STUDIES IN THERMOCAPILLARY CONVECTION OF THE MARANGONI-BENARD TYPE

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INTRODUCTION

Previous research by the authors [1-3] has shown that either oscillatory, nonplanar shear or temporal modulation of a boundary temperature can stabilize significantly the onset of thermocapillary convection in a fluid layer originally at rest and heated from below. Within the context of control theory, both methods are forms of open loop control. We now extend our investigation of the control of Marangoni-Benard convection by considering the effects of closed loop control using feedback. The basic model for the linearized problem is similar to one employed by Tang and Bau [4] who showed that Rayleigh-Benard convection can be stabilized by the use of linear feedback control. We also use the temperature of the lower boundary as an actuator but, instead of sensing the temperature within the layer, we assume that the depth of the layer can be measured in order to input information to the actuator.

Marangoni-Benard convection can occur in one of two distinct ways. One mode of instability, typical of situations involving moderate Bond numbers, was studied first by Pearson [5]. I occurs for a critical wavelength of the order of the depth of the layer and typically involves negligible surface deformation. The preferred pattern of convection is one of hexagons. A quite different mode of instability occurs at the very small Bond numbers typical of a microgravity situation. It was studied first by Scriven and Sternling [6] and by Smith [7]. The critical wavelength for this mode is much larger than the depth and considerable surface deformation occurs. A recent experimental investigation [8] has shown that the onset of this long wavelength mode can often lead to rupture of the fluid layer. We therefore felt that control of this mode of instability would be a worthy test of the use of feedback control because rupture of the layer is usually detrimental to any application where thermocapillary convection is possible.

The long wavelength mode of Marangoni Benard convection differs from Rayleigh-Benard convection not only by its length-scale but also by the fact that a sub-critical instability is dominant rather than the supercritical bifurcation characteristic of Rayleigh-Benard convection (which gives rise to convective rolls). Due to this difference, a control strategy considerably more complicated than that used by Tang and Bau [4] must be employed. In fact, we show that a nonlinear control strategy is essential in order to eliminate the subcritical instability and to stabilize the layer. Another major difference between our analysis and that of Tang and Bau, who considered the control of an individual normal mode, is that we can consider direct control of arbitrary disturbances as long as they are composed of large wavelength components.

ANALYSIS

Only a brief review of the problem formulation and analysis will be given here because a detailed presentation [9] has recently been submitted for journal publication. We consider a gas-liquid layer model contained between two parallel walls contained at constant but unequal temperatures, with the wall bounding the liquid having the higher temperature. In particular, the simplified two-layer model of VanHook et al. [8] is used, in which the effects of heat transfer in the gas layer are modeled but dynamical effects of the gas are ignored. Because we restrict the analysis to large wavelength disturbances, both linear and nonlinear effects can be investigated by means of a small wavenumber expansion [8,10]. We further restrict the analysis to values of the Marangoni number (M) close to the critical value (M_c). This restriction allows us to explore the problem on a weakly nonlinear basis for arbitrary values of the surface tension, which is usually assumed to be large for nonlinear studies where M is well above M_c. For the large wavelength case, it is convenient to define and use the parameter $D = M/G$, where G is the Galileo number. VanHook et al. [8] refer to D^{-1} as being the dynamic Bond number. At lowest order in an expansion involving the nondimensional wavenumber $q(<<1)$, the critical condition for the case without control is determined to be $\frac{3}{2}D(1|F) = 1$, where F is the two-layer Biot number as defined in [8]. We can then define $\in \frac{3}{2}D(1|F) - 1$ as a measure of sub- or supercriticality. If η represents the deflection of the interface relative to the undisturbed level of the interface, the weakly nonlinear regime is explored by means of the scalings $c = q^2R$ and $\eta = q^2A$, where R and A are $O(1)$. After defining a suitable nondimensional time τ, an amplitude equation arises at $O(q^2)$ in which nonlinear quadratic terms appear, namely,

$$\frac{\partial A}{\partial \tau} \Big| \nabla \cdot \left(R \Big| \frac{1}{B} \nabla^2 \Big| (2F-1)A \right) \Big| \nabla A = 0 \qquad (1)$$

where B is the ordinary Bond number. Because the amplitude equation involves quadratic terms, subcritical instability with $R < 0$ of the transcritical type occurs. Although subcritical equilibrium solutions are possible, they turn out to be unstable. Equilibrium solutions are not possible for $R > 0$. We then have a picture of a very unstable situation, with instability occurring for $R < 0$ as well as $R > 0$ and growth being possibly limited only by higher-order nonlinear terms. The above scaling fails if $F = 1/2$, in which case cubic terms in A must be considered. For that case, it can be shown that the cubic term augments the instability instead of limiting growth.

We now seek to stabilize this very unstable situation by utilizing feedback control. The interfacial deflection is assumed to be measured, and a control temperature T_c at the lower wall is assumed of the form

$$T_c = K_1 \eta | K_2 \eta^2 | K_3 \eta^3. \qquad (2)$$

Higher-order terms could be included in the control law but would not appear in our weakly nonlinear analysis. The lower boundary now has an inhomogeneous temperature, which is assumed to vary smoothly along the wall. For the linear problem, the critical condition for disturbances with $q \rightarrow 0$ is found to be $c = K_1$ where $K_j = 3DK_j/2(1 \mid H)$, $j = 1,2,3$, and where H is the one-layer Biot number [9]. Thus, linear feedback control has a stabilizing effect, obtained basically by heating or cooling the fluid in such a way that the thermocapillary effect at the interface is in part cancelled. In terms of the ratio of the critical Marangoni number with control to that without, we obtain

$$M_c \frac{(K_1 \neq 0)}{(K_1 = 0)} = \frac{1}{1 - K_{1,eff}}. \qquad (3)$$

where $K_{1,eff} = K_1/(1+\mathrm{F})(1+\mathrm{H})$. As $K_{1,eff}$ increases from zero, the ratio (3) increases in magnitude and approaches infinity as $K_{1,eff} \rightarrow 1$. In this limit, the thermocapillary effect arising at the interface is exactly cancelled by the control. However, stabilization by linear control does not affect the fact that subcritical instability can occur. Because this instability is associated with the quadratic terms in (1), we must have $K_2 \neq 0$ in (2) in order to eliminate the possibility of subcritical instability. With control, the coefficient of the quadratic term turns out to be

$$2F - 1 - a_1 \hat{K}_1 - 2\hat{K}_2 \qquad (4)$$

where $a_1 = 2(1 - (1 \mid F)H)$. We want to make this term small by suitable selection of $\hat{K}_2$. However, exact cancellation might not occur, and so we let

$$2F - 1 - a_1 \hat{K}_1 - 2\hat{K}_2 = \alpha q \qquad (5)$$

where α is an $O(1)$ parameter. With the quadratic term in (1) almost eliminated, we can then balance the linear terms with the next order nonlinear (cubic) term by defining $c - \hat{K}_1 = q^2 R$ and $\eta = qA$ to obtain the following cubic amplitude equation:

$$\frac{\partial A}{\partial \tau} \Big| \nabla \cdot \left(R \Big| \frac{1}{B} \nabla^2 \Big| \alpha A \mid \beta A^2 \right) \nabla A = 0 \qquad (6)$$

which can be compared to (1). The cubic term can now control the growth of the instability for $R > 0$ if $\beta < 0$ where $\beta = \beta\,(H, F, \hat{K}_1, \hat{K}_3)$ is a function defined by the analysis. When this is achieved, the original subcritical instability has been converted into a supercritical bifurcation.

Values of K_1, K_2 and K_3 required to achieve a supercritical bifurcation at $\in > 0$(i.e., $M_c(K_1, K_2, K_3) > M_c(0)$) are given in Table 1. As is clear, the values of K_1, K_2 and K_3 and all $O(1)$ even for an increase of 100% in the critical Marangoni number.

F	r	β	D_c	K_1	K_2	K_3
0.33	0.50	0.00	0.75	0.49	–0.59	0.54
0.67	0.50	0.00	0.60	0.61	–0.30	1.18
0.33	0.50	–0.50	0.75	0.49	–0.59	0.70
0.67	0.50	–0.50	0.60	0.61	–0.30	1.38
0.33	1.00	–0.50	1.00	0.73	–0.76	0.86
0.67	1.00	–0.50	0.80	0.92	–0.61	1.49

Table 1: Values of Control Gains Computed from Selected Parameters

CONCLUSIONS

We conclude that feedback control can be used to postpone the onset of Marangoni-Benard long-wavelength convection and even to convert the subcritical bifurcation characteristic of the case without control into a supercritical bifurcation. In order to achieve this goal, however, a nonlinear control strategy of the type given by (2) is required. The gain K_1 is selected to give the desired increase in M_c, K_2 is chosen so as to eliminate the possibility of a transcritical bifurcation, and K_3 is then selected to give a forward pitchfork (supercritical) bifurcation. With K_2 and $K_3 = 0$, convection could start for M well below M_c due to finite amplitude disturbances even though $M_c(K_1) > M_c(0)$.

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