PARTICLE SEGREGATION IN COLLISIONAL SHEARING FLOWS.

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INTRODUCTION

The size segregation of flowing or shaken grains is a commonly observed phenomenon in industrial processes and in nature. In many industrial processes a homogeneous aggregate is desired; in these, size segregation is undesirable. However, in the mining industry, for example, segregation by size is exploited in some processing operations. Also, grain segregation is useful in understanding the origin of natural grain deposits; here it provides an indication of whether an aggregate of grains was deposited dry, with the larger grains above, or under water, with the larger grains below [1].

In systems that do not involve much agitation of the grains, several mechanisms that involve gravity have been identified as leading to such segregation. These include the preferential downward percolation of smaller particles in relatively slow inclined shear flows [2], the upward frictional ratchetting of large particles [3], and the preferential filling of space beneath larger particles by smaller particles in a system that is occasionally shaken [4,5].

In highly agitated flows, there is a mechanism independent of gravity that is available to drive separation of different grains. This is associated with spatial gradients in the energy of their velocity fluctuations. In steady, fully-developed flows, the balance of momentum exchanged in collisional interaction between and among different species of grains requires, in general, that spatial gradients of concentration be balanced by spatial gradients of particle fluctuation energy.

In sheared or vibrated systems of colliding grains, gravity also influences mixtures of grains of different sizes. Here, buoyant forces act to separate grains that differ in size and, consequently, in the local volume that they displace. In reduced gravity, buoyancy is suppressed and attention can be focused on the simpler balance involving the gradients in concentration and the gradients in fluctuation energy. Reduced gravity also eliminates the possibility that a collisional flow will condense into a slower, denser flow dominated by enduring contacts rather than by collisions.

Because collisions between grains inevitably dissipate energy, collisional granular shear flows are usually of limited extent in the direction transverse to the flow. One consequence of this is that shear flows are strongly influenced by their boundaries. Because grains, on average, slip relative to boundaries, a bumpy or frictional boundary can convert slip energy into fluctuation energy. However, because each collision between a grain and the boundary dissipates fluctuation energy, there is a competition between production and dissipation.

In principle, it is possible to design the geometry of the boundary - for example, the size and spacing of the bumps - so that the boundary either produces or dissipates fluctuation energy [6]. This permits the control of the component of the spatial gradient of the fluctuation energy that is normal to the boundary. The gradients in fluctuation energy established by such boundaries may be exploited to drive the separation by size or other properties in a binary mixture of spherical grains.

Microgravity makes the visual observations possible by permitting us to employ moderate rates of shear. On earth, the effects of gravity can be minimized by shearing so rapidly that the particle pressure overwhelms gravity.

However, in this event, separation takes place too rapidly for visual observation, buoyancy and/or condensation associated with the centripetal acceleration must be accounted for, and the particles can be severely damaged. Because, in the absence of gravity, the only available time scale is proportional to the speed of the moving boundary, this speed can be made arbitrarily slow to permit observations and to avoid particle damage, without altering the phenomenon under study.

The primary goal of this research is to carry out a physical experiment in which particle segregation is induced and maintained in a collisional flow of a binary mixture of two different types of spheres. The segregation will be driven in the absence of gravity by a spatial gradient in the kinetic energy of the velocity fluctuations of the mixture. The flow is to take place in a shear cell in the form of a race track in which the grains are sheared by the motion of the inner boundary relative to the outer. The gradient of the kinetic energy is to be maintained using boundaries with different geometric features that result in different rates of conversion of the mean slip velocity into fluctuation energy at their surfaces.

The planned experiments will isolate and investigate two different sub-mechanisms of collisional segregation that usually occur together: the first is associated with differences in the inertia of the spheres, the second is associated with differences in the geometry of the spheres. We will attempt to neutralize a third sub-mechanism associated with differences in collisional dissipation between the two types of spheres. Inertial segregation will be studied in a system of spheres with different masses, but equal diameters; geometric segregation will be studied in a system with different diameters, but equal masses.

In the experiment, a steady shearing flow will be maintained in shear cell by the relative motion of parallel, bumpy boundaries. The resulting profiles of mean velocity, fluctuation velocity, and concentration will be measured in a region of fully-developed flow. They will be compared with those predicted by theory and those measured in computer simulations. We will also test simple analytical results for the mean velocity and fluctuation energy, obtained in a dense limit of the kinetic theory, against the experiment. The extent of agreement between the experiments, the theory, and the computer simulations will provide a test of the assumptions upon which the theory and the computer simulations are based.

For the theory, these include the assumptions that the collisions are instantaneous and binary, that there is no correlation in pair position and/or velocity, that terms quadratic in the gradients may be neglected, that friction may be incorporated into the flow theory and boundary conditions in relatively simple ways, that there is equipartition of energy between the two species, that the inertia associated with diffusion may be neglected, and that the species viscosity does not influence segregation. The computer simulations are based on a hybrid hard-particle overlap algorithm for collisional interactions and the implementation of a simple three parameter model of a collision.

Computer simulations play an important role in the research. They have guided the design of the shear cell shown in Fig. 1, and they have assisted us in determining the strategy for flow visualization and in establishing our requirements for microgravity.

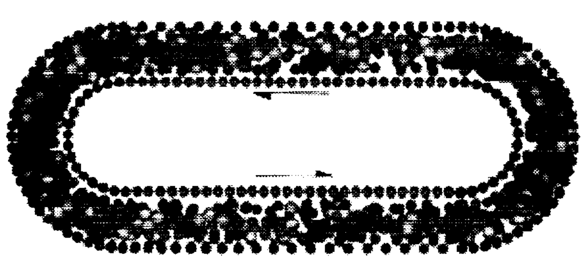

Figure 1 A shear cell of modest size. The outer boundary is fixed. The inner boundary moves with speed U in the direction shown.

The link between the computer simulations, the theory, and the physical experiments is provided by our ability to measure the collision parameters in an accurate and reproducible experiment. Our hope is to provide a demonstration of the power and utility of computer simulations and to establish them as an equal partner to theory and physical experiment. In this event, fewer physical experiments need be done, and the simulations can be used with confidence as the basis of the theoretical modeling. Also, with advances in computing power, such simulation may eventually be employed to study the behavior of complete systems of technological importance that involve collisional granular flow.

The success of experiment will indicate that the general theoretical framework provided by the kinetic theory is the appropriate one for describing collisional granular flow. It will reinforce our confidence in our modeling of the boundary conditions, the frictional interactions, and in the simplifying assumptions that were made to describe segregation. The experiments may also show that the dense limit of the kinetic theory for a single species provides an adequate description of the mixture velocity, mixture fluctuation energy, and mixture concentration. Then relatively simple analytical formula may be used to predict these, at least in steady, fully-developed flows. Similar simplifications of the theory for segregation are likely to emerge from our study of simple limiting cases.

We anticipate that the knowledge gained in this activity will contribute to our understanding of particle segregation in reduced gravity, where segregation mechanisms other than gravitational begin to dominate and collisional flows are more easily sustained. Examples of such flows are those involved in the transport of materials in Lunar and Martian environments and in manufacturing processes in space. We also anticipate that we will gain insight into the role played by collisional segregation mechanisms in grain flows in Earth gravity.

BACKGROUND

Most experimental studies of particle segregation are carried out on flows involving a free surface that are condensed by gravity into layers of grains in relative motion [2]. In each layer grains interact with at least several of their neighbors through rubbing and bumping. Phenomenological models of segregation in such flows that take into account their significant features often provide a satisfactory description [2]. Because the interactions of a single particle with its neighbors are so complicated and because the statistical characterizations of the interactions certainly involve strong correlations between the positions and velocities of particles, it is likely that a more detailed treatment is not possible.

Careful observations of particle segregation in less condensed free surface flows are less common. Such flows involve interactions between particles that are more violent than rubbing and bumping; these interactions can often be realistically modeled as binary collisions. Also, in these more agitated flows, the particle interactions are more random and, as a consequence, their statistical description is simpler.

Collisional flows can most easily be maintained in a shear cell. Typically, in such a cell, the flow takes place in a cylindrical annulus in which the bottom and sides are rotated and the top is held fixed [7]. The weight of the top is supported by collisions with the agitated grains of the shearing flow. This weight can be adjusted in order to maintain a fixed flow thickness or to vary the thickness in a controlled way. Knowledge of how the cell has been filled and the thickness of the flow permit the calculation of the average volume fraction. A measurement of the force necessary to keep the top from rotating permits the determination of the shear stress required to maintain the relative velocity of the bottom and top.

At high enough rates of shear, the weight of the grains in the cell is often a small fraction of the weight of the top of the cell. In this event, the influence of gravity on the flow may be neglected. However, the centripetal acceleration is always important; at the highest shear rates, it is typically more than five times the gravitational acceleration. Also, at the high shear rates necessary to render gravity negligible, collisions between the particles and between the particles and the boundaries may become so violent as to cause significant damage.

Theories for the prediction of particle segregation in collisional shearing flows make use of the analogy between the molecules of a dense gas and the agitated macroscopic grains that exists provided that the collisions between grains do not dissipate too much energy [8]. The most refined theory for segregation in dense molecular gases is a kinetic theory for mixtures of elastic spheres based on the assumption of molecular chaos and the correct extension to mixtures of Enskog's characterization of the influence of the finite volume of the particles on their frequency of [9, 10]. The derivation of the theory employs the Chapman-Enskog expansion to obtain the velocity distributions to sixth order. The results of the theory are consistent with irreversible thermodynamics, have been presented graphically, and are available numerically.

In our application of this theory to mixtures of inelastic grains, we have characterized the amount of permitted dissipation and we have obtained analytical results at second order in the Chapman-Enskog expansion for binary mixtures [8]. Analytical expressions permit the rational approximation of the coefficient governing transport, segregation, and dissipation over a range of mixture and species volume fractions and particle size and mass ratios.

The presence of dissipation in collisions permits collisional shearing flows of inelastic grains to achieve a steady balance in which the rate at which the kinetic energy of the velocity fluctuations increases, due to collisions driven by gradients in the mean velocity of the mixture, is equal to the rate at which it decreases, due to the energy lost in each collision, and the rate at which it is transported, due to inhomogeneities in the flow. The experimental study of special, simple types of binary mixtures of spheres in inhomogeneous steady shearing flows permits various aspects of the phenomena of segregation to be studied separately and helps in developing intuition about the behavior of more complicated mixtures.

In order to carry out the analysis of such experiments, we have extended existing theory for shearing flows of identical spheres in several ways. We have accounted for friction in collisions between particles in the flow by incorporating the additional energy loss into an effective coefficient of restitution [11]. We have extended existing boundary conditions for a single species interacting with either a bumpy or a frictional boundaries to apply to a binary mixture interacting with a boundary that is both bumpy and frictional [12]. We have generalized existing theory for a dense shearing flow of a single species slightly to apply to the prediction of the profiles of mixture mean velocity and mixture mean kinetic energy in dense shearing flows of a binary mixture between bumpy, frictional boundaries.

In order to incorporate friction into the kinetic theory, a simple but realistic model of a frictional collision must be employed. The simplest such model distinguishes between collisions in which the relative velocity of the points of contact is momentarily zero during a collision and those in which it is not. The former are called sticking collisions, the latter are called sliding collisions. The model employs three parameters: a coefficient of normal restitution for both sticking and sliding collisions, a coefficient of friction for sliding collisions, and a coefficient of tangential restitution for sticking collisions [13]. We have employed the model to interpret the results of experiments on binary collisions between identical spheres and collisions between a single sphere and a flat or a bumpy boundary. The parameters determined in this way provide an excellent fit to the data [14, 15].

Computer simulations have played important roles in the development of the field of granular mechanics [16, 17]. Such simulations of dissipative but frictionless disks and spheres have, in some instances, supported simple predictions of the kinetic theory and, in other instances, shown that collisional granular flows of frictionless, inelastic particles could behave in ways far more complicated than predicted. A corresponding contribution from computer simulations for frictional particles was not possible until a simple, physically realistic model of the frictional interaction between two colliding spheres could be verified in physical experiments. Using the simple model, numerical simulations have contributed to an understanding of the flow of inelastic frictional spheres down frictional inclines and in the determination of boundary conditions for frictional, inelastic spheres interacting with a flat, frictional elastic wall [18, 19]. Such simulations can now be employed with confidence to interpret physical experiments and to inform the development of theory.

PRELIMINARY RESULTS

The objectives of the experiment are to control the distributions of mixture fluctuation energy across the cell using boundaries with the appropriate geometric features and collision properties; to measure profiles of mixture mean velocity, mixture number density, and mixture fluctuation energy across the cell in order to compare them with the predictions of both the dense and the full theory and to compare the flows and fields of fluctuation energy and species' number densities with those predicted in the computer simulations.

As Figs. 2 demonstrates with an image acquired on the KC 135 microgravity experimental aircraft, the vision algorithm can determine the location of the sphere centers accurately.

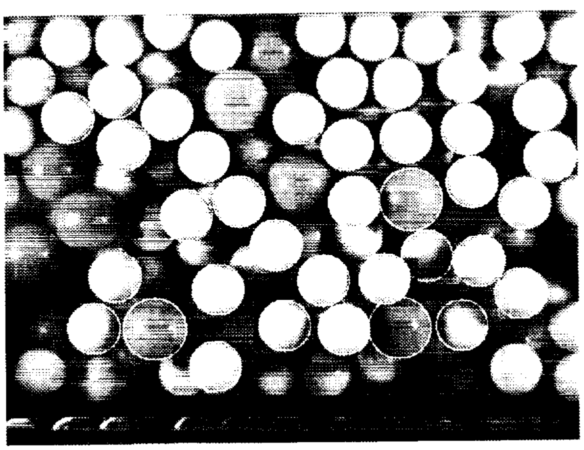

Figure 2 View of the fully-developed region of the prototype cell with v_A = 10% and v_B = 30%. Circles indicate detected spheres.

Figs. 3 and 4 show that the computer vision algorithm produces profiles of mean velocity and temperature that are in reasonable agreement with the computer simulations.

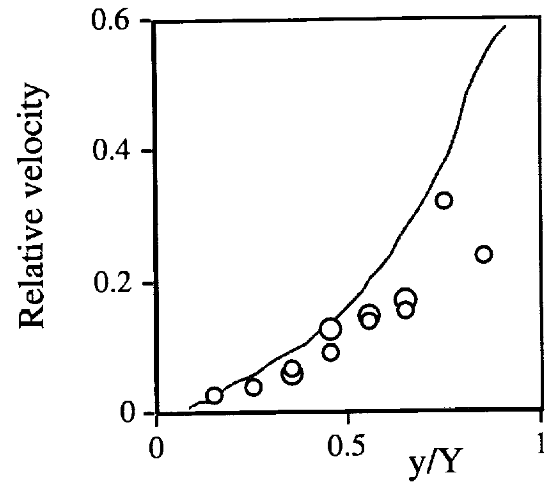

Figure 3 Mean transverse grain velocity profile relative to the speed of the boundary. The small and large circles represent data for the small and large spheres, respectively. The lines are predictions of the simulations.

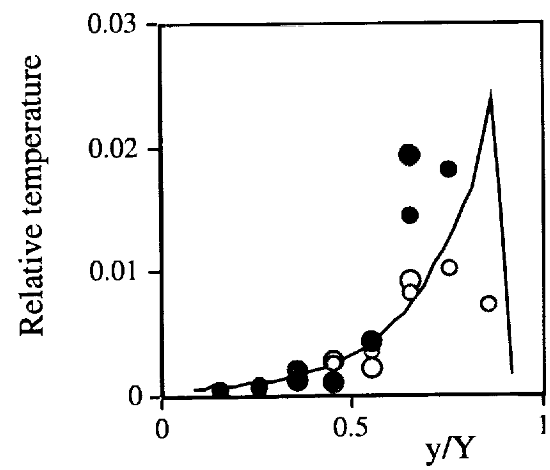

Figure 4 Temperature made dimensionless with $m_B U^2$. The filled and open circles represent temperature in the x- and y-directions, respectively.

The computer simulations were carried out in a cell that was periodic in the flow direction with a square cross-section of about eight diameters of the smaller spheres in the flow. On its inner and outer boundaries, cylinders with the diameter of the smaller spheres in the flow were spaced with their centers one and one and one-half diameters apart, respectively. The side walls were flat. The spheres in the flow were of the same material with their diameters in a ratio of four to three. The coefficient of friction and the coefficient of tangential restitution in all collisions were taken to be 0.1 and 0.4, respectively. The coefficient of normal restitution between spheres in the flow was taken to be 0.9; that for collisions between spheres in the flow and the boundaries was taken to be 0.8. These are typical values measured in experiment [15].

Note that Figs. 3 and 4 indicate that the chain was run too fast in the KC 135 tests for the frame rate employed. As a consequence, the velocity statistics near the moving boundary were not captured as accurately as they can be.

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