URC97070

Steady-State Solution of a Flexible Wing

Reza Karkehabadi[1]
Suresh Chandra[2]
Ramesh Krishnamurthy[3]
NASA Center for Aerospace Research
North Carolina A&T State University
Greensboro, NC 27411

A fluid-structure interaction code, ENSAERO, has been used to compute the aerodynamic loads on a swept-tapered wing. The code has the capability of using Euler or Navier-Stokes equations. Both options have been used and compared in the present paper. In the calculation of the steady-state solution, we are interested in knowing how the flexibility of the wing influences the lift coefficients. If the results of a flexible wing are not affected by the flexibility of the wing significantly, one could consider the wing to be rigid and reduce the problem from fluid-structure interaction to a fluid problem.

INTRODUCTION

With the advent of more powerful computers and more efficient algorithms, there has been significant advances in both computational fluid dynamics (CFD) and computational structural dynamics (CSD). These advances have played a major role and contributed significantly to the area of aeroelasticity. To increase the efficiency of aircrafts, particularly for high speed aircrafts, reducing the weight plays a major role and in doing so, this leads to a more flexible structure. Aeroelasticity has a major role in the design of aircrafts. The interaction of the flow with flexible structure could limit the performance of an aircraft and it also can cause dangerous situations. For example, due to the presence and movement of shock waves in the transonic range, undesirable aeroelastic behavior might occur. Also, a highly swept wing of an aircraft might experience vortex-induced aeroelastic oscillations [1].

There have been significant advances in the area of composite materials. Composite materials technology provides structural designers with a capability to specify many of the stiffness properties of modern aircraft structures. A structure can be designed in such a way that the deformation of the structure is prescribed by selecting the material and orienting the composite piles. This technology provides various possibilities for improving the aerodynamic performance of an aircraft under different loading conditions. To be able to solve fluid and structural equations simultaneously, helps in achieving these improvements.

Experimental tests, for aeroelastic wings, requires wind-tunnel experiments that are highly expensive. The experimental tests cannot be ruled out and it is necessary to be complemented with the numerical solutions. In order to calculate the aeroelastic response of a structure, the fluid and structural equations have to be solved simultaneously. NASA Ames Research Center has developed a code, ENSAERO, that is able to accurately couple the Euler and Navier-Stokes equations with the structural equations. This code calculates the aeroelastic response by simultaneously integrating the Euler/Navier-Stokes equations and structural equations of motion. The fluid equations are solved by using finite-difference technique and the structural equations are solved using finite-element method. An early version of ENSAERO [2] was applied to an elastic rectangular wing and the results demonstrated the accuracy of the code in predicting the flutter dynamic pressure of the wing. [t should be noted that the Euler equation were used. ENSAERO was extended to solve Navier-Stokes equations [3]. The code has the ability to model moving control surfaces [4]. Also ENSAERO has the option of using Euler or Navier-Stokes equations and it is able to simulate transonic flows on wing-body combination [5]. Since grid generation techniques for aeroelastic calculations involve moving components, ENSAERO has the capability of using moving grids.

In the present paper, ENSAERO is used to compute the aerodynamic loads on a swept-tapered wing. The steady-state results are obtained by considering the wing to be elastic and the results are compared with the one obtained considering the wing to be rigid. Since we are interested in the steady-state solution and using a fluid-structure interaction code is computationally expensive. Is it possible to consider the wing to be rigid? Can we ignore the property of the material in obtaining the steady-state solution? We attempt to answer these questions.

---

[1] Research Associate
[2] Research Professor
[3] Research Associate

AERODYNAMIC EQUATIONS

The compressible N-S equations in Cartesian coordinates can be written as

$$\frac{\partial Q}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\frac{\partial G}{\partial z}=0 \qquad \text{(I)}$$

where Q, E, F, and G are flux vectors given by

$$Q=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho w\\ E_t\end{bmatrix}$$

$$E=\begin{bmatrix}\rho u\\ \rho u^2+P-\tau_{xx}\\ \rho uv-\tau_{xy}\\ \rho uw-\tau_{xz}\\ (E_t+p)u-u\tau_{xx}-v\tau_{xy}-w\tau_{xz}+q_x\end{bmatrix}$$

$$F=\begin{bmatrix}\text{P'}\\ \rho uv-\tau_{xy}\\ \rho v^2+p-\tau_{yy}\\ \rho uw-\tau_{yz}\\ (E_t+p)v-u\tau_{xy}-v\tau_{yy}-w\tau_{yz}+q_y\end{bmatrix}$$

$$G=\begin{bmatrix}\rho w\\ \rho uw-\tau_{xz}\\ \rho vw-\tau_{yz}\\ \rho w^2+p-\tau_{zz}\\ (E_t+p)w-u\tau_{xz}-v\tau_{yz}-w\tau_{zz}+q_z\end{bmatrix}$$

and the shear stresses and heat-flux equations are given as

$$\tau_{xx}=\frac{2}{3}\mu(2\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z})$$

$$\tau_{yy}=\frac{2}{3}\mu(2\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}-\frac{\partial w}{\partial z})$$

$$\tau_{zz}=\frac{2}{3}\mu(2\frac{\partial w}{\partial z}-\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y})$$

$$\tau_{xy}=\tau_{yx}=\mu(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y})$$

$$\tau_{xz}=\tau_{zx}=\mu(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z})$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$q_x = -k\frac{\partial T}{\partial X},\ q_y = -k\frac{\partial T}{\partial Y},\ q_z = -k\frac{\partial T}{\partial Z}.$$

ρ is the fluid density and u, v, w are the fluid velocity in x, y, and z directions. Et is the total energy per unit volume, p is the pressure, μ is the viscosity, and k is the coefficient of thermal conductivity, The governing equation can be transformed "into general curvilinear coordinate where

$$\tau = t$$
$$\xi = \xi(x,y,z,t)$$
$$\eta = \eta(x,y,z,t)$$
$$\zeta = \zeta(x,y,z,t)$$

and the resulting transformed equations can be written in non-dimensional form as

$$\frac{\partial \hat{Q}}{\partial \tau} + \frac{\partial \hat{E}}{\partial \xi} + \frac{\partial \hat{F}}{\partial \eta} + \frac{\partial \hat{G}}{\partial \zeta} = 0 \qquad (2)$$

where ^ indicates the transformed quantities. In order to solve Navier-Stokes equations, extensive computational time and storage is required. For this reason, the reduced equations, known as "thin-layer" or "parabolized" Navier-Stokes equations are being used. In the thin-layer approximation the viscous terms containing derivatives in the parallel direction to the surface of the body are neglected from Navier-Stokes equations. The thin-layer model requires a boundary-layer type coordinate system. Equation 2 simplifies to

$$\partial_\tau \hat{Q} + \partial_\xi \hat{E} + \partial_\eta \hat{F} + \partial_\zeta \hat{G} = \mathrm{Re}^{-1}\,\partial_\zeta \hat{S}$$

where all the viscous terms are contained in S. In order to solve the aerodynamic equations, ENSAERO has capability of both central-difference and upwind scheme [6]. The central difference scheme used is based on the implicit factorization algorithm of Beam and Warming [7] with the modifications by Pulliam and Chaussee [8] for diagonalization.

AEROELASTIC EQUATIONS OF MOTION

The aeroelastic equations of motion for a flexible wing are solved by using the Rayleigh-Ritz method, Using this technique, the aeroelastic displacements at any time are expressed as a function of assumed modes. The number of assumed modes are Finite and the amount of contribution that each mode has on the total motion is derived by Lagrange's equation. For further detail and more information see [2]-[6].

The finite element matrix form of the aeroelastic equations of motion for the elastic wing are given in general form as

$$[M]\{\ddot{q}\} + [G]\{\dot{q}\} + [K]\{q\} = \{F\}$$

where [M], [G], and [K] are the mass, damping, and stiffness matrices, respectively. $\{q\}$ is the displacement vector and $\{F\}$ *is* the aerodynamic loading vector and is computed by solving Euler or Navier-Stokes equation. Dots denote the derivative with respect to time.

RESULTS

A swept-tapered wing, shown in Figure 1, is used in our calculations. The aspect ratio of the wing is 3, the tapered ratio is 0.14, and the leading edge sweep angle is 51.34 degrees. The Mach number is 2.5 and Reynolds number is 5×10^7. As a first step, we consider the wing to be elastic and solve the fluid and structure equations simultaneously by using ENSAERO. Total lift coefficient for this wing as a function of time is shown in Figure 2. The angle of attack is 10 degrees. The figure compares the results obtained using Euler with Navier-Stokes equations. As the figure indicates, the difference in total lift between these two options is not significant.

Figures 3, 4, and 5 compare the sectional lift obtained using Navier-Stokes and Euler equations. The wing is set into motion from the impulsive start until it reaches steady-state. The sectional lift coefficients are shown for different sections on the wing, 28%, *61%*, and 90% from the root of the wing. The lift coefficients oscillate for all cases because the wing is elastic and, in our calculations, we consider the elasticity of the wing. Again, there is not a significant difference in the sectional lift coefficients between Euler and Navier-Stokes equations. As one would expect, the lift coefficients of the section that is closer to the root reaches steady-state faster than the other sections. This is more clear by comparing Figures 3,4, and 5 where the lift coefficients are shown for the different sections of the wing. At time= 94.3 seconds the section 28% away from the root has little oscillation and the one that is 90°/0 away from the root has the most.

In order to obtain the steady-state solution for an elastic wing, we have considered the flexibility of the wing in our calculations. This requires solving fluid and structural equations simultaneously. Calculating the aerodynamic loads on an elastic wing requires more computation and therefore, it is computationally expensive and time consuming. In calculating the steady-sate solution, often the wing is considered to be rigid. We consider the above wing to be rigid and calculate the aerodynamic loads. Again, the angle of attack is 10 degrees. The total lift coefficient on the wing is shown in Figure 6. The solid line is the Navier-Stokes solution and the dashed line is the Euler solution. Comparing the lift coefficients of the rigid wing (Figure 6) with the elastic wing (Figure 2) shows that the steady-state results differ significantly and the difference is about 19%. The sectional lift coefficient for the rigid wing at 28°/0 from the root is shown in Figure 7. The difference with the elastic case is about 13°/0. In Figure 8 the sectional lift coefficient is shown for the section of the wing located at 6 1% from the root. Comparing this figure with Figure 4, one sees that the steady-state lift coefficient is 0.165 for rigid and 0.128 for elastic case. The difference between the rigid and elastic is about 29%.

There is more flexibility near the tip; hence, the difference with the rigid case is highest near the tip. In Figure 9 the sectional lift coefficient is shown for the wing at 90% from the root, which is very close to the tip. Again, the wing is considered to be rigid. Figure 5 demonstrates the sectional lift coefficient for the same section with the same condition except the wing is elastic. The difference between these two are significantly large, 60%. In all cases, the Navier-Stokes solution is given as well as Euler solution. The Figures indicate that the lift coefficients using either options are not being significantly affected.

CONCLUSION

It was shown that the computed values of lift coefficients are not greatly affected, for the configuration we have considered, by using either Navier-Stokes or Euler equations. The elasticity of the wing contributes to the lift coefficients. A more rigid wing has higher lift coefficients. In case of a flexible wing, part of the energy is used to oscillate the wing; hence, the value of the lift decreases with the increase in flexibility of the wing. The section" of the wing that is near the tip has the highest amount of flexibility; therefore, the difference (with the rigid one) is highest in this section. In order to obtain the lift coefficients on a wing one has to consider the elasticity of the wing and in reducing the problem to a rigid wing some accuracy might be lost.

ACKNOWLEDGMENTS

The authors would like to thank NCSC for the computer time that they have provided. Our thanks are extended to NASA for their support under the grant NAGW-2924.

REFERENCES

1. Dobbs, S. K., and Miller, G. D., "Self-Induced Oscillation Wind-Tunnel Test of a Variable Sweep Wing," AIAA Paper 85-0739, April 1985.
2. Guruswamy, G. P., "Unsteady Aerodynamic and Aeroelastic Calculations for Wings Using Euler Equations," AIAA Journal, Vol. 28, 1990, PP. 461-469.
3. Guruswamy, G. P., "Navier-Stokes Computations on Swept-Tapered Wings, Including Flexibility," AIAA Paper 90-1152, Apr. 1990.
4. Obayashi, S., and Guruswamy, G. P., "Navier-Stokes Computations for Oscillating Control Surface," AIAA Paper 92-4431, Aug. 1992.
5. Obayashi, S., and Guruswamy, G. P., and Tu, E. L., "Unsteady Navier-Stokes Computations on a Wing-Body Configuration in Ramp Motions," AIAA Paper 9 l-2865, Aug. 1991.
6. Obayashi, S., and Guruswamy, G. P., and Goorjian, P. M., "Streamwise Upwind Algorithm for Computing Unsteady Transonic Flows Past Oscillating Wings," AIAA Journal, Vol. 29, No. 10, Oct. 1991, PP 1668-1677.
7. Pulliam, T. H., and Chaussee, D. S., "A Diagonal Form of an Implicit Approximate Factorization Algorithm," Journal of Computational Physics, Vol. 39, No. 2, Feb. 1981, pp 347-363.
8. Beam and Warming, R. F., "An Implicit Finite-Difference Algorithm for Hyperbolic Systems in Conservation Law Form," Journal of Computational Physics, Vol. 22, No. 9, Sept. 1976, pp. 87-110.

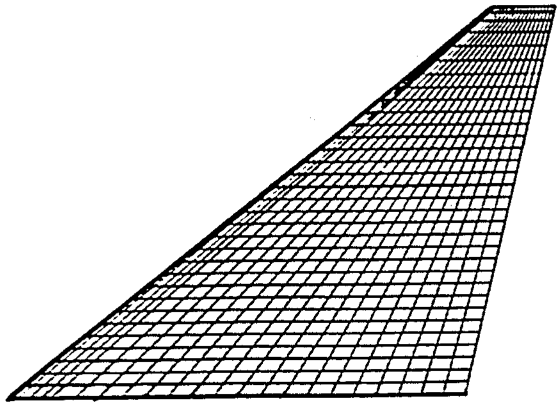

Fig. 1: 2-D view of the wing used in this paper.

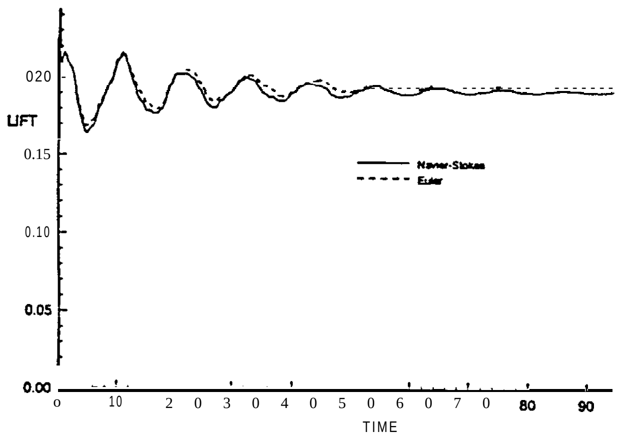

Fig. 2: Total lift coefficients of a flexible wing as a fiction of time.

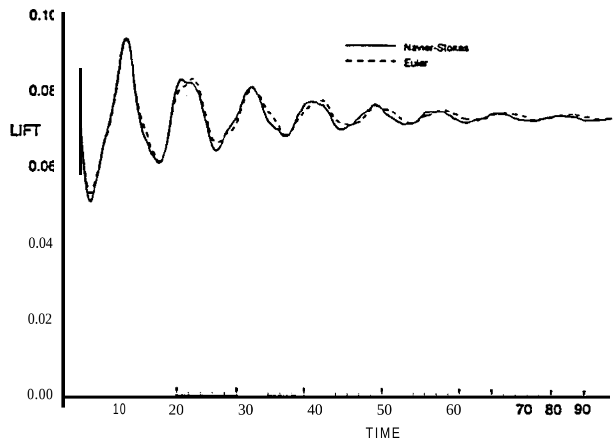

Fig. 3: Lift coefficients of a flexible wing as a function of time for the section 28% from the root.

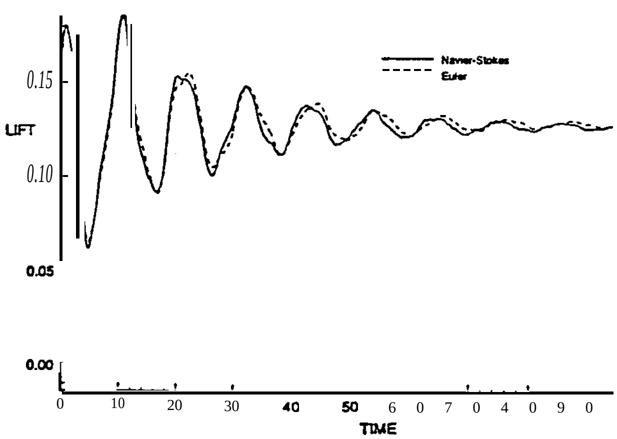

Fig. 4: Lift coefficients of a flexible wing as a function of time for the section 61’%. from the root.

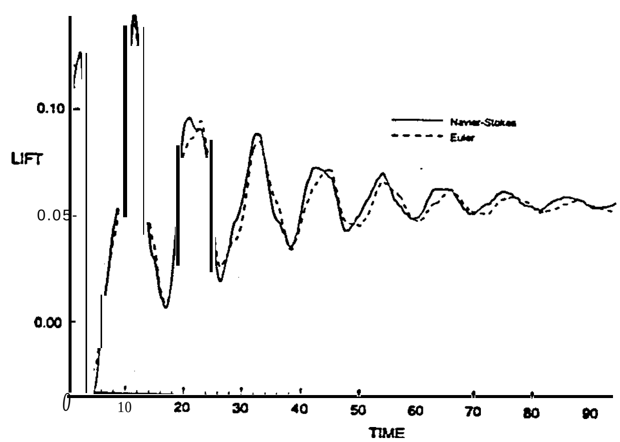

Fig. 5: Lift coefficients of a flexible wing as a function of time for the section 90% from the root.

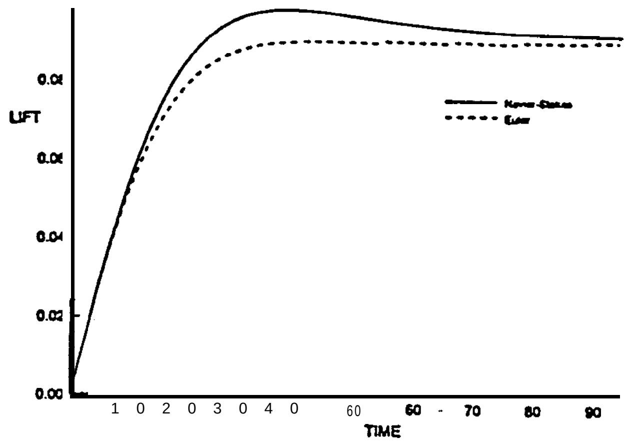

Fig. 6: Total lift coefficients of a rigid wing as a function of time.

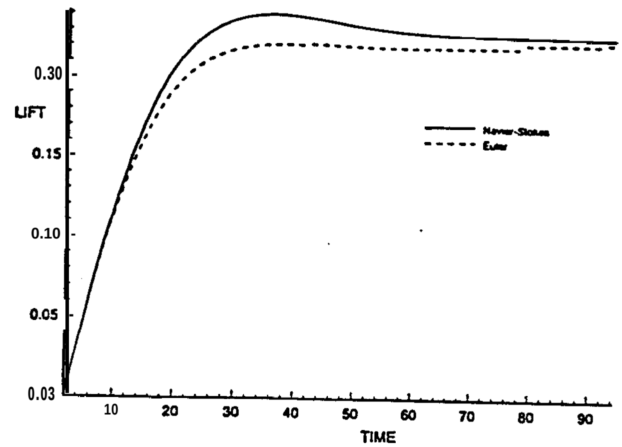

Fig. 7: Lift coefficients of a rigid wing as a function of time for the section 28% from the root.

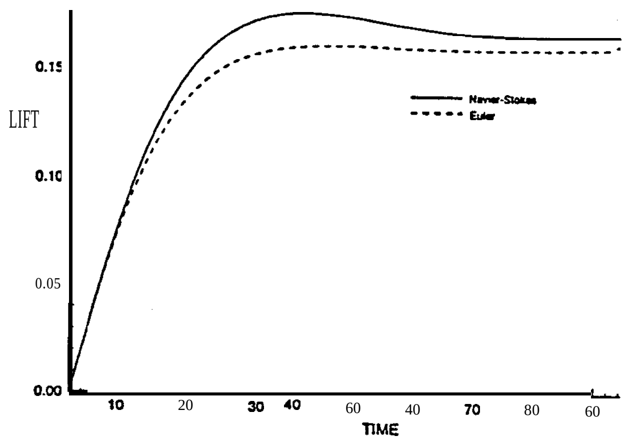

Fig. 8: Lift coefficients of a rigid wing as a function of time for the section 61% from the root.

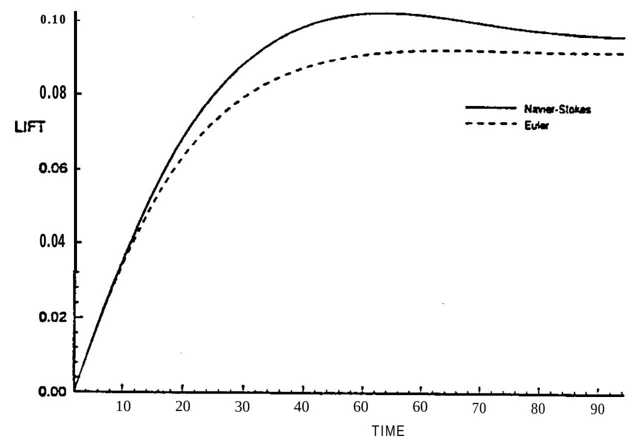

Fig. 9: Lift coefficients of a rigid wing as a function of time for the section 90% from the root.