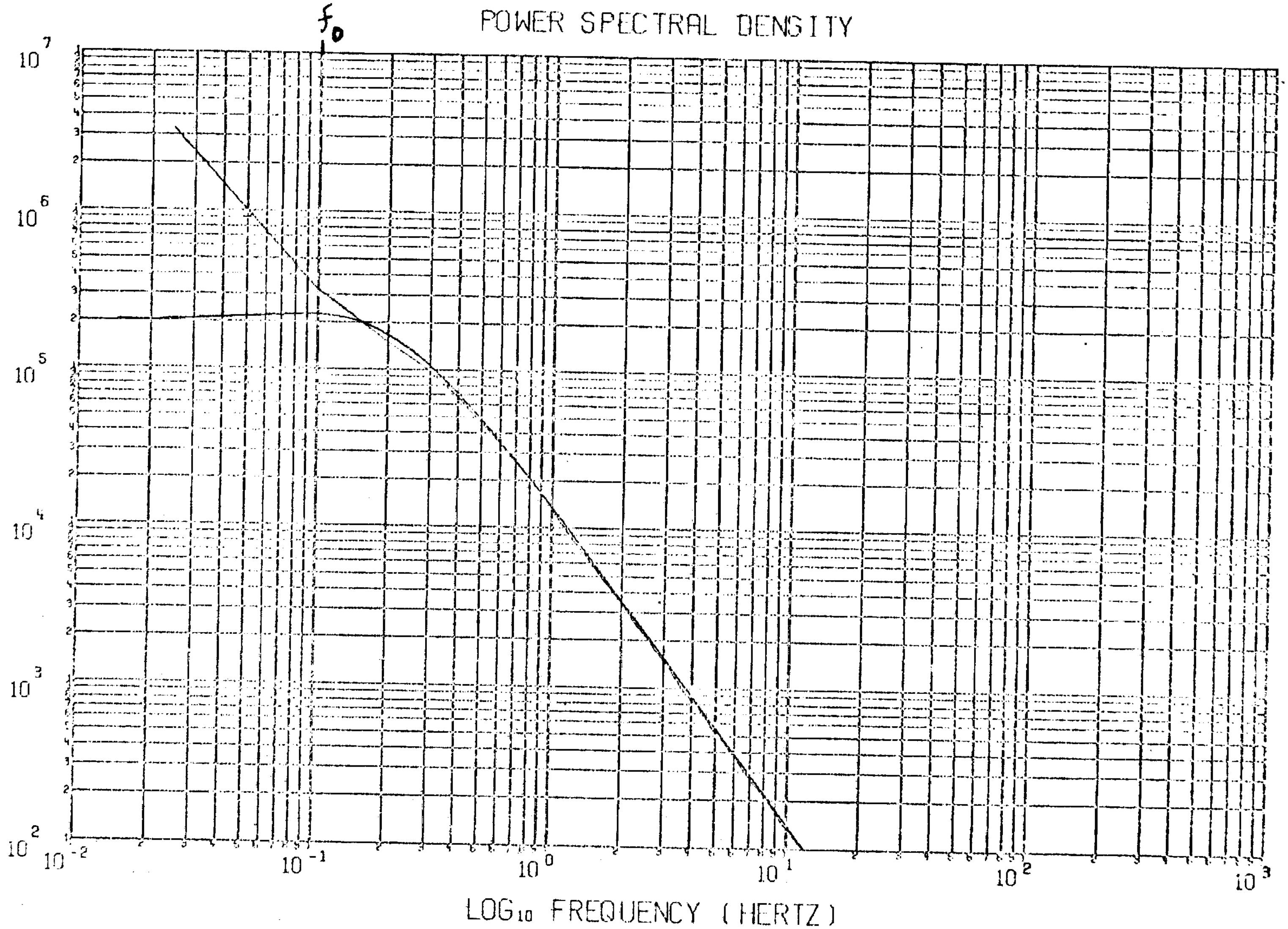
WITH PARZEN AUTOCORRELATION WINDOW OVER 513 LAGS

RESULTS ENSEMBLE AVERAGED OVER 4 TIME SEGMENTS

FIRST DIFFERENCE PRE-WHITENING AND POST-DARKENING APPLIED

$B_e = 0.0488$ Hz. $\approx f_o/2$; $n \approx 250$ dof

FIGURE 5. SIMULATED SPECTRUM COMPUTED WITH 1ST-DIFF. PRE-WHITENING ($B_e = 0.0488$ Hz)



SERIAL NUMBER = 099999

DELTA T = .04000

DELTA F = 97656 x 10^{-6}

15871 INPUT DATA POINTS

129 POWER POINTS PLOTTED

ALGORITHM:

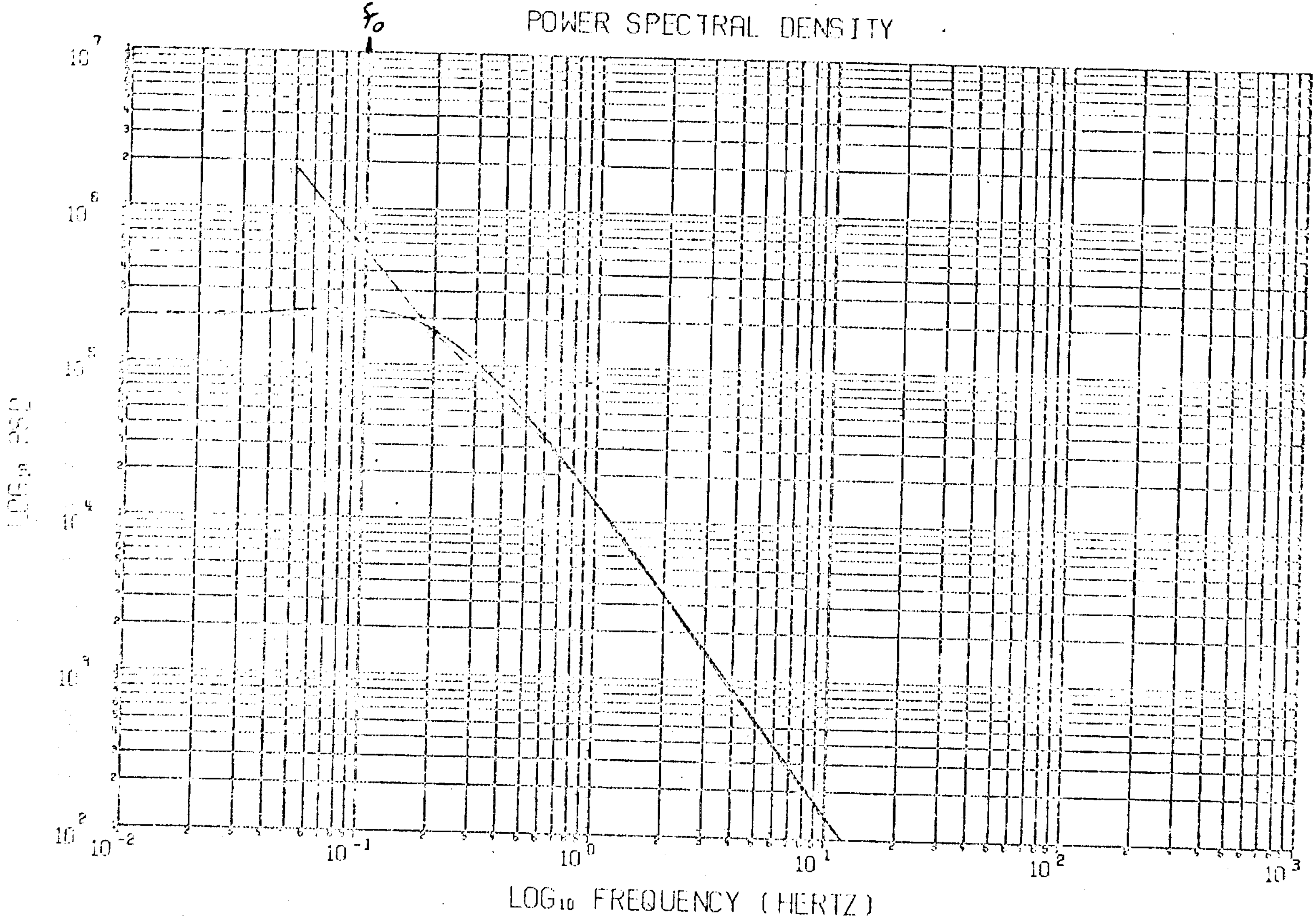
FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH PARZEN AUTOCORRELATION WINDOW OVER 129 LAGS

FIRST DIFFERENCE PRE-WHITENING AND POST-DARKENING APPLIED

$B_e = 0.195$ Hz $\approx 2f_0$; $n \approx 250$ d.o.f.

FIGURE 6. SIMULATED SPECTRUM COMPUTED WITH 1ST-DIFF. PRE-WHITENING ($B_e = 0.195$ Hz)



SERIAL NUMBER = 099999
DELTA T = .04000
DELTA F = 19531 x 10^{-5}
15871 INPUT DATA POINTS
65 POWER POINTS PLOTTED

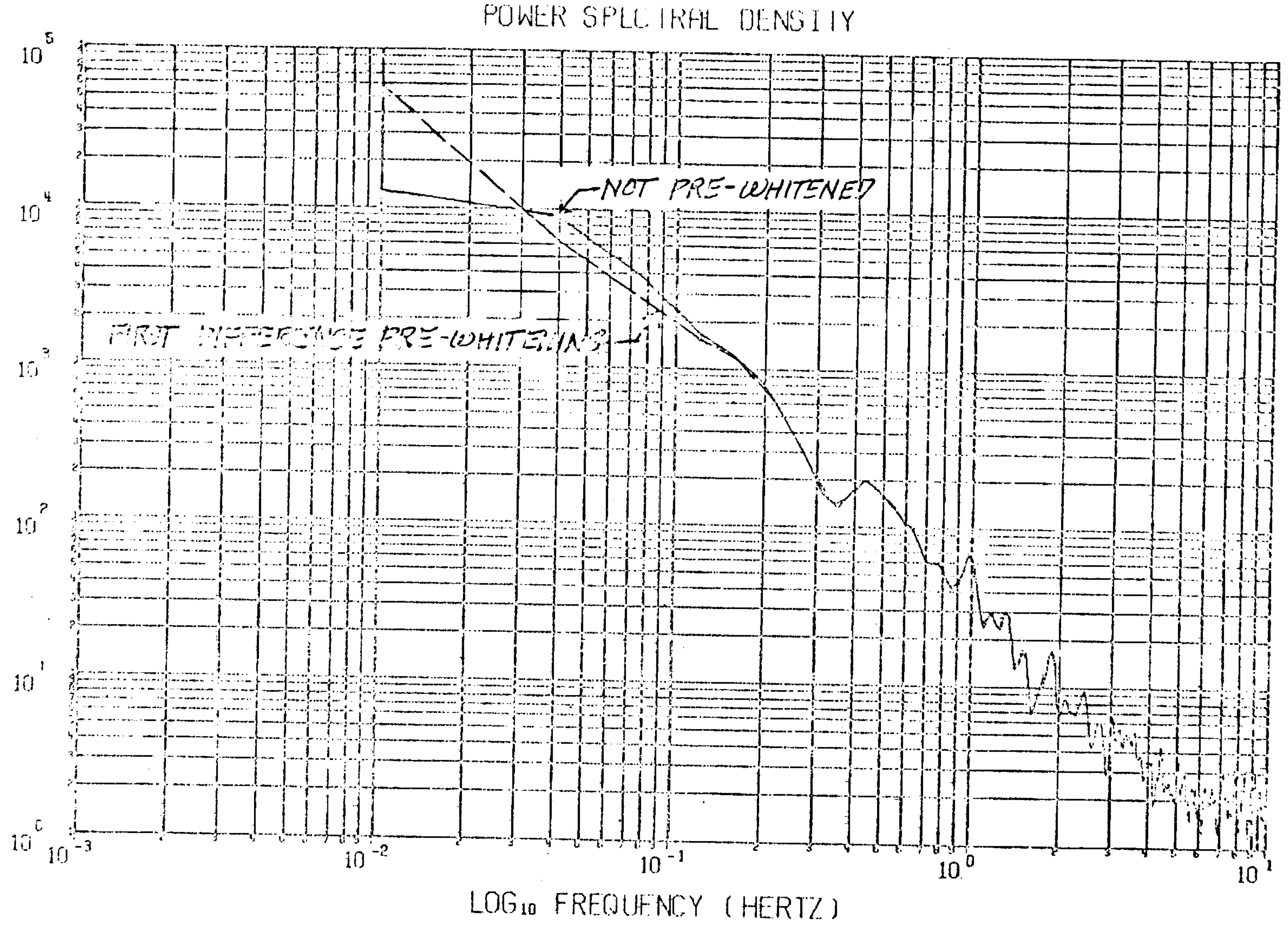
ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY
WITH PARZEN AUTOCORRELATION WINDOW OVER 65. LAGS
FIRST DIFFERENCE PRE-WHITENING AND POST-DARKENING APPLIED
$B_e = 0.390$ Hz. ≈ $4f_0$; $n \approx 500$ d.o.f.

FIGURE 7. SIMULATED SPECTRUM COMPUTED WITH 1ST-DIFF. PRE-WHITENING (B_e = 0.390 Hz)

The above results leave little doubt that the lack of an apparent "knee" in much past gust velocity spectral data may have been due to bias errors introduced by first-difference pre-whitening. This point is illustrated in Figs. 8 through 10 which present spectra for actual gust velocity data computed with and without pre-whitening for three different resolution bandwidths. It is clear in all cases that a "knee" probably exists in the data, but the pre-whitening errors blur it out. This is true even at the narrowest bandwidth of B_e = 0.078 Hz (Fig. 8) where the benefits of pre-whitening at the frequencies above the "knee" are minimal.

In conclusion, one could theoretically contrive a pre-whitening filter with an inverse "knee" which would avoid the difficulties caused by the first-difference type pre-whitening filter. The results of spectral analyses performed on the Von Karman data using such a "knee" type pre-whitening filter are presented for two extreme resolution bandwidths (B_e = 0.195 and 0.390 H_z) in Figs. 11 and 12. It is clear that the bias errors have been suppressed substantially at all frequencies by this type of pre-whitening. However, as noted in the earlier evaluation[1/], such pre-whitening operations are hazardous. Specifically, if one chooses the knee at an incorrect frequency, the result will be both a peak and a notch in the pre-whitened spectrum which will aggravate the bias error problem.



SERIAL NUMBER = 70004

DELTA T = .05000

DELTA F = 33062 x 10-6

3896 INPUT DATA POINTS

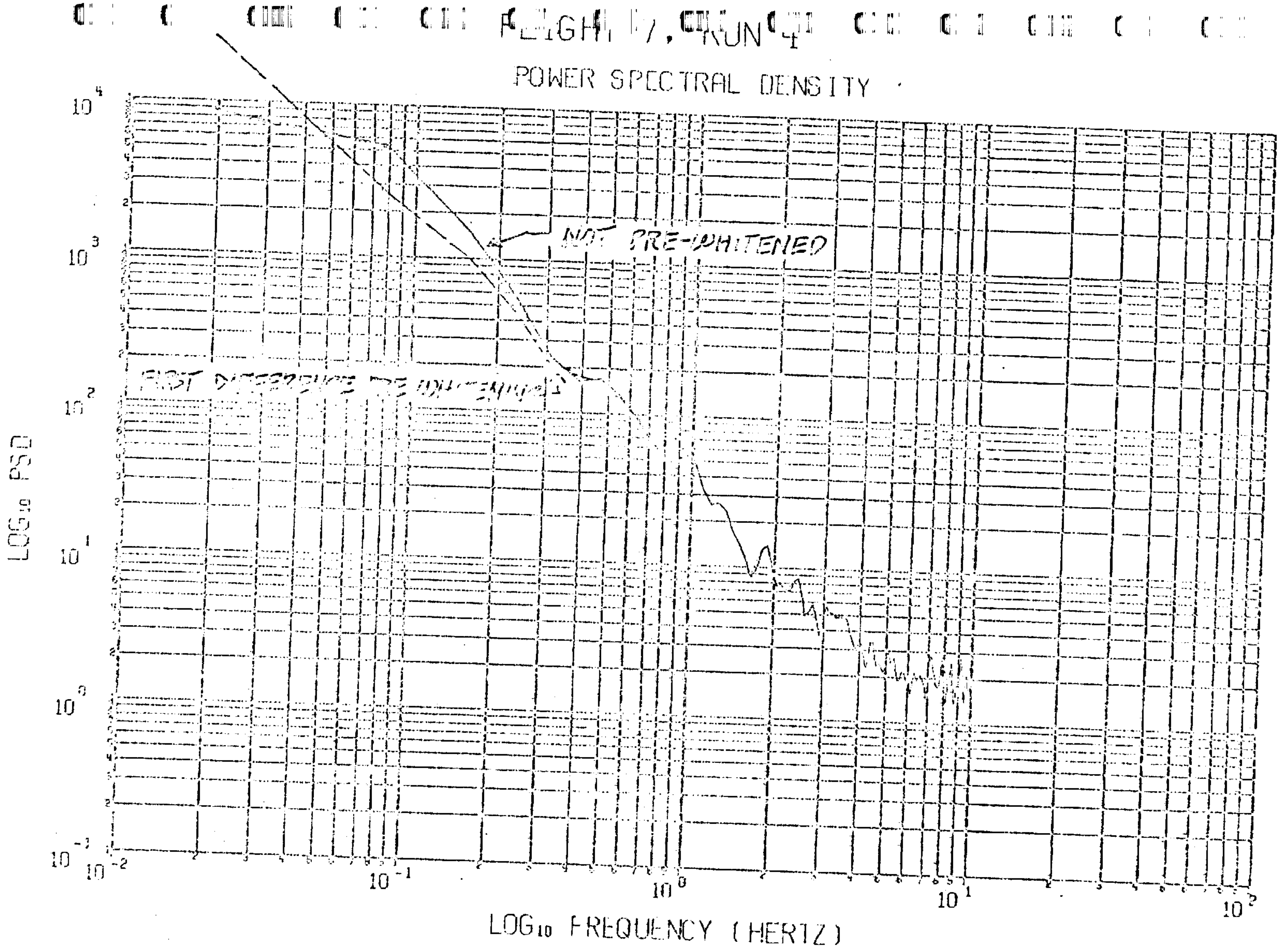
ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH HANN AUTOCORRELATION WINDOW

$B_e = 0.0781$ Hz, ; $n \approx 30$ d.o.f.

FIGURE 8. GUST VELOCITY SPECTRA WITH AND WITHOUT PRE-WHITENING ($B_e = 0.0781$ Hz)



SERIAL NUMBER = 70004
DELTA T = .05000
DELTA F = 78125 x 10^{-6}
3896 INPUT DATA POINTS

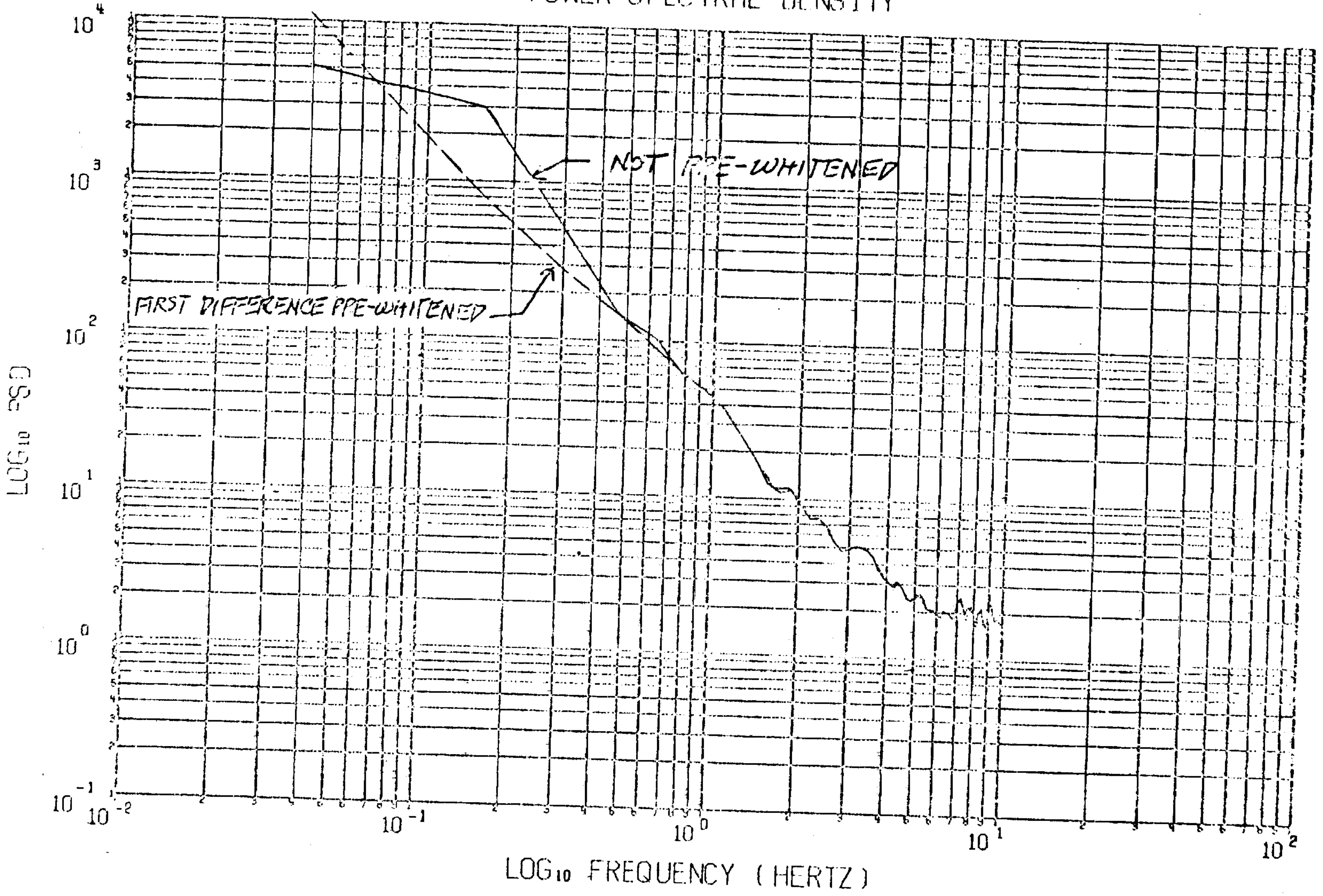
ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY
WITH HANN AUTOCORRELATION WINDOW
$B_e = 0.156$ Hz. ; $n \approx 61$ d.o.f.

FIGURE 9. GUST VELOCITY SPECTRA WITH AND WITHOUT PRE-WHITENING ($B_e = 0.156$ Hz)

FLIGHT 7, RUN 4

POWER SPECTRAL DENSITY



SERIAL NUMBER = 70004

DELTA T = .05000

DELTA F = 15625 x 10^{-5}

3836 INPUT DATA POINTS

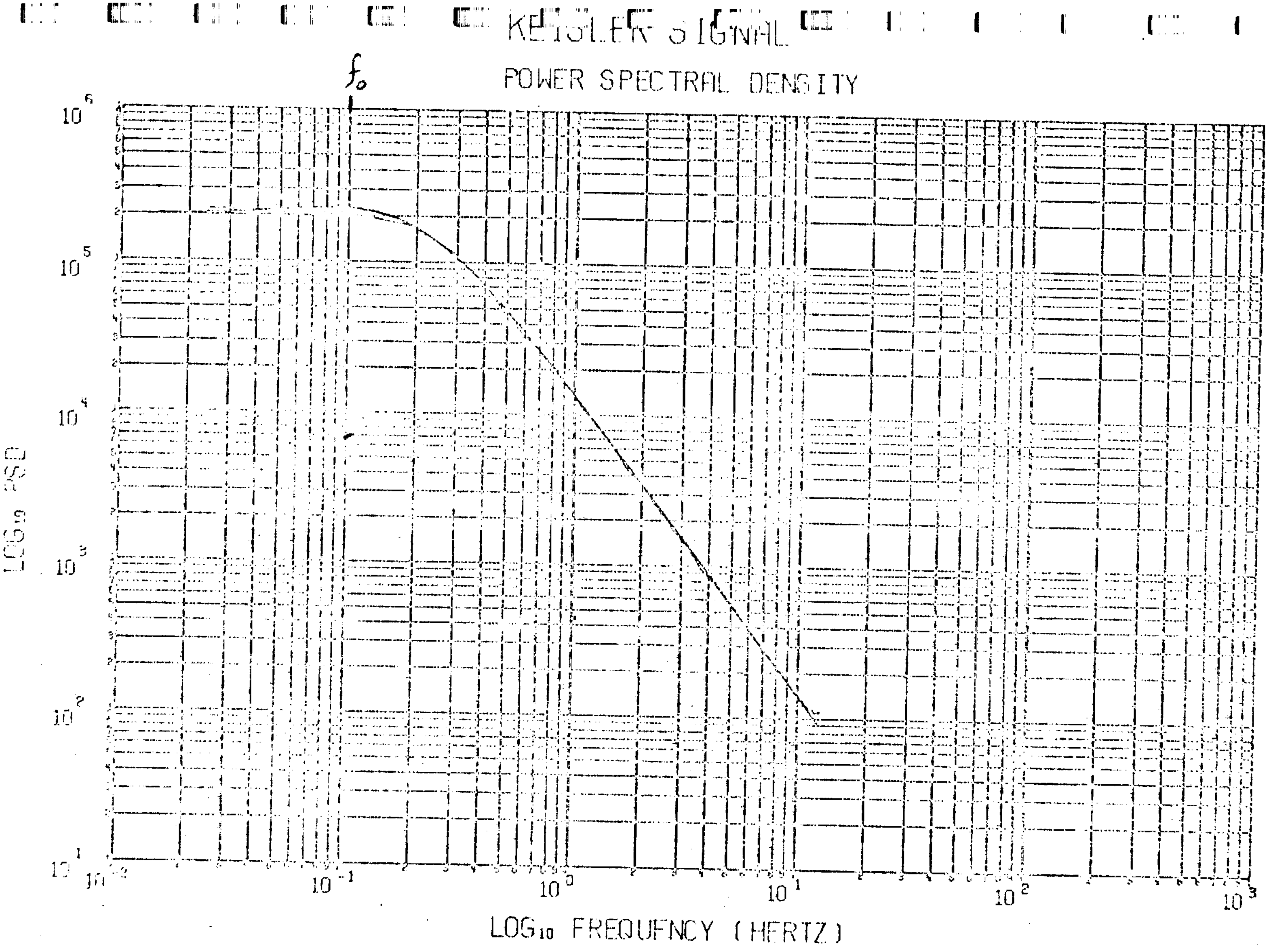
ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH HANN AUTOCORRELATION WINDOW

$B_3 = 0.312$ Hz. ; $n \approx 122$ d.o.f.

FIGURE 10. GUST VELOCITY SPECTRA WITH AND WITHOUT PRE-WHITENING ($B_e = 0.312$ Hz)



SERIAL NUMBER = 003999

DELTA T = .04000

DELTA F = 97656 x 10^{-6}

15871 INPUT DATA POINTS

129 POWER POINTS PLOTTED

ALGORITHM:

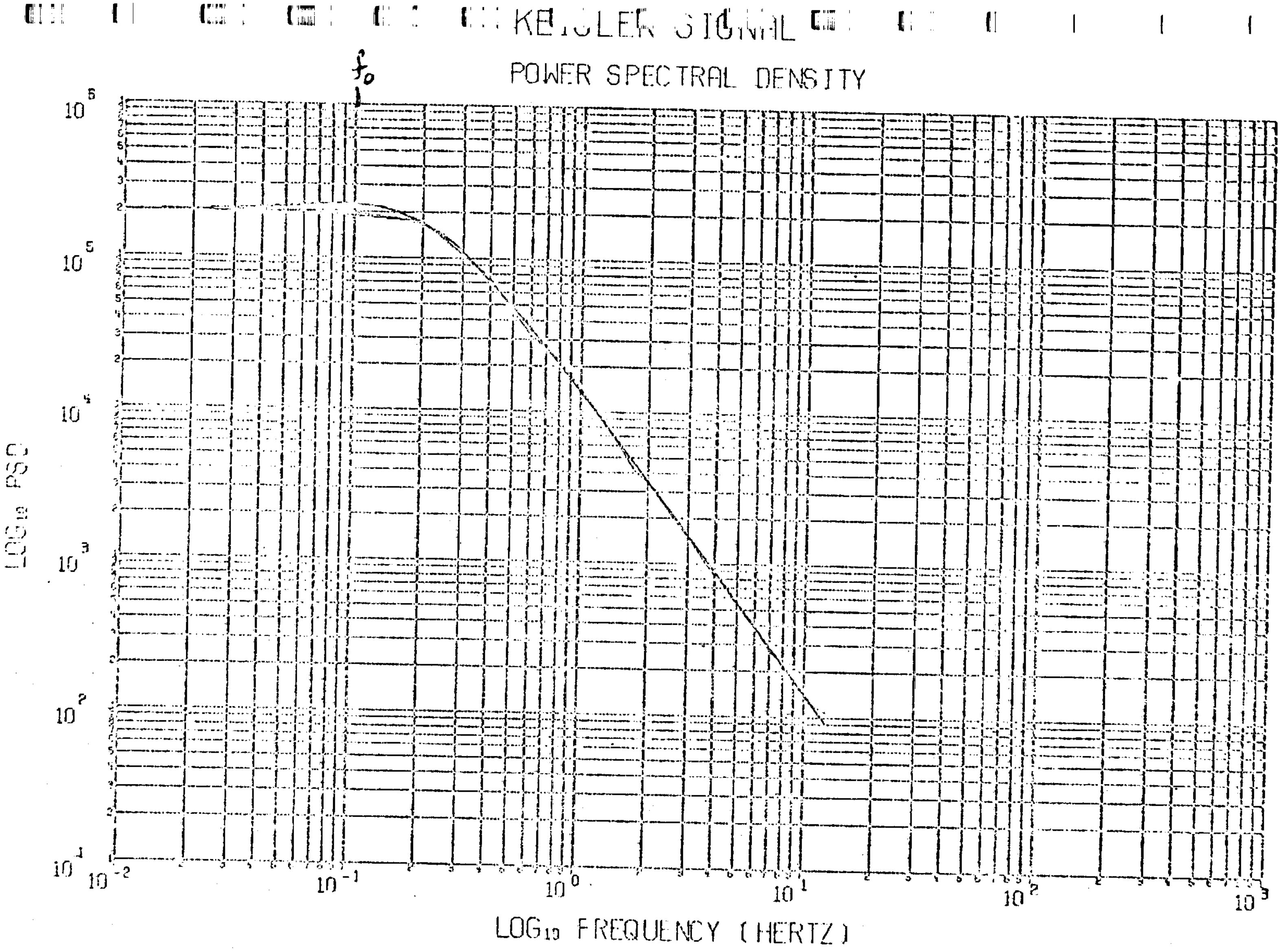
FREQUENCY DOMAIN BLACKMAN-TUKEY

WITH PARZEN AUTOCORRELATION WINDOW OVER 129 LAGS

FREQUENCY DOMAIN PRE-WHITENING AND POST-DARKENING APPLIED

B_2 = 0.175 Hz. ; n ≈ 250 d.o.f.

FIGURE 11. SIMULATED SPECTRUM COMPUTED WITH KNEED PRE-WHITENING (B_2 = 0.175 Hz)



SERIAL NUMBER = 093399
DELTA T = .04000
DELTA F = 19531 x 10^{-5}
15871 INPUT DATA POINTS
65 POWER POINTS PLOTTED

ALGORITHM:

FREQUENCY DOMAIN BLACKMAN-TUKEY
WITH PARZEN AUTOCORRELATION WINDOW OVER 65 LAGS
FREQUENCY DOMAIN PRE-WHITENING AND POST-DARKENING APPLIED
B_0 = 0.390 Hz

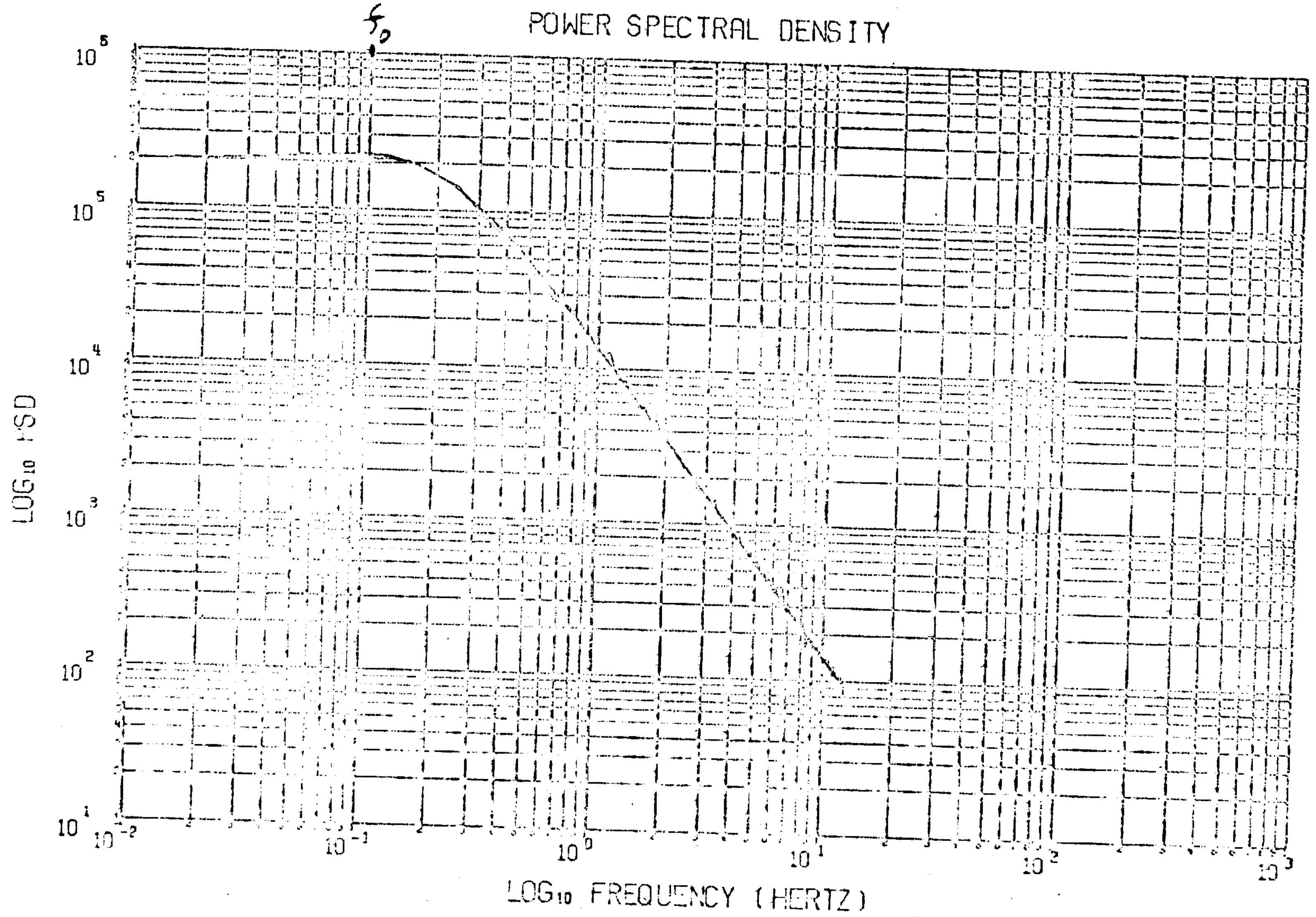
FIGURE 12. SIMULATED SPECTRUM COMPUTED WITH KNEED PRE-WHITENING (B_0 = 0.390 Hz)

3. FREQUENCY VERSUS ENSEMBLE AVERAGING

When spectra are computed using direct Fourier transform operations, one has the choice of averaging the raw spectral estimates either by frequency averaging the estimates computed from a single long record, or ensemble averaging the estimates computed from a collection of subrecords. The pros and cons of the two procedures have previously been discussed.[1/] In summary, if the zero frequency estimate is not used, frequency averaging provides a major advantage in that spectral components can be estimated at frequencies down to about ½ B_e, rather than down to only B_e as for the case of ensemble averaging. An additional benefit from frequency averaging, however, has evolved from the current studies.

When frequency averaging is used, all of the spectral components available from the original record are reflected in the analysis. For example, if the record length is T and 16 adjacent components are averaged together to obtain 32 degrees-of-freedom in the final estimate, then the lowest frequency estimate will reflect information at frequencies from 1/T to 16/T with uniform weight (assuming the estimate at 0 Hz is zero). However, if the record is divided into 16 segments for ensemble averaging to obtain the same 32 degrees-of-freedom, then the lowest frequency estimate will reflect information at frequencies from about 0/T to 32/T with a triangular weight centered at 16/T (again assuming the estimate of 0 Hz is zero). In other words, segmenting the original record for ensemble averaging throws away information at the lowest frequencies which is available when frequency averaging is used.

The above point is illustrated for an extreme case in Figs. 13 and 14, which present spectra estimated by the direct Fourier transform procedure with a resolution bandwidth of about $B_e = f_o$.

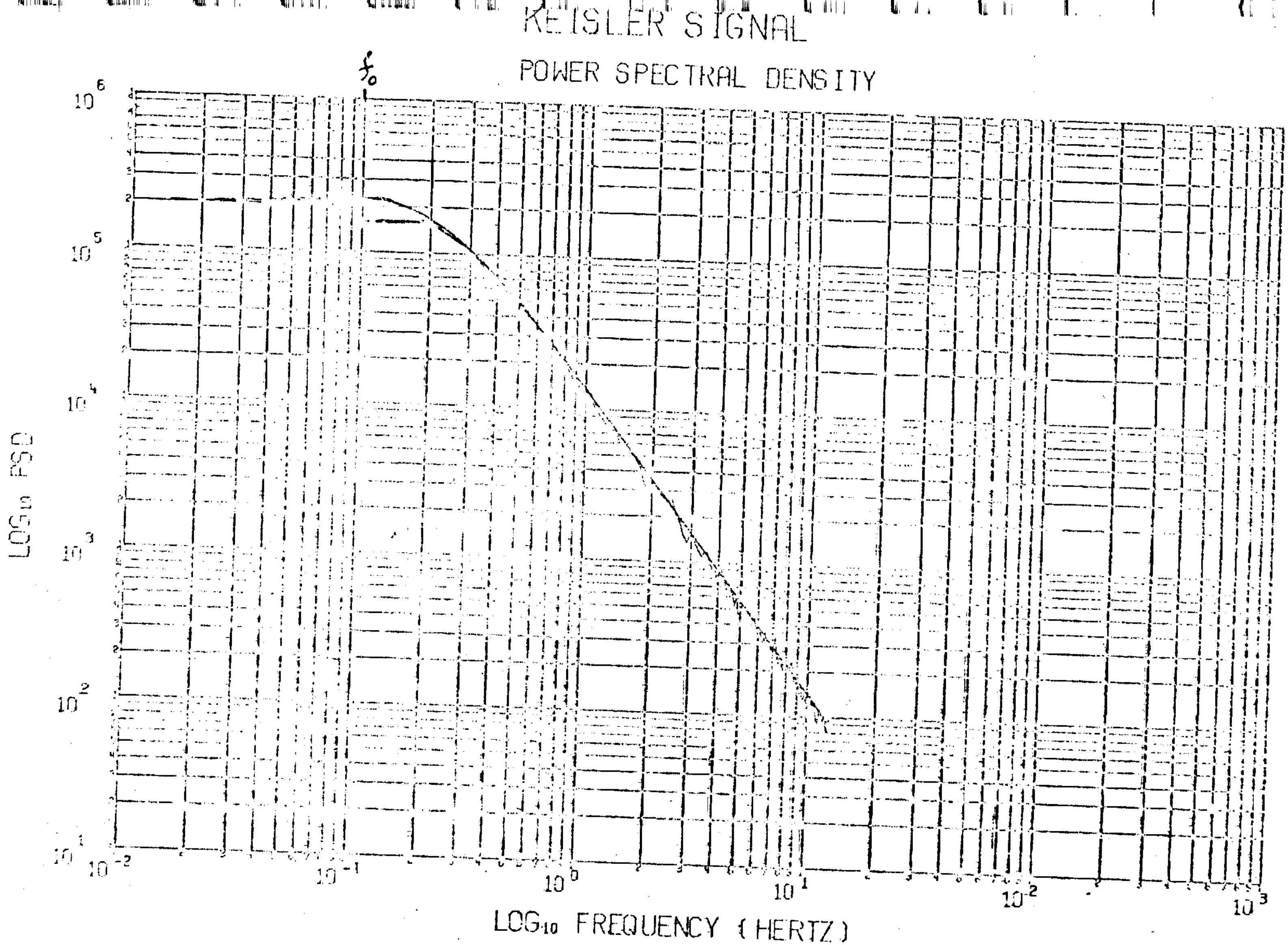


SERIAL NUMBER = 09339
DELTA T = .04000
DELTA F = 97656 x 10^{-5}
16384 INPUT DATA POINTS
128 POWER POINTS PLOTTED

ALGORITHM:

TRANSFORM OF WINDOWED TIME SIGNAL
USING HANN WINDOW
RESULTS FREQUENCY AVERAGED OVER 64 POINTS
B_e = 0.0976 Hz.

FIGURE 13. SIMULATED SPECTRUM COMPUTED USING FREQUENCY AVERAGING (B_e=0.0976 Hz)



SERIAL NUMBER = 099999
DELTA T = .04000
DELTA F = 97656 x 10^{-6}
16384 INPUT DATA POINTS
128 POWER POINTS PLOTTED

ALGORITHM:

TRANSFORM OF WINDOWED TIME SIGNAL
USING HANN WINDOW
RESULTS ENSEMBLE AVERAGED OVER 64 TIME SEGMENTS
$B_3 = 0.0976$ Hz.

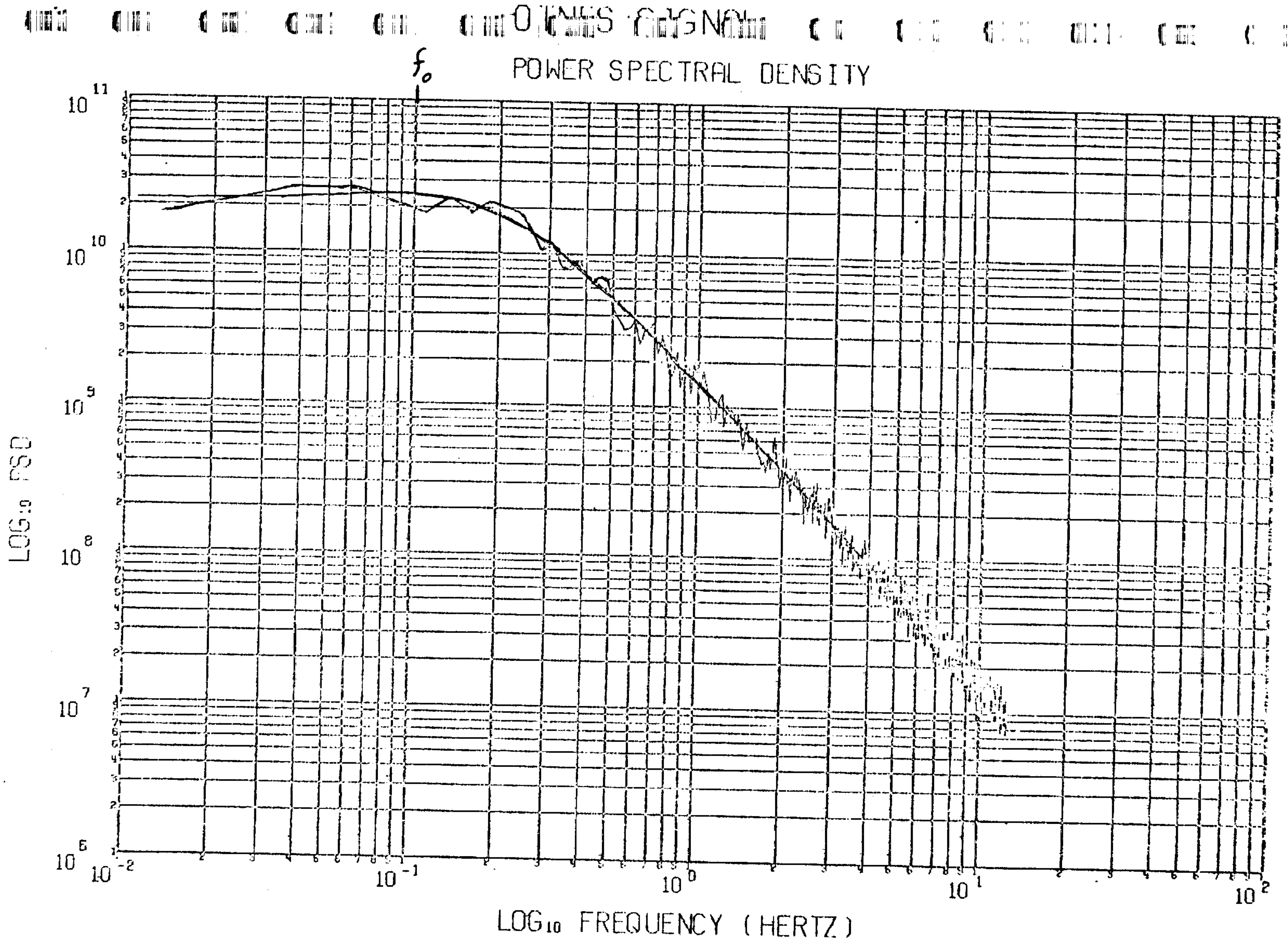
FIGURE 14, SIMULATED SPECTRUM COMPUTED USING ENSEMBLE AVERAGING ($B_3 = 0.0976$ Hz)

In Fig. 13, the estimate was obtained by frequency averaging the components of a single record over 64 adjacent components. In Fig. 14, the estimate was obtained by sub-dividing the record into 64 segments, computing the components for each segment, and then ensemble averaging the results. It is clear that the ensemble averaging procedure under-estimates the spectral density at the lowest non zero spectral component.

The above conclusions should not detract from the merits of ensemble averaging. In particular, the need to suppress spurious low frequency trends in the data, or to pool data records from different flights may still demand ensemble averaging inspite of the above noted problem. Final decisions on this issue require further information on the actual properties of the acquired data.

4. SPECTRAL WINDOW

The earlier evaluation[1/] suggested that the type of spectral window used to suppress side-lobe leakage was probably not a major concern. This suggestion appears to have been confirmed by the results of the current study. Unfortunately, an example involving an extreme (wide resolution) case was not run. However, one example was computed with a resolution bandwidth of about $f_o/4$ using both a Hann smoothing window and no smoothing at all (a boxcar window). The results are presented in Figs. 15 and 16. It is seen from these figures that even the total omission of smoothing causes no significant error in this case. Of course, some error would be expected from the boxcar window as the resolution bandwidth increases. However, the difference in the degree of suppression of this error provided by the Hann versus the Parzen window should not be significant. The Hann window is recommended for the LRC analyses because it provides slightly better resolution than the Parzen window.



SERIAL NUMBER = 099999

DELTA T = .04000

DELTA F = 244.14 x 10^{-6}

65536 INPUT DATA POINTS

512 POWER POINTS PLOTTED

ALGORITHM:

TRANSFORM OF WINDOWED TIME SIGNAL

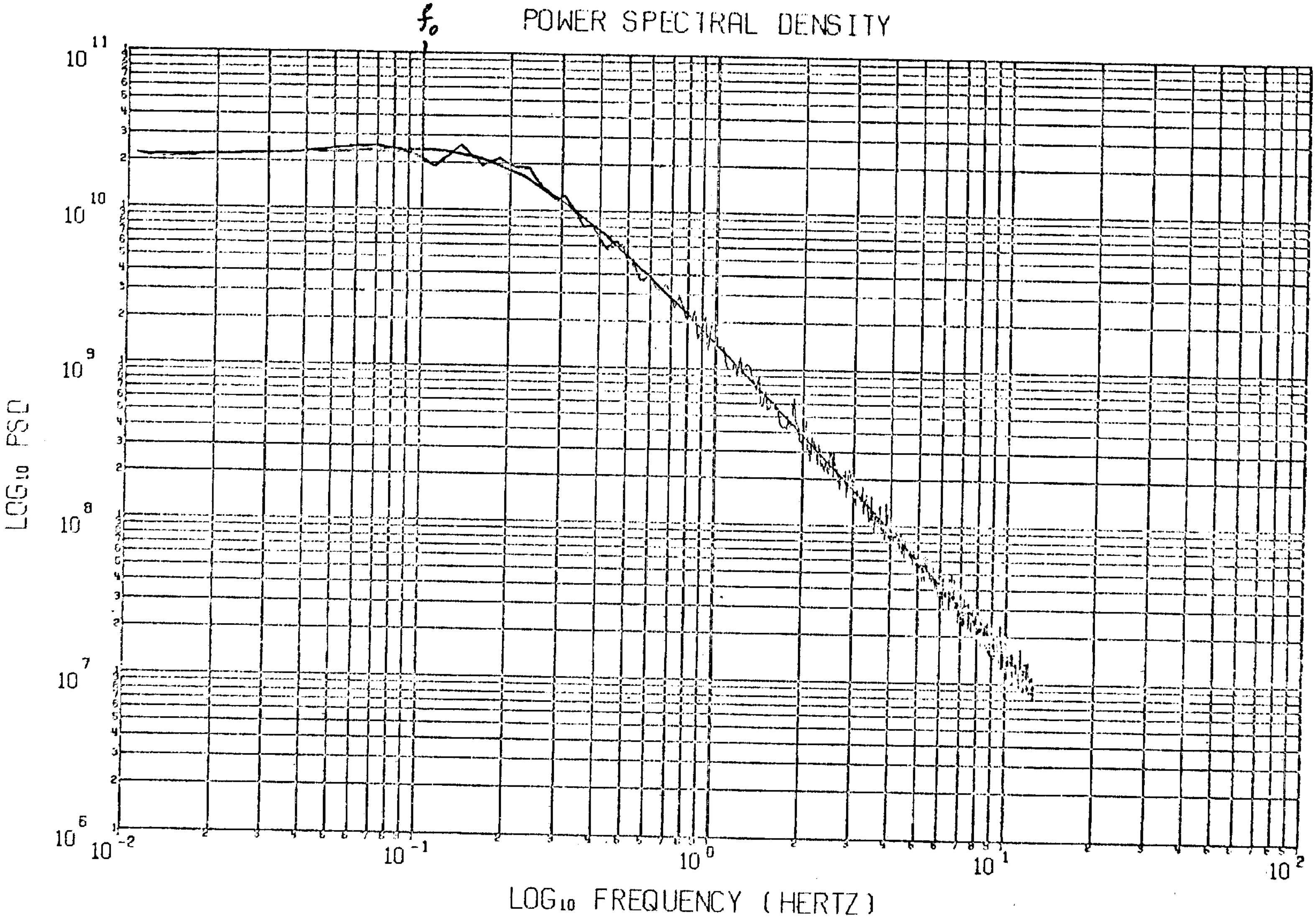
USING HANN WINDOW

RESULTS FREQUENCY AVERAGED OVER 16 POINTS

RESULTS ENSEMBLE AVERAGED OVER 4 TIME SEGMENTS

$B_e \approx 0.0244$ Hz $\approx f_o/4$; $n = 128$ d.o.f.

FIGURE 15. SIMULATED SPECTRUM COMPUTED USING HANN SMOOTHING ($B_e = 0.0244$ Hz)



SERIAL NUMBER = 099999

DELTA T = .04000

DELTA F = 24414 x 10^{-6}

65536 INPUT DATA POINTS

512 POWER POINTS PLOTTED

ALGORITHM:

TRANSFORM OF WINDOWED TIME SIGNAL

USING BOXCAR WINDOW

RESULTS FREQUENCY AVERAGED OVER 16 POINTS

RESULTS ENSEMBLE AVERAGED OVER 4 TIME SEGMENTS

$B_e \approx 0.0244$ Hz $\approx f_0/4$; $n = 128$ dof.

FIGURE 16. SIMULATED SPECTRUM COMPUTED USING NO SMOOTHING ($B_e = 0.0244$ Hz).

5. ZERO FREQUENCY ESTIMATES

When direct Fourier transform procedures are used to compute power spectra, the estimate at zero frequency will always be zero due to the standardization operations on the data prior to analysis. Hence, when a record of length T is subdivided into m segments for analysis by ensemble averaging procedures, the lowest spectral component in the results will appear at the frequency m/T. If frequency averaging is used, an estimate at zero frequency can be generated by averaging over the first m/2 components and assigning the result a frequency of zero. The more desirable approach, however, is to average over the first m components and assign the result a frequency of (m+1)/2T. This is approximately ½ the lowest frequency available from the ensemble averaged data, but not zero frequency.

When Blackman-Tukey procedures (involving Fourier transforms of truncated correlation functions) are used, a nonzero spectral estimate will generally be obtained at zero frequency. This estimates represents the net area under the truncated autocorrelation function, which is not necessarily forced to zero by the pre-analysis standardization procedures. At first glance, this would appear to constitute a major advantage for the Blackman-Tukey approach, since the zero frequency estimate relates directly to the scale of turbulence[3/], an important quantity to be estimated from the data. Two facts should be noted, however. First an equivalent zero frequency estimate can be generated, if desired, by the direct Fourier transform procedure using frequency averaging, as previously described. Second, due to the integrated accelerometer noise problem in gust velocity data,[1/] and the detrending operations required to suppress this problem, any estimate at zero frequency must be viewed with great suspicion. In fact, it is believed by this author that such zero frequency estimates should be routinely excluded from the data presentation.

6. CONCLUSIONS AND RECOMMENDATIONS

The basic conclusions to be drawn from the LRC studies using simulated Von Karman gust velocity signals are as follows:

1. The spectral analysis of a Von Karman signal without pre-whitening will result in an over-estimate of the spectral values at frequencies above the "knee," and an under-estimate of spectral values at frequencies near and below the "knee." The magnitude of these bias errors increases with the resolution bandwidth of the analysis, varying from negligible errors for a bandwidth of ¼ the "knee" frequency, to errors of perhaps 50% for a bandwidth of 4 time the "knee" frequency.

2. The use of conventional first-difference pre-whitening will suppress the bias errors at frequencies above the "knee" to a negligible level, *but will generally increase the bias errors at frequencies near the "knee"* for resolution bandwidths of greater than ½ the "knee" frequency. Specifically, pre-whitening leads to a severe over-estimation of the spectral densities below the "knee." The net effect is to blur out the "knee."

3. The problems posed by first-difference pre-whitening can be eliminated by using a pre-whitening filter with an inverse "knee" at least crudely matched to the "knee" in the data. However, such a pre-whitening procedure is hazardous since any mismatch in the frequencies of the data and filter "knees" will only aggrevate the bias error problem.

4. For analysis by direct Fourier transform procedures, the frequency averaging approach appears to provide more accurate estimates at the lowest frequencies than does the ensemble averaging approach.

5. The type of smoothing window used for the analysis (Hann, Parzen, GEO, etc.) does not appear to make any significant difference.

The specific recommendations which evolve from these conclusions are as follows:

1. At least initially, all gust velocity data should be analyzed both with and without pre-whitening. First-difference type pre-whitening is suggested to permit comparisons to prior data. However, some analysis with a "kneed" pre-whitening filter is also acceptable as long as the results are interpreted with caution.

2. Direct Fourier transform analysis procedures are suggested with frequency averaging employed when feasible. A final decision of the averaging technique must await the results of initial error measurements on the data acquisition system.

3. Side-lobe suppress should be accomplished using the Hann window.

The results of these studies have resolved many of the problems of concern in the forthcoming data acquisition and analysis program. Important problems which still require attention are those related to time history detrending and nonstationary data properties. Appropriate action on detrending must await the results of initial hanger tests which will establish the magnitude of the low frequency trend problem. Likewise, the manner in which nonstationary sample records are dealt with depends heavily upon the actual results obtained in the program. Various procedures for approaching both of these problems have already been summarized in the earlier evaluation.1/

In closing, it should be mentioned that BBN has been working on new techniques for evaluating nonstationary random processes which might be applicable to the LRC gust data. If so, they could eliminate many of the problems associated with extracting information from nonstationary gust velocity records, and greatly improve the quality of the results obtained from the LRC atmospheric turbulence program. These techniques, however, are not fully developed for such applications. Some support from LRC would be required to establish their usefulness for this program.

7. REFERENCES

1. Piersol, A. G., "Evaluation of LRC Gust Velocity Data Acquisition and Analysis System," BBN Report No. 2435, Bolt Beranek and Newman Inc., Canoga Park, California, June 1973.

2. Bendat, J. S., and A. G. Piersol, *Random Data: Analysis and Measurement Procedures*, pp. 187, 189-193, Wiley-Interscience, New York, 1971.

3. Houbolt, J. C., "Atmospheric Turbulence," *AIAA Journal*, 11, No. 4, pp. 421-437, April 1973.