PARAMAGNETIC LIQUID BRIDGE IN A GRAVITY-COMPENSATING MAGNETIC FIELD*,†

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Abstract

Magnetic levitation was used to stabilize cylindrical columns of a paramagnetic liquid in air between two solid supports. The maximum achievable length to diameter ratio R_{max} was ~ (3.10 ± 0.07), very close to the Rayleigh-Plateau limit of π. For smaller *R*, the stability of the column was measured as a function of the Bond number, which could be *continuously* varied by adjusting the strength of the magnetic field.

Liquid bridges supported by two solid surfaces have been attracting scientific attention since the time of Rayleigh [1] and Plateau. For a cylindrical bridge of length *L* and diameter *d*, it was shown theoretically that in zero gravity the maximum slenderness ratio R [$\equiv L/d$] is π [1]. The stability and ultimate collapse of such bridges is of interest because of their importance in a number of industrial processes and their potential for low gravity applications. In the presence of gravity, however, the cylindrical shape of an axisymmetric bridge tends to deform, limiting its stability and decreasing the maximum achievable value of *R*. Theoretical studies have discussed the stability and possible shapes of axisymmetric bridges [2-6]. Experiments typically are performed in either a Plateau tank, in which the bridge is surrounded by a density-matched immiscible fluid [7-9], or in a space-borne microgravity environment [10]. It has been shown, for example, that the stability limit *R* can be pushed beyond π by using flow stabilization [6], by acoustic radiation pressure [7,9], or by forming columns in the presence of an axial electric field [8]. In this work magnetic levitation was used to simulate a low gravity environment and create quasi-cylindrical liquid columns in air. Use of a magnetic field permits us to *continuously* vary the Bond number $B \equiv \frac{g\rho d^2}{4\sigma}$, where g is the gravitational acceleration, ρ is the density of the liquid, and σ is the surface tension of the liquid in air. The dimensionless Bond number represents the relative importance of external forces acting on the liquid column to those due to surface tension. Our central result is that in a large magnetic field gradient we could create and stabilize columns of mixtures of water and paramagnetic manganese chloride tetrahydrate ($MnCl_2 \bullet 4H_2O$), achieving a length to diameter ratio very close to π.

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The principle of magnetic compensation of gravity is straightforward. For a material of volumetric magnetic susceptibility χ in a magnetic field H, the energy per unit volume is given by $U = -\frac{1}{2}\chi H^2$, and the force per unit volume is $-\nabla U$. To compensate gravity it is required that $\frac{1}{2}\chi\nabla H^2_{comp} \approx \rho g$, where H_{comp} corresponds to the magnetic field whose gradient just compensates gravity. For ∇H^2 larger or smaller than $2\rho g/\chi$, the liquid will rise or sag in the column, ultimately causing the column to collapse if ∇H^2 deviates too significantly from its gravity-compensating value. Thus the effective force on the column may be controlled by varying the current in the magnet.

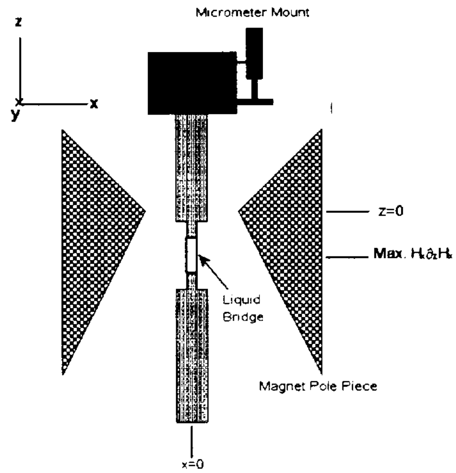
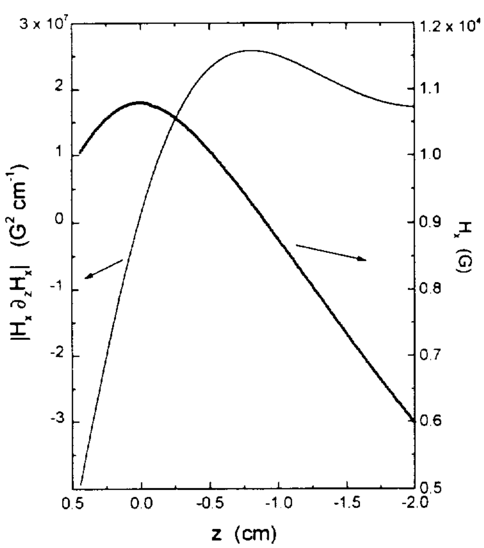


Fig. 1 Schematic view of the experimental setup. The fluid injection system, involving a hypodermic needle that injects material from the side, is not shown.

An electromagnet fitted with special Faraday pole pieces was used to produce ∇H^2 uniform to approximately 6% over the length of a 1 cm-long column. In order to determine the field profile, a Bell model 9500 Gaussmeter utilizing a Hall effect probe was used to measure H_x as a function of vertical position z along the symmetry plane (x = 0) of the magnet (Fig. 1). For all practical purposes, the field profile is translationally invariant along the y-axis, and therefore the y coordinate does not enter into the problem. Experimental values of both H_x and the product $H_x\partial_z H_x$ are shown as functions of z in Fig. 2. Note that along the plane x = 0 the z-component of field H_z vanishes, although a small component of H_z exists for $x \neq 0$. This small component may give rise to a slight distortion of the cylinder perpendicular to its symmetry axis, and will be discussed below. Nevertheless, over the small diameter of the columns $H_z\partial_z H_z$ remains small, and ∇H^2 is dominated by $H_x\partial_z H_x$; we shall therefore consider the z-component of force to be $\chi H_x\partial_z H_x$.

Fig. 2 Magnetic field H_X (right axis) and $H_X\partial_Z H_X$ (left axis) *vs.* vertical position z at $H = H_{comp}$. z = 0 corresponds to the position of closest approach of the pole pieces (see Fig. 1). The quantity $H_X\partial_Z H_X$ is maximum at z = -0.8 cm.



Manganese chloride tetrahydrate was obtained from Aldrich Chemicals and used as received. A high-concentration mixture of 62.5 wt.% $MnCl_2 \bullet 4H_2O$ in distilled water was prepared. By weighing a known volume of the mixture, its density was determined to be $\rho = (1.45 \pm 0.01)$ gm cm^{-3}. The surface tension σ in air was measured to be (116 ± 6) ergs cm^{-2} by the pendant drop method [11,12] To establish a confidence level for this technique, the measurements were repeated with both pure water and glycerol, where the measured values of σ were found to scatter within $\pm$5% of accepted values in the literature.

Fig. 1 shows a sketch of the apparatus. Two ½-

inch diameter aluminum rods were machined to have cylindrical tips at their ends that are d = 0.32 cm diameter and 1.27 cm long. The pair was placed vertically in the magnet at x =0, such that the small tips faced each other. The upper rod was attached to a precision micrometer to facilitate adjustment of its position along the z-axis relative to the lower tip, and the tip of the lower rod was placed at approximately 0.4 cm below z = -0.8 cm. [z = 0 corresponds to the point where the pole pieces reach their closest approach, and z = -0.8 cm is the position of maximum $H_x \partial_z H_x$]. The lower tip was placed at this position so that the center of the liquid column would be at the approximate maximum in $H_x \partial_z H_x$. A boroscope attached to a CCD camera was positioned along the y-axis to view the liquid bridge, and the images were recorded with a video cassette recorder. The magnetic field was adjusted so that ∇H^2 approximately corresponded to ∇H^2_{comp} , and liquid was injected into the gap (typically starting at 0.1 cm) between the tips using a 25 gauge butterfly hypodermic needle and syringe. The upper tip was then translated upward using the micrometer, thereby creating a liquid cylinder between the two tips. As the upper tip was further translated, a waist formed in the column and more liquid had to be added to maintain a uniform cylinder. During this procedure the magnetic field also had to be fine-tuned to prevent sagging. This procedure was continued until a uniform cylinder of a desired length L (and thus a given slenderness ratio R = L/d) was achieved. For the longest cylinders (0.8 < L < 1.0 cm), the shape of the cylinder was found to be extremely sensitive to magnetic field: We found that if $H_x \partial_z H_x$ were to deviate from 2.57×10^7 $G^2\ cm^{-1}$ [defined as $(H_x \partial_z H_x)_{comp}$] by more than 1%, a noticeable bulge in the cylinder would appear near the top (for too large a field) or the bottom (for too small a field). Thus, knowing the density ρ and $(H_x \partial_z H_x)_{comp}$, we were able to extract the volumetric magnetic susceptibility (per cm^3) $\chi = \frac{\rho g}{(H_x \partial_z H_x)_{comp}} = (5.54 \pm 0.05) \times 10^{-5}$. We note that during the course of the experiment the relative humidity was kept near 100% to minimize evaporation of the water. Had we not done so, water evaporation would have increased the concentration of the paramagnetic salt in the column, and therefore changed the susceptibility.

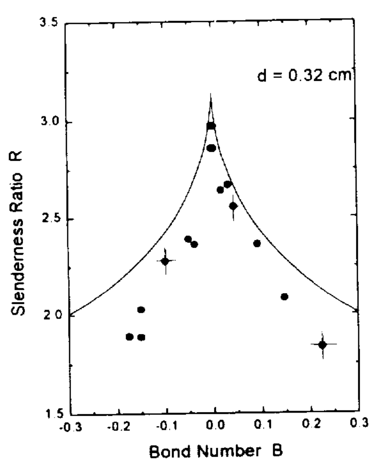
Let us now turn to the stability of the column as a function of the Bond number. For our experiment the Bond number B must be redefined to include the effects of the magnetic field, *viz.*,

$$B \equiv \frac{(\rho g - \chi H_x \partial_z H_x) d^2}{4\sigma}. \qquad (1)$$

As described above, columns of a given slenderness ratio *R* were created and stabilized in a magnetic field gradient, such that $(H_x \partial_z H_x)_{comp} = 2.57 \times 10^7$ G^2 cm^{-1}; this corresponds to B = 0. Then B was varied either positively or negatively by decreasing or increasing the magnetic field from its value H_{comp}. For a given *R* there was some maximum and minimum field, corresponding to a negative and positive Bond number, beyond which the column could no longer be sustained and catastrophically collapsed. In Fig. 3 we show two images of the stable bridge: One is at Bond number B approximately equal to zero. The second is at a lower field where the Bond number is just within the stability limit. [If the field is further reduced, the bridge collapses]. The Bond numbers corresponding to the limits of stability of the bridge were measured as a function of *R*, and are shown in Fig. 4. In a similar manner we also examined cylinders of smaller diameter d = 0.16 cm, finding comparable results for *R vs. B*; these are shown in Fig. 5. [Note that for d = 0.16 cm, the lower tip was placed ~ 0.2 cm below the position of maximum $H_x \partial_z H_x$, *i.e.*, below z = -0.8 cm] In both figures we also show theoretical numerical results for the stability limits calculated by Coriell *et al.* [2].

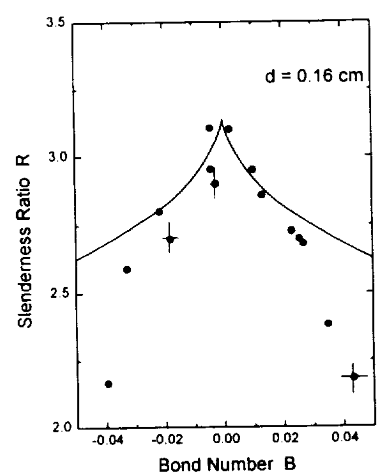
Fig. 3 Photograph of d = 0.32 cm liquid bridge with R = 2.39. a) Stable bridge with $H_x \partial_z H_x$ adjusted to approximately $(H_x \partial_z H_x)_{comp}$, so that B is close to zero. b) Stable bridge with $H_x \partial_z H_x$ reduced, so that B = 0.09 (*cf.* Eq. 1). Note that when $H_x \partial_z H_x$ is further reduced, so that B ~ 0.093, the bridge collapses.

Fig. 4 Slenderness ratio *R vs.* Bond number B at the stability limits for d = 0.32 cm bridges. The region below the inverted "V" corresponds to the region of stability; outside this region the column collapses. Vertical error bars correspond to experimental uncertainty in determining *R* and horizontal error bars to uncertainty in reproducing collapse of the column at a given Bond number. The solid line represents the theoretical stability limits according to Ref. 2.



Although our experimental results for the stability limits are apparently symmetric about B = 0 and are in reasonable agreement with theory, there are clearly deviations from the theoretical curves. One problem is the uniformity of the magnetic force (see Fig. 2). Over very small length scales $H_x \partial_z H_x$ is quite uniform, although as the cylinder length approaches 1 cm there are significant variations in the magnetic force along the z-axis. For this reason it is likely that the stability of the longer columns may be compromised. It should also be remembered that although the magnetic fields corresponding to the stability limits are determined directly, the Bond number is *derived* from these fields, as well as from experimental measurements of ρ and σ. The surface tension, in particular, has a not-insignificant uncertainty. Thus, in addition to the

Fig. 5 Same as Fig. 4, except for d = 0.16 cm.



experimental error bars that appear in Figs. 4 and 5, there may be an additional *systematic* error of up to 7% in B (the abscissa). Such an error could be partially responsible for the small disagreement between experiment and theory. Additionally, we note that for the d = 0.16 cm columns the stability limits are reduced from theory at smaller *R*. We do not yet understand this phenomenon. We note, however, that as the liquid sags (rises) near the field stability limit, a thin film of liquid would often wet the sides of the lower (upper) rod, especially for shorter, smaller *R*, columns. Thus liquid would be drawn off from the column, reducing its apparent stability. This observation, in conjunction with possible inhomogeneities in the rod itself, may be partially responsible for the observed deviations. Yet another issue is the liquid volume V, where deviations from $V = \pi L d^2/4$ could affect the apparent stability [13,14] of the column. In our experiments we did not measure the volume of the bridge directly, but rather adjusted the volume to obtain an apparently right circular cylinder at B = 0. Finally, we need to consider two effects which can alter the cross section of the cylinder in the *xy*-plane from a circle to an ellipse. The dominant component of magnetic field is along the *x*-axis. We have performed a magnetic boundary value calculation, including the surface tension, to determine the distortion on an infinitely long paramagnetic column arising from a transverse magnetic field. We found that for our values of χ and H_x, the eccentricity is ~0.004 at H_{comp}, compared to zero at $H_x = 0$. This represents a tiny deviation from a circular cross-section, and would be nearly impossible to detect with our imaging scheme. An additional issue is that because H_x varies with x — we have also mapped out this variation with our Gaussmeter — there is a weak transverse force on the liquid cylinder in the x-direction. This force vanishes at the symmetry plane (x = 0), but grows linearly with *x* away from the midpoint between the pole pieces. We have calculated the distortion on the circular cross section arising from this nonuniform magnetic force, and again found that the eccentricity is of order 0.02. The effects on the stability are therefore likely to be small. Both calculations will be published elsewhere [15].

To summarize, we have demonstrated that magnetic levitation may simulate a low gravity environment to enable the formation of stable liquid bridges. Such a technique permits the continuous variation of the Bond number and obviates the need for using density-matched liquids. Despite these advantages, the inhomogeneities of the magnetic forces tend to reduce slightly the limits of stability. On the basis of these results we intend to examine the effects of a modulated magnetic field on the stability. Additionally, we may also consider the stability of *diamagnetic* fluids in much higher fields, as we have already demonstrated the principle of levitation in these systems [16].

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