

# On Decision-Making Among Multiple Rule-Bases in Fuzzy Control Systems

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## Abstract

Intelligent control of complex multi-variable systems can be a challenge for single fuzzy rule-based controllers. This class of problems can often be managed with less difficulty by distributing intelligent decision-making amongst a collection of rule-bases. Such an approach requires that a mechanism be chosen to ensure goal-oriented interaction between the multiple rule-bases. In this paper, a hierarchical rule-based approach is described. Decision-making mechanisms based on generalized concepts from single-rule-based fuzzy control are described. Finally, the effects of different aggregation operators on multi-rule-base decision-making are examined in a navigation control problem for mobile robots.

## 1 Introduction

Many fuzzy controllers proposed in the literature utilize a monolithic rule-base structure. That is, the precepts that govern desired system response are encapsulated as a single collection of *if-then* rules. In most instances, the rule-base is designed to carry out a single control policy or goal. As structure and task constraints are removed from the problem domain, the need for increased system autonomy mandates the development of more sophisticated controllers. Complex intelligent systems must be capable of achieving multiple goals whose priorities may change with time. When employing fuzzy logic, it becomes difficult to formulate monolithic rule-bases which comply with multiple interacting goals, as this requires formulation of a large and complex set of fuzzy rules. In this situation a potential limitation to the utility of the monolithic fuzzy controller becomes apparent. Since the size of complete monolithic rule-bases increases exponentially with the number of input variables [1], multi-input systems can potentially suffer degradations in speed of response. Alternatively, controllers can be designed to realize a number of distributed special-purpose capabilities that can be integrated to achieve different control objectives. This can be done by organizing fuzzy systems into hierarchical rule structures. It has been demonstrated that such rule structures can be employed to overcome the limitation of monolithic structures by reducing the rate of rule increase to linear or piecewise-linear [1]–[2]. Hierarchical rule structures have also been proposed for controlling systems with interacting goals [3].

This paper describes a fuzzy control architecture for complex systems in which distributed intelligence can be represented as hierarchical or decentralized structures, e.g. autonomous mobile vehicles, multi-agent systems, electric power systems, and other large-scale systems. Decision-making mechanisms based on generalized concepts from monolithic fuzzy control are described. The effects of different aggregation operators on multi-rule-base decision-making are examined in an example application to a motion control

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problem for mobile robots. These include the following t-conorms: bounded sum, arithmetic maximum, probabilistic sum, and the Sugeno  $S_\lambda$  family [4].

## 2 Hierarchical Distributed Fuzzy Control

Fuzzy controllers are intelligent control systems that smoothly interpolate between rules, i.e. rules fire to continuous degrees and the multiple resultant actions are combined into an interpolated result. The underlying theory is based on fuzzy sets [5] which are represented by a mathematical formulation known as the membership function. This function gives a degree or grade of membership within a fuzzy set. Over a given universe of discourse  $X$ , the membership function of a fuzzy set  $\tilde{A}$ , denoted by  $PA(z)$ , maps the elements  $x \in X$  into a numerical value in the unit interval, i.e.

$$\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]. \quad (1)$$

Within this framework, a membership value of zero corresponds to an element which is definitely not a member of the fuzzy set, while a value of one corresponds to the case where an element is definitely a member of the set. Partial membership is indicated by values between 0 and 1, continuous. Implementation of a fuzzy controller requires assigning membership functions for both inputs and outputs, thus the membership values are actually measures of degree of causality in an input-output mapping. Inputs to a fuzzy controller are usually measured variables, associated with the state of the controlled plant, that are fuzzified (assigned membership values) before being processed by an inference engine. The heart of the controller inference engine is a *rule-base* of if-then rules whose antecedents and consequences are made up of linguistic variables and associated fuzzy membership functions. Consequences from different rules are numerically aggregated by fuzzy set union operation and are then collapsed (defuzzified) to yield a single real number output that serves as the control signal for the plant.

In our hierarchical approach, each rule-base is encoded with a distinct control policy governed by fuzzy inference. Thus, each rule-base is similar to the conventional fuzzy controller in that it performs an inference mapping from some input space to some output space. If  $X$  and  $Y$  are input and output universes of discourse of a behavior with a rule-base of size  $n$ , the usual fuzzy if-then rule takes the following form

$$IF \ x \ is \ \tilde{A}_i \ THEN \ y \ is \ \tilde{B}_i \quad (2)$$

where  $x$  and  $y$  represent input and output fuzzy linguistic variables, respectively, and  $\tilde{A}_i$  and  $\tilde{B}_i$  ( $i = 1..n$ ) are fuzzy subsets representing linguistic values of  $x$  and  $y$ . Typically,  $x$  refers to sensory data or goal information and  $y$  to control outputs (inputs to the controlled system). In general, the rule antecedent consisting of the proposition “ $x$  is  $\tilde{A}_i$ ” could be replaced by a compound fuzzy proposition consisting of a conjunction (and/or disjunction) of similar propositions. Similarly, the rule consequent “ $y$  is  $\tilde{B}_i$ ” could include additional FLC output propositions.

Overall system behavior is decomposed into a bottom-up hierarchy of increased complexity in which activity at a given level is dependent upon activities at the level(s) below. A collection of primitives typically resides at the lowest level which we refer to as the primitive level. These are simple, self-contained sets of rules that serve a single purpose. They perform nonlinear mappings from different subsets of the sensor suite to subsets of control actions. Alone, each primitive rule-base would be insufficient for achieving complex tasks. Primitive rule-bases are building blocks for more intelligent and higher-level competence. That is, they can be combined synergistically to produce composite capabilities suitable for accomplishing goal-directed operations.

Rule-bases at different levels of the hierarchy are generally interconnected. For example, consider the simple two-level structure illustrated in Figure 1 consisting of primitive rule-bases  $R_{1a}$  and  $R_{1b}$ , and composite rule-base  $R_{2a}$ . The interconnection of  $R_{2a}$  with the primitives implies that it can be decomposed as a function of the primitives such that the interaction of  $R_{1a}$  and  $R_{1b}$  produce the desired task-oriented function of  $R_{2a}$ .

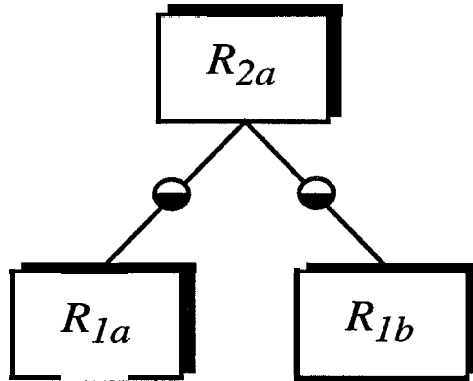


Figure 1: Portion of a rule-base hierarchy.

Note that the overall hierarchy can consist of additional fuzzy rule-bases, the number of which are indicative of the problem complexity. The circles on the adjoining lines in the figure represent weights of the associated primitive rule-bases that fluctuate according to their applicability in the current situation. In general, these weights can be threshold activated. For a given system, the rule-bases and the associated hierarchical arrangement are arrived at following a subjective analysis of the problem and the task domain.

### 3 Multi-Rule-base Decision-Making

Consensus among multiple rule-bases is achieved using a weighted decision-making strategy embodied in a concept called the *degree of applicability* (DOA). The DOA is a measure of the instantaneous level of activation of a set of rules. The fuzzy rules that make up composite rule-bases are formulated such that the DOA,  $\alpha_j \in [0, 1]$ , of primitive rule-base  $j$  is specified in the consequent of *applicability rules* of the form

$$IF \mathbf{x} \text{ is } \tilde{A}_i \text{ THEN } \alpha_j \text{ is } \tilde{D}_i \quad (3)$$

where  $\tilde{A}_i$  is defined as in (1).  $\tilde{D}_i$  is a fuzzy set specifying the linguistic value (e.g. “high”) of  $\alpha_j$  for the situation prevailing during the current control cycle. This feature allows competence of certain rule-bases to influence the overall system response to a greater or lesser degree depending on the current situation. It serves as a form of adaptation since it causes the control policy to dynamically change in response to goals, sensory input, and internal state. Thus, higher-level rule-bases consist of meta-rules that provide forms of inhibition and dominance. Rule-bases with maximal applicability ( $\alpha_{max} \leq 1$ ) can be said to dominate, while those with partial applicability ( $0 < \alpha < \alpha_{max}$ ) can be said to be inhibited. The hierarchy facilitates decomposition of complex problems as well as run-time efficiency by avoiding the need to evaluate rules from rule-bases that do not currently apply. As a result, the number of rules consulted during each control cycle varies dynamically as governed by the DOAs of the rule-bases involved.

Coordination and conflict resolution are achieved within the framework of fuzzy logic theory –via operations on fuzzy sets [8]. Fuzzy rules of each applicable primitive behavior are processed yielding respective output fuzzy sets. These fuzzy sets are equivalent to the result produced by rule-base evaluation in monolithic fuzzy controllers *before* applying the defuzzification operator. Following consultation of applicable rule-bases, each fuzzy output is weighted (multiplied) by its corresponding DOA, thus effecting its activation to the level prescribed by the higher-level rule-base. The resulting fuzzy sets are then aggregated using an appropriate fuzzy set union operator, and defuzzified to yield a crisp output that is representative of the intended coordination. Since control recommendations from each applicable rule-base are considered in the final decision, the resultant control action can be thought of as a consensus of recommendations offered by multiple experts. This coordination procedure is a generalization of the idea of rule weighting in a monolithic rule base to rule-base weighting among multiple rule-bases. In a similar manner, we use fuzzy set theory to

generalize rule conflict resolution in monolithic rule bases, for resolving conflicts among multiple conflicting rule-bases.

### 3.1 Aggregation of multi-rule-base outputs

One area of flexibility in this approach is in choosing an appropriate operator for consolidating the multiple control recommendations. We focus on the t-conorm, or generalized union operator of fuzzy set theory. Recall that primitive rule-base outputs are fuzzy sets, and an aggregation across rule-bases must be performed to produce an overall control output. The chosen t-conorm may be different than that used to aggregate the individual *rule* outputs in each rule-base. As the selection of the t-conorm used for rule-base aggregation dictates how anything approaching a consensus will be made, available options should be considered.

We consider the following t-conorms: bounded sum, arithmetic maximum, probabilistic sum, and the Sugeno  $S_\lambda$  family. Their definitions follow, where  $\mu_{\tilde{R}}$  and  $\mu_{\tilde{S}}$  are membership values describing the fuzzy sets,  $R$  and  $\tilde{S}$ , which are undergoing a union operation.  $U(\mu_{\tilde{R}}, \mu_{\tilde{S}})$  denotes the t-conorm, or fuzzy union operator.

Bounded sum:

$$U(\mu_{\tilde{R}}, \mu_{\tilde{S}}) = \min(1, \mu_{\tilde{R}} + \mu_{\tilde{S}}) \quad (4)$$

Maximum:

$$U(\mu_{\tilde{R}}, \mu_{\tilde{S}}) = \max(\mu_{\tilde{R}}, \mu_{\tilde{S}}) \quad (5)$$

Probabilistic sum:

$$U(\mu_{\tilde{R}}, \mu_{\tilde{S}}) = \mu_{\tilde{R}} + \mu_{\tilde{S}} - \mu_{\tilde{R}}\mu_{\tilde{S}} \quad (6)$$

Sugeno  $S_\lambda$  family:

$$U(\mu_{\tilde{R}}, \mu_{\tilde{S}}, \lambda) = \min(1, \mu_{\tilde{R}} + \mu_{\tilde{S}} + \lambda\mu_{\tilde{R}}\mu_{\tilde{S}}) ; \lambda \geq -1 \quad (7)$$

Note that the bounded sum is a special case of the Sugeno family of t-conorms, namely  $S_0$ . Also, the probabilistic sum is very similar to  $S_{-1}$ . The  $S_\lambda$  family is one of a variety of parameterized families of aggregation operators; others can be found in [6, 7]. The selection of the above set of t-conorms was based on their computational simplicity (i.e. no division or exponent operations required),

## 4 Example

For illustration, we apply the hierarchy of distributed rule-bases to a mobile robot navigation example. The task is collision-free and goal-directed navigation from some location to a designated goal inside a hypothetical warehouse environment. A hierarchy of rule-bases for this indoor navigation is arranged as in Figure 2 which implies that goal-directed navigation can be decomposed as a function of goal-*seek* and route-*follow*. In the context of mobile robot fuzzy control, these rule-bases are referred to as fuzzy behaviors [8]. These behaviors can be further decomposed into the primitives shown. *Avoid-collision* and *wall-follow* are self-explanatory. The doorway behavior guides a robot through narrow passageways in walls; *go-to-xy* directs motion along a straight line trajectory to a particular location. In order to demonstrate the decision-making aspects of the controller in the simplest manner possible consider only the composite behavior — goal-*seek*. As illustrated in Figure 2, its effect arises from synergistic interaction between *go-to-xy* and *avoid-collision*. The simulated mobile robot is modeled after LOBOt, a custom-built base with a 2-wheel differential drive and two stabilizing casters. It is octagonal in shape, 75 cm tall and 60 cm in width. The output of the primitive behaviors are right and left wheel speeds; the inputs of the hierarchy are the goal location and subsets of sensor readings. The robot's sensor suite includes optical encoders on each driven wheel and 16 ultrasonic transducers arranged primarily on the front, sides, and forward-facing obliques. The initial state of robot is at  $(x \ y \ \theta)^T = (11.7m \ 12.3m \ \frac{\pi}{2}rad)^T$ . The goal is located at  $(1.5m, 1m)$ .

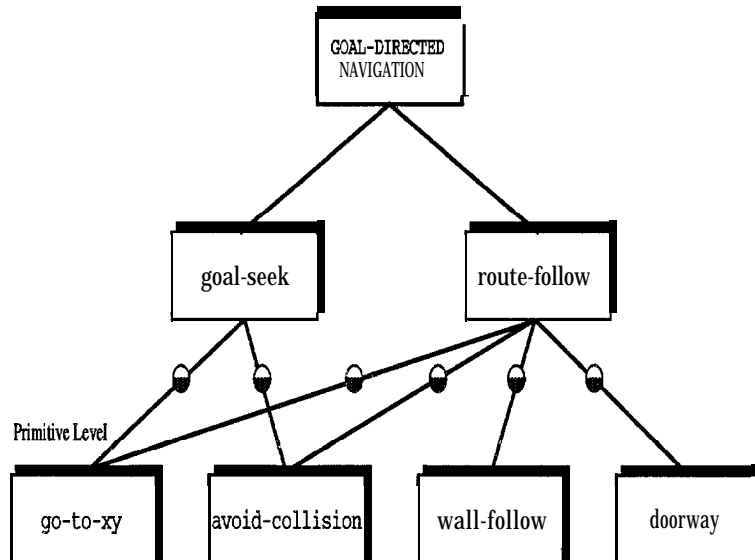


Figure 2: Hierarchical decomposition of mobile robot behavior.

We ran the simulated navigation using each of the t-conorms defined in Section 3.1 to examine the relative impact that each has on motion decisions made during the run. That is, the fuzzy outputs of `go-to-xy` and `avoid-collision` were aggregated using (2)-(5). The resulting path taken by the robot using the bounded sum t-conorm is shown in Figure 3a; dimensions are meters. The robot simultaneously achieves the goals of reaching the target location and avoiding collisions. The paths resulting from using maximum and probabilistic sum as t-conorms were very similar to the bounded sum case. However, the decisions made as a result of applying the  $S_\lambda$  family for  $\lambda \geq 1$  were clearly different as revealed by the alternative path shown in Figure 3b for  $\lambda = 1$ . In this case, the ensemble of control decisions made over the course of the run have led to a more direct path to the goal. The results were similar for  $\lambda > 1$ . Thus, possible variations in system behavior can be determined through examination of the effects of t-conorm selection on multi-rule-base decision-making. Of course, the selection of an appropriate t-conorm will be system dependent and based on desired system response.

## 5 Conclusion

Decision-making in distributed intelligent systems is facilitated using a collection of fuzzy rule-based decision systems and controllers. Consensus among multiple rule-bases is achieved using a weighted decision-making strategy based on degrees of applicability  $y$  associated with each rule-base. When conditions for activation of a single rule-base (or several) are satisfied, there is no need to consult rule-bases that do not apply. The approach is suitable for fuzzy control of complex, systems that can be represented as hierarchical or decentralized structures. By generalizing decision-making concepts of monolithic fuzzy controllers, it is possible to coordinate multiple distributed rule-bases in a single multi-level control system. When the proposed decision-making mechanisms are employed, it is beneficial to examine different aggregation operators over rule-base outputs to determine the most appropriate operator for the application.

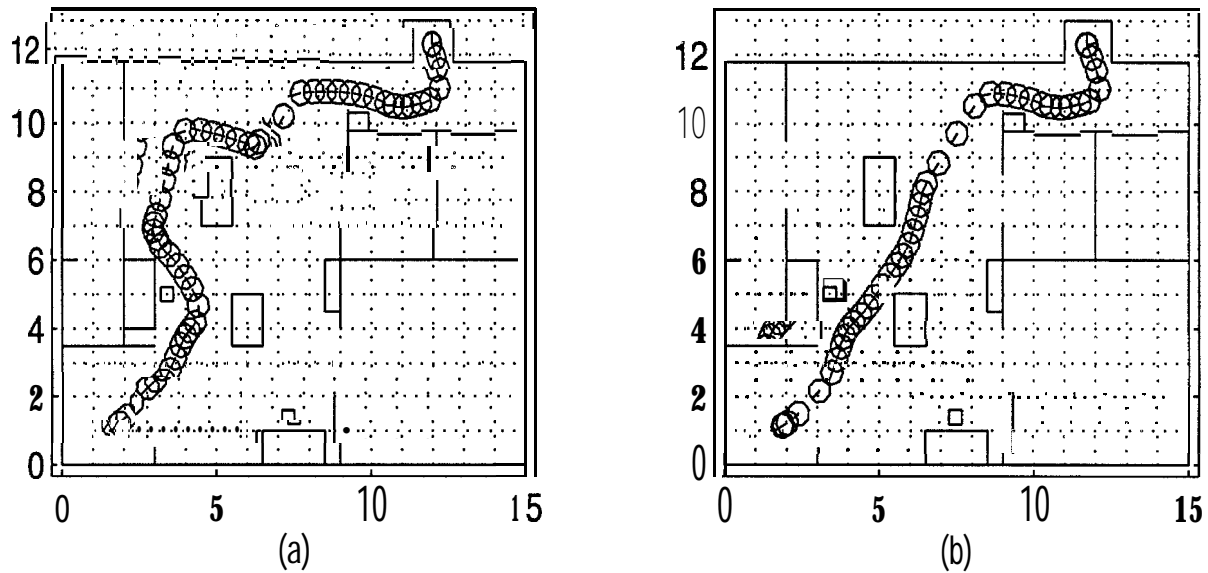


Figure 3: Simulated navigation control using different t-conorms.

## References

- [1] Raju, G. V. S., J. Zhou and R. A. Kisner, "Hierarchical Fuzzy Control", *International Journal of Control*, 54 (5), pp. 1201-1216, 1991.
- [2] Jamshidi, M. *Large-Scale Systems: Modeling, Control, and Fuzzy Logic*, Prentice Hall PTR, Upper Saddle River, NJ, 1997.
- [3] Berenji, H.R. et al, "A Hierarchical Approach to Designing Approximate Reasoning-Based Controllers for Dynamic Physical Systems", *6th Conference on Uncertainty in AI*, pp. 362-369, 1990.
- [4] Driankov, D., H. Hellendoorn and M. Reinfrank *An Introduction to Fuzzy Control*, Springer-Verlag, Berlin, 1993.
- [5] Zadeh, L.A. "Fuzzy sets", *Information and Control* 12, pp. 338-353, 1965.
- [6] Klir, G.J. and T.A. Folger *Fuzzy Sets, Uncertainty and Information*, Prentice Hall, Englewood Cliffs, NJ, 1988.
- [7] Yager, R.R. and D.P. Filev *Essentials of Fuzzy Modeling and Control*, Wiley & Sons, New York, 1994.
- [8] Tunstel, E. "Coordination of Distributed Fuzzy Behaviors in Mobile Robot Control", *IEEE International Conference on Systems, Man and Cybernetics*, Vancouver, BC, pp. 4009-4014, October 1995.