COMPUTATION OF SUPERSONIC JET MIXING NOISE USING PARC CODE WITH A κ–ε TURBULENCE MODEL

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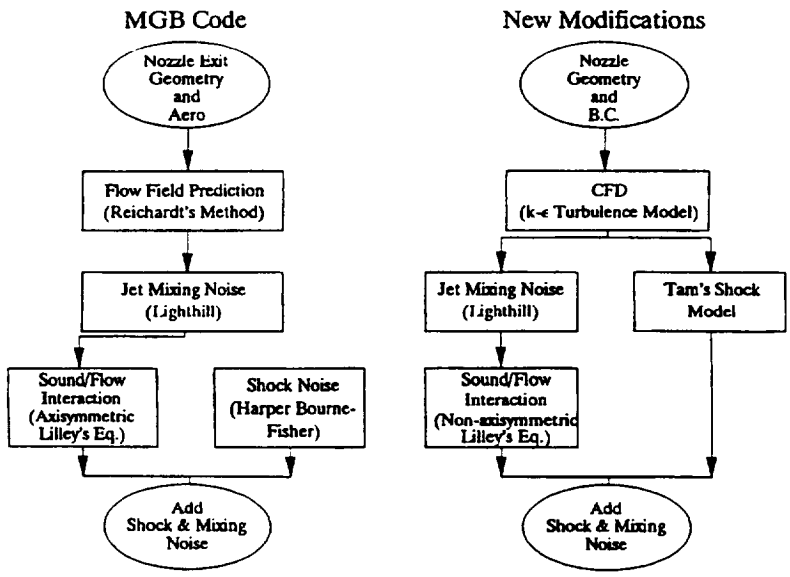
BACKGROUND



A number of modifications have been proposed in order to improve the jet noise prediction capabilities of the MGB code. This code which was developed at General Electric, employees the concept of acoustic analogy for the prediction of turbulent mixing noise. The source convection and also refraction of sound due to the shrouding effect of the mean flow are accounted for by incorporating the high frequency solution to Lilley's equation for cylindrical jets (Balsa and Mani). The broadband shock-associated noise is estimated using Harper-Bourne and Fisher's shock noise theory. The proposed modifications are aimed at improving the aerodynamic predictions (source/spectrum computations) and allowing for the non-axisymmetric effects in the jet plume and nozzle geometry (sound/flow interaction). In addition, recent advances in shock noise prediction as proposed by Tam can be employed to predict the shock-associated noise as an addition to the jet mixing noise when the flow is not perfectly expanded. Here we concentrate on the aerodynamic predictions using the PARC code with a k-ϵ turbulence model and the ensuing turbulent mixing noise. The geometry under consideration is an axisymmetric convergent-divergent nozzle at its design operating conditions. Aerodynamic and acoustic computations are compared with data as well as predictions due to the original MGB model using Reichardt's aerodynamic theory.

MODELING APPROACH

- **Source Spectrum Calculations**
  - **Lighthill's Acoustic Analogy**
  - **Ribner and Batchelor Assumptions**
  - **Calculation of Source Strength and its Spectrum Using CFD**
- **Sound/Flow Interaction**
  - **High Frequency Asymptotic Solution to Lilley's Equation for a Multipole Source Convecting in an Axisymmetric Parallel Flow (Balsa and Mani)**
  - **Calculation of Directivity Factors Based on CFD Results**

GOVERNING EQUATIONS

Lighthill's Equation

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

$$T_{ij} = \rho V_i V_j + \delta_{ij}(p - c^2 \rho) - e_{ij}$$

$$e_{ij} = \mu(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial V_k}{\partial x_k})$$

$$\frac{e_{ij}}{\rho V i V_j} \sim O(\frac{1}{Re}), \qquad Re = \frac{\rho U L}{\mu}$$

$$\frac{1}{c^2} dp - d\rho = (\frac{\partial \rho}{\partial s})_p ds$$

- The effects of source convection and refraction are included in the source term.

Lilley's Equation

$$\frac{D}{Dt}\left(\frac{D^2\sigma}{Dt^2} - \frac{\partial}{\partial x_i}c^2\frac{\partial\sigma}{\partial x_i}\right) + 2\frac{\partial V_j}{\partial x_i}\frac{\partial}{\partial x_j}c^2\frac{\partial\sigma}{\partial x_i} = -2\frac{\partial V_j}{\partial x_i}\frac{\partial V_k}{\partial x_j}\frac{\partial V_i}{\partial x_k} + \frac{D}{Dt}\left[\frac{D}{Dt}\left(\frac{1}{c_p}\frac{Ds}{Dt}\right)\right] + \text{viscous terms}$$

$$\sigma = \frac{1}{\gamma}ln\frac{p}{p_o}$$

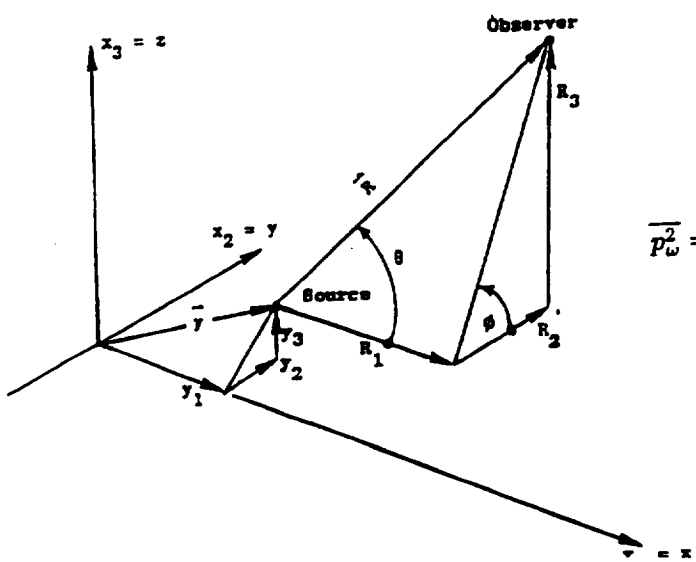
$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_k\frac{\partial}{\partial x_k}$$

- The effects of source convection and refraction are included in the operator term of Lilley's eq.

Source Spectrum Calculations

- Mean-square sound pressure autocorrelation in the far field due to a finite volume of turbulence (in absence of convection and fluid shielding)

$$\overline{p^2}(R,\theta,\phi) = \frac{R_i R_j R_k R_\ell}{16\pi^2 C_a^4 R^6} \int_{\vec{y}} \int_{\vec{\xi}} \frac{\partial^4}{\partial \tau^4} \overline{(\rho V_i V_j)(\rho' V_k' V_\ell')} d\vec{\xi} d\vec{y},$$



$$\overline{p^2_\omega} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overline{p^2} e^{i\omega\tau} d\tau$$

SOURCE SPECTRUM CALCULATIONS

- Fourth-order velocity correlation tensor

$$S_{ijk\ell} = \overline{V_i V_j V_k' V_\ell} = \int_{-\infty}^{+\infty} (V_i V_j)(V_k' V_\ell')dt$$

- Source strength (Quasi-incompressible turbulence)

$$I_{ijk\ell} = \rho^2 \int_{\vec{\xi}} \frac{\partial^4}{\partial \tau^4} S_{ijk\ell} d\vec{\xi}$$

- Reduction in order of correlation tensor (Ribner)

$$S_{ijk\ell} = S_{ik}S_{j\ell} + S_{i\ell}S_{jk} + S_{ij}S_{k\ell}$$

$$S_{ij}(\tau, \vec{\xi}) = \int_{-\infty}^{+\infty} V_i V_j' dt$$

- Separable second-order tensors

$$S_{ij}(\tau, \vec{\xi}) = R_{ij}(\vec{\xi})G(\tau)$$

- Isotropic turbulence model of Batchelor

$$R_{ij}(\vec{\xi}) = Te^{-\pi(\xi/L_x)^2} \times \left\{[1 - \pi(\xi/L_x)^2]\delta_{ij} + \pi\xi_i\xi_j/L_x^2\right\}$$

$$T = \frac{1}{3}\overline{V_i V_i}, \qquad \xi^2 = \xi_1^2 + \xi_2^2 + \xi_3^2$$

- Gaussian correlation time delay

$$G(\tau) = e^{-(\tau/\tau_o)^2}$$

- Source spectrum component

$$I_{1111}(\Omega) \sim \rho^2 k^{\frac{7}{2}} (\Omega\tau_o)^4 e^{\frac{-(\Omega\tau_o)^2}{8}}$$

$$L_x \sim \frac{k^{3/2}}{\epsilon}, \qquad \tau_o \sim \frac{L_x}{\sqrt{k}}, \qquad k = \frac{1}{2}\overline{V_i V_i}$$

SOURCE SPECTRUM CALCULATIONS

- Characteristic time delay of correlation

$$\tau_o \sim \frac{1}{(\partial U/\partial r)} \quad or \quad \tau_o \sim \frac{k}{\epsilon}$$

- Doppler shifted frequency

$$\Omega = 2\pi f \overline{C}, \qquad M_c = .5M + \beta_c M_j$$

$$\overline{C} = \sqrt{(1 - M_c cos\theta)^2 + (\alpha_c k^{.5}/C_\infty)^2}$$

The proportionality constant in finding τ_o and the convection constants α_c and β_c are determined empirically.

SOUND/FLOW INTERACTION

- Mean square pressure in the far-field

$$\overline{p^2}(R, \theta, \Omega) = \int_{\vec{y}} \Lambda(a_{xx} + 4a_{xy} + 2a_{yy} + 2a_{yz}) d\vec{y}$$

- Source term

$$\Lambda \sim \frac{(\frac{\rho_\infty}{\rho})^2 I(\Omega)}{(4\pi R C_\infty C)^2 (1 - M cos\theta)^2 (1 - M_c cos\theta)^2}$$

- Shielding function

$$g^2(r) = \frac{(1 - M cos\theta)^2 (\frac{C_\infty}{C})^2 - cos^2\theta}{(1 - M_c cos\theta)^2}$$

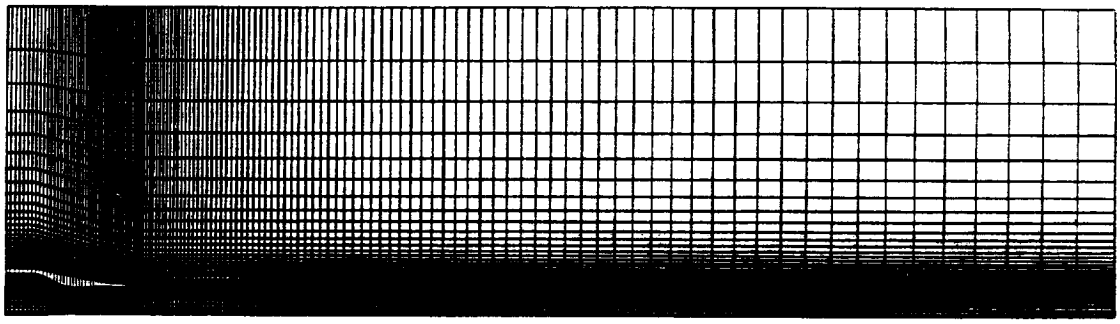
$$M(r) = U(r)/C_\infty, \qquad M_c = U_c/C_\infty$$

Directivity factors $a_{xx} \ldots$ are functions of the Shielding factor $g^2(r)$.

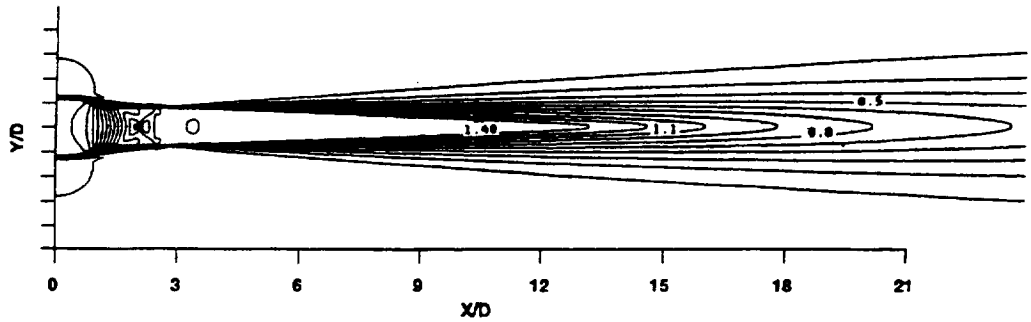
Geometry and Computational Grid

Design Parmaters for C-D Nozzle

Throat diameter	5.1 in.
Exit diameter	5.395 in.
Distance from throat to exit	5.525 in.
Exit velocity	2409 fps
Ambient velocity	400 fps
Pressure ratio	3.121
Stagnation temperature	1716 °R

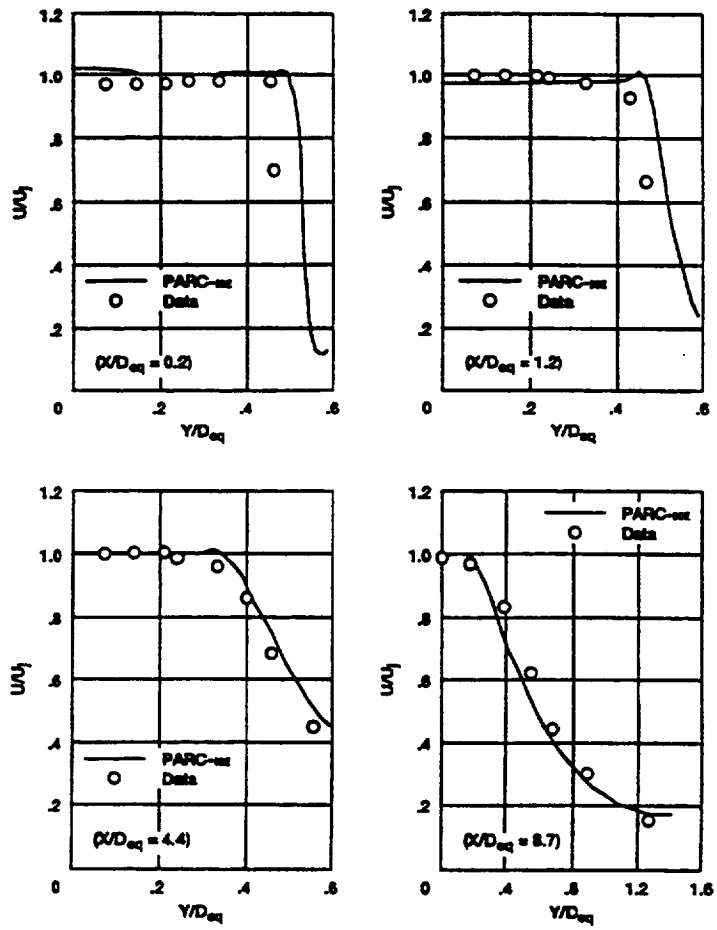


MACH NUMBER



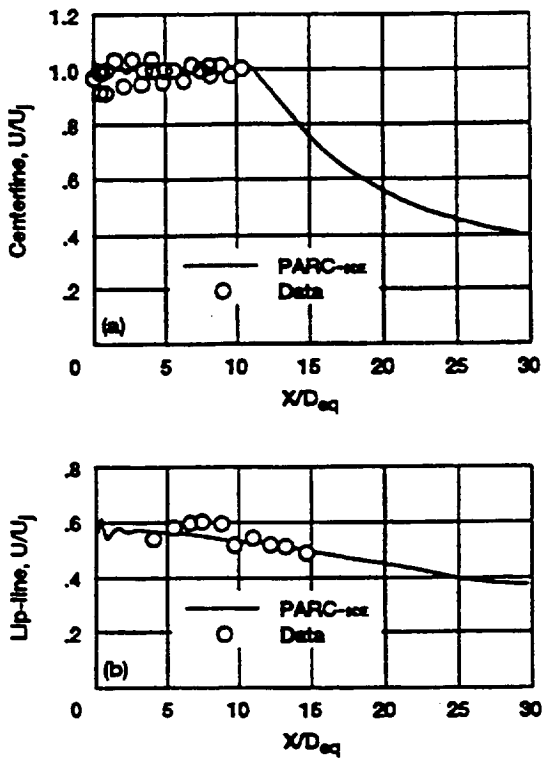
Mach number contour plot for the convergent-divergent nozzle at the design condition (NPR = 3.12), using PARC code with a k-ϵ turbulence model. The upstream conditions are specified at three diameters within the nozzle. The flowpath for this nozzle has been designed to obtain an isentropic, uniform and parallel flow at the exit for the design Mach number of 1.4.

COMPARISON OF VELOCITY PROFILE WITH DATA

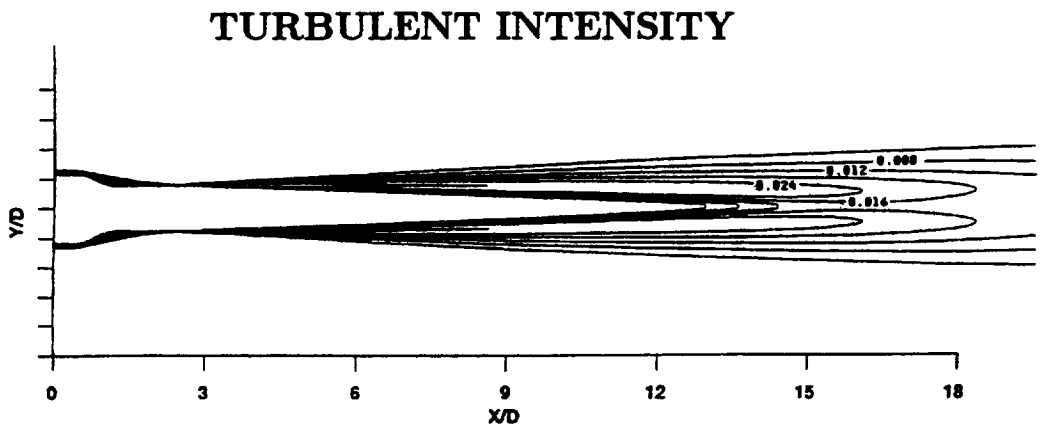


Comparison of velocity profiles with data at four different axial locations. The axial and radial dimensions are normalized with respect to the jet throat diameter D_{eq}.

VELOCITY PROFILE

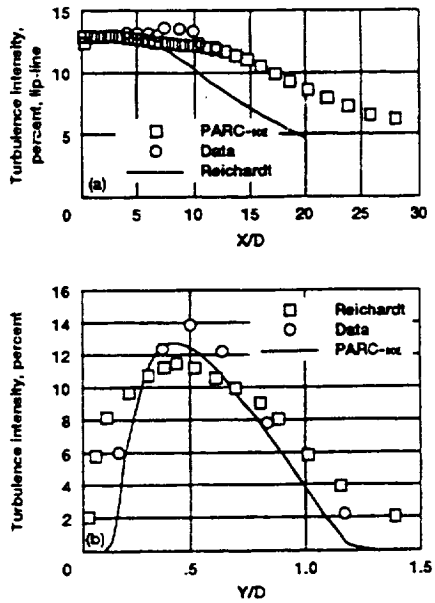


Comparison of velocity profiles with data on the (a) centerline (b) lip-line.



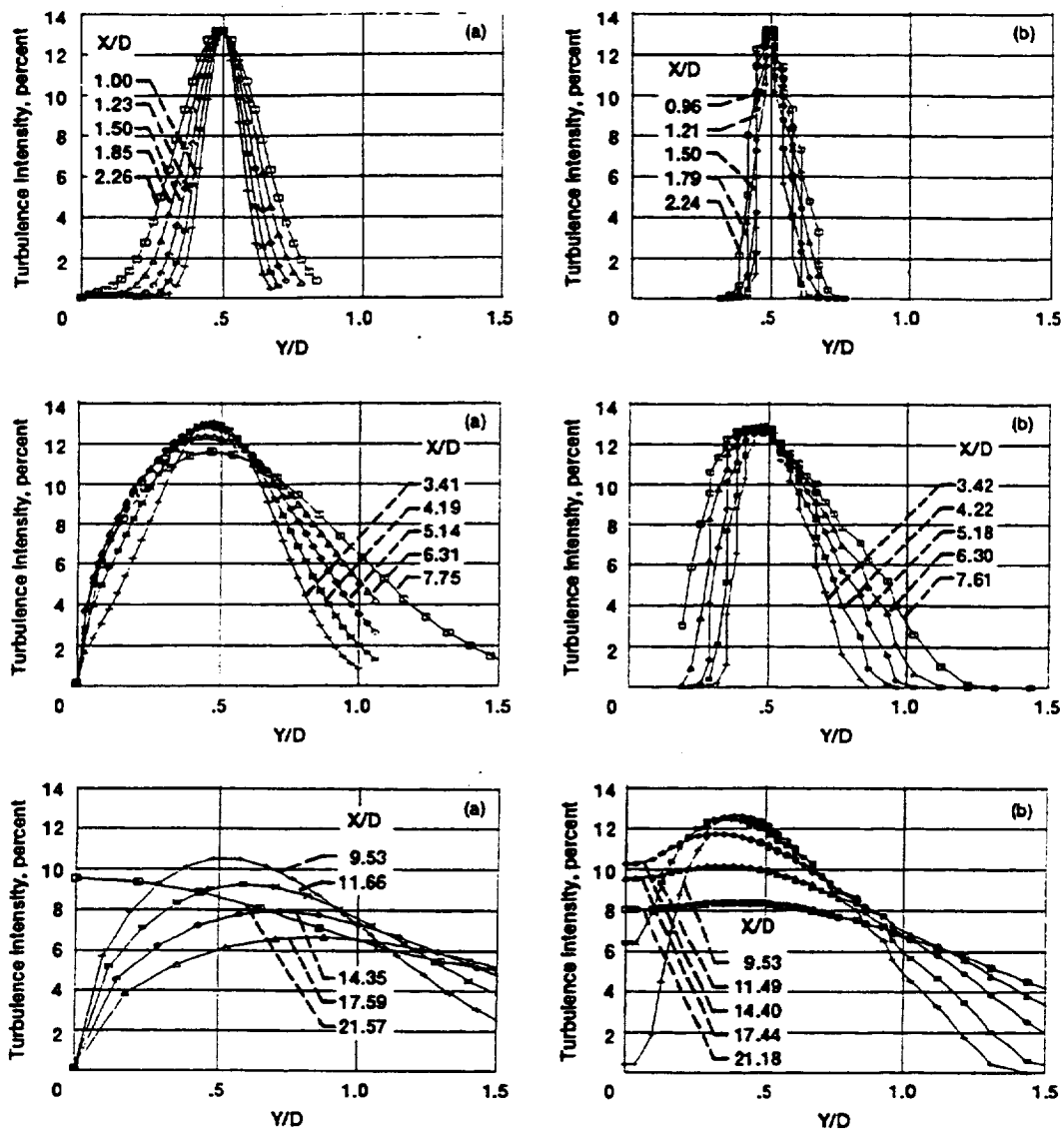
Turbulent intensity contour plot. The contour levels are normalized with respect to square of ambient sound speed.

COMPARISON OF PARC TURBULENT INTENSITY WITH DATA



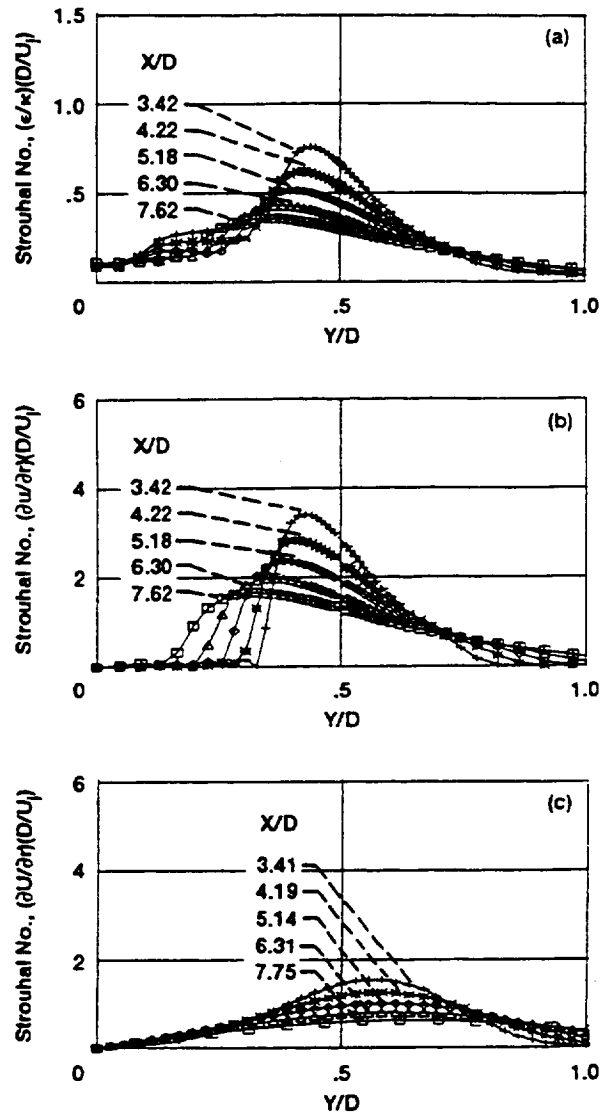
Comparison of PARC turbulent intensity profiles with data and predictions due to Reichardt's theory on the (a) lip-line (b) X/D = 8.21. Radial distance Y is measured from the centerline and all percentages are based on the jet exit velocity U_j. Figure (a) shows that a maximum level of 13% is predicted by both prediction methods although the Reichardt's theory shows a much faster decay along the lip-line. The agreement between PARC and data is reasonably acceptable.

COMPARISON OF PARC TURBULENT INTENSITY WITH REICHARDT'S SOLUTION



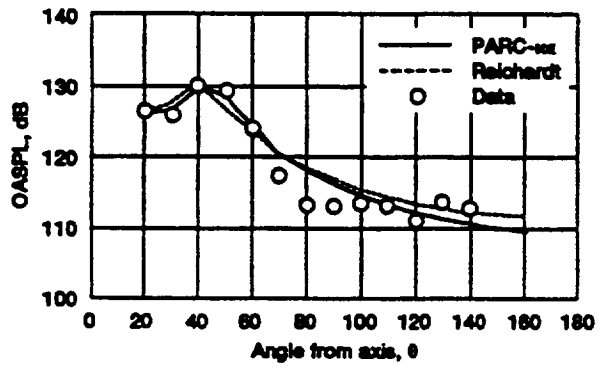
Comparison of turbulent intensity between (a) Reichardt's theory (b) PARC-kϵ. The radial profiles of turbulence can be compared at various axial locations. Figures (b) show that the centerline value of turbulent intensity peaks at X/D = 14.4 which is nearly twice the length of the potential core for this jet and decays farther along the jet. This is in agreement with experimental observations. Reichardt's model, on the other hand, fails to properly predict radial profiles of turbulence farther downstream of the potential core.

STROUHAL NUMBER



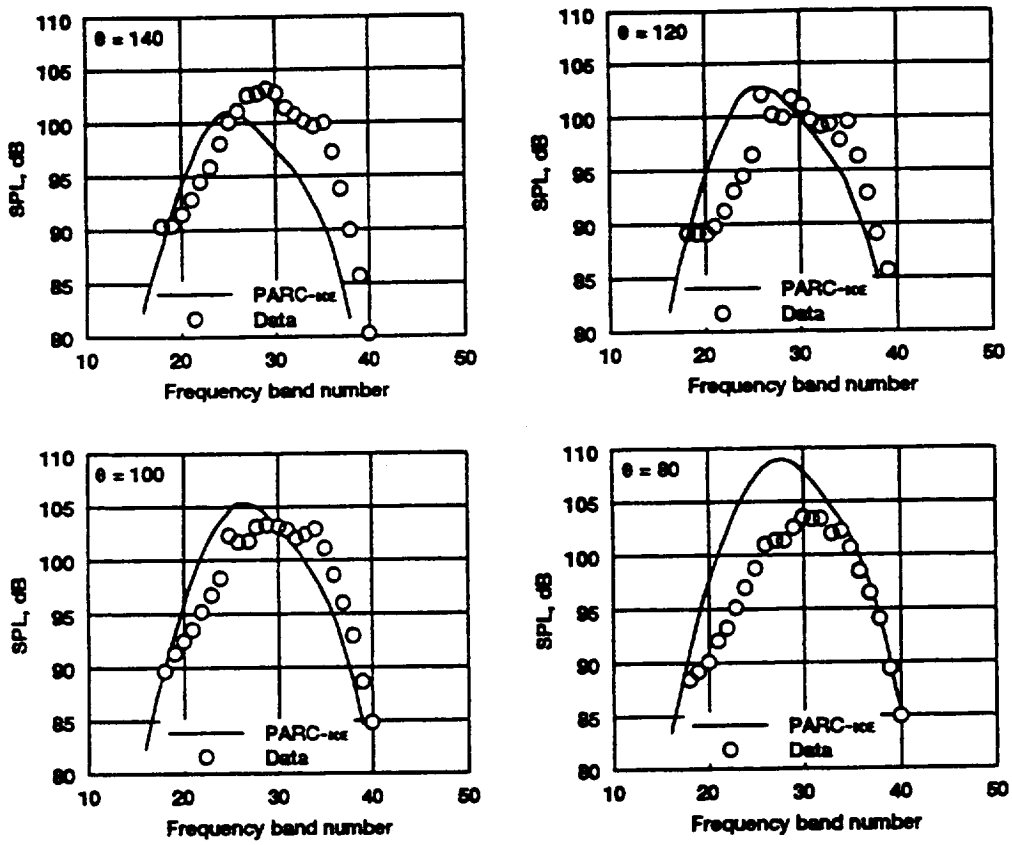
The Strouhal number based on the inverse of the characteristic time delay of correlation nondimensionalized with respect to jet exit velocity and diameter. Figures (a) and (b) are concluded from PARC results using the two definitions ϵ/k and $\partial U/\partial r$ respectively. They show that, outside a proportionality constant, similar results can be obtained for the correlation time factor as a function of source location. Figure (c) is based on Reichardt's aerodynamic predictions and should be compared with Figure (b).

COMPARISON OF OVERALL SOUND PRESSURE LEVEL DIRECTIVITY WITH DATA



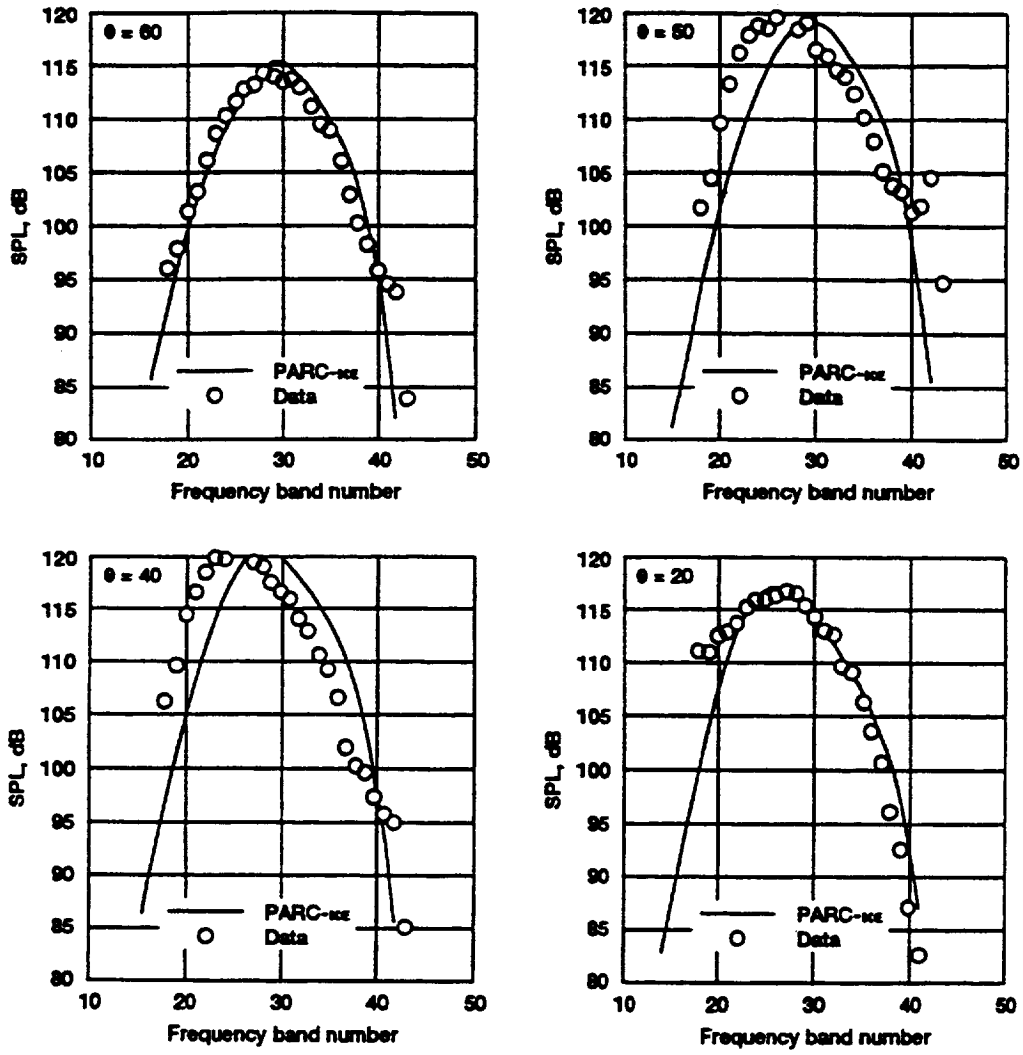
The overall sound pressure level directivity (OASPL) as estimated from PARC-kϵ is compared with data and predictions of Reichardt aerodynamic theory (on a 40 foot radius). The characteristic time delay τ_o is obtained from $1/\tau_o = 2(\epsilon/k)$. The convection constants α_c and β_c are determined empirically.

COMPARISON OF SPECTRAL COMPONENTS OF NOISE WITH DATA



Comparison of noise spectra with data (based on 1/3 octave center frequency) at various observation angles. Band number 24 corresponds to 1 kHz.

COMPARISON OF SPECTRAL COMPONENTS OF NOISE WITH DATA



Comparison of noise spectra with data (based on 1/3 octave center frequency) at various observation angles. Band number 24 corresponds to 1 kHz.

SUMMARY

- Source Strength has been successfully predicted using PARC code with a k–ϵ turbulence model
- The limitation on aerodynamic grid selection has been removed by adopting a two-stage aerodynamic and acoustic algorithm
- The time-delay of correlation was calculated directly from kinetic energy of turbulence and its dissipation rate
- Constants used in supersonic convection factor need to be determined empirically
- The SPL directivity and spectra demonstrate good agreement with data especially at angles where mixing noise is dominant