Determination of the Accommodation Coefficient Using Vapor/Gas Bubble Dynamics in an Acoustic Field

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Hertz-Knudsen-Langmuir Formula (1882) for Kinetics of Phase Transitions:

$$\xi = \frac{\beta(T_a)}{\sqrt{2\pi R_v T_a}}\left[p_s(T_a) - p_v\right]$$

ξ the rate of evaporation (condensation),

β the accommodation (condensation) coefficient,

T_a the temperature of the interface,

p_s the saturation pressure,

p_v the vapor pressure,

R_v the gas constant of the vapor.

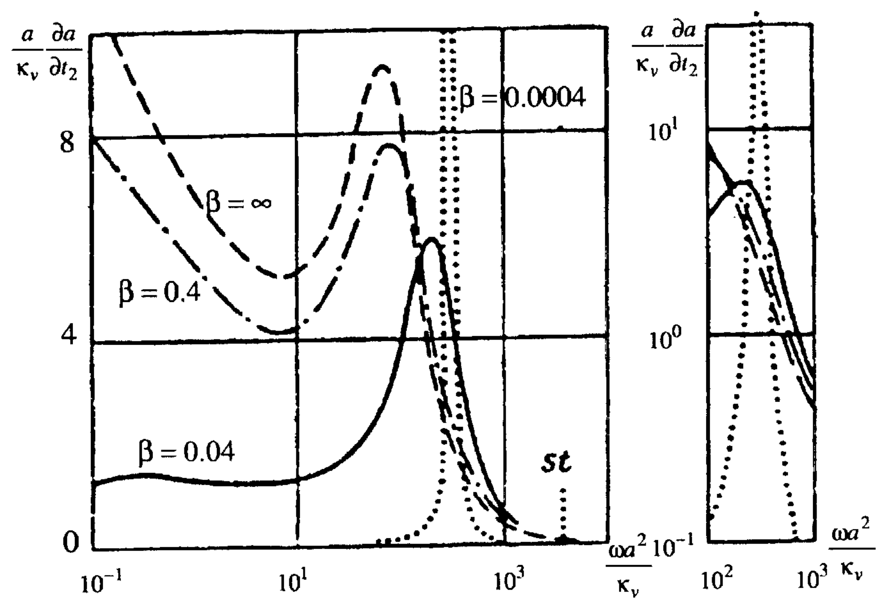
Non-Equilibrium Evaporation/Condensation:

- Vacuum Evaporation;
- Processing of Molten Metals;
- Vapor Explosions;
- High Velocity Jets;
- Atmospheric Small Droplet Clouds;
- Sound Propagation in Vapor/Droplet Mixtures;
- Bubbles/Droplets in Acoustic Fields;
- Laser Vaporization;
- Other High-Speed Processes.

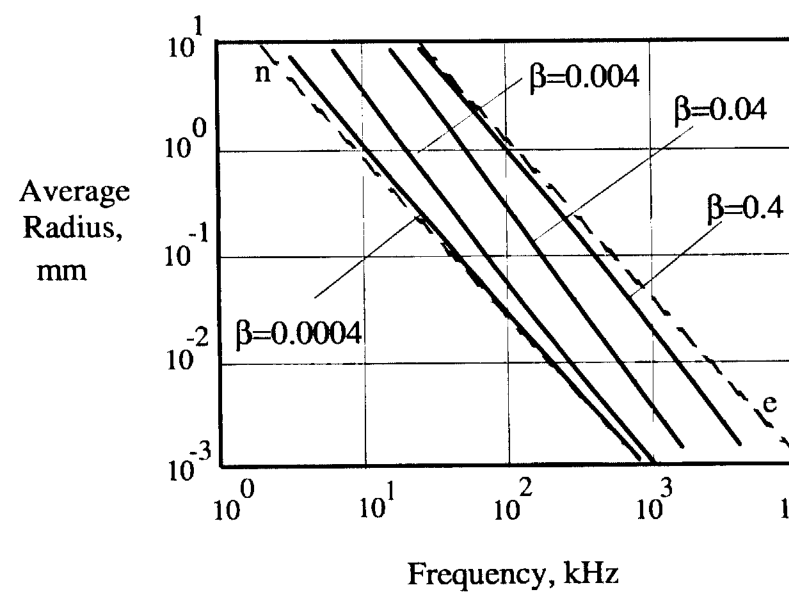
Rectified Heat and Mass Transfer To a Vapor Bubble

Rectified heat and mass transfer to vapor bubbles can strongly depend on the value of the accommodation coefficient

Dependence of the growth rate of a water vapor bubble in 10 kHz acoustic field on the bubble size (p=1 bar, T=373 K).



Stable time-averaged radius of a water vapor bubble at various values of the accommodation coefficient (p=1 bar, T=373 K).



N.A. Gumerov, Weakly non-linear oscillations of the radius of a vapour bubble in an acoustic field. *J. Appl. Maths Mechs* 55 (2), 205-211, 1991.

Bubble Motion in Standing Acoustic Waves

Standing wave:

$$p_\infty = p_{\infty 0}\left[1 + \varepsilon \cos \omega t \sin kx\right], \quad \rho \frac{dU_\infty}{dt} = -\nabla p_\infty, \quad \omega = Ck.$$

Bubble motion ($ka << 1$):

$$F = F_b + F_m + F_\mu = 0,$$

$$F_b = \frac{4}{3}\pi a^3 \rho \frac{dU_\infty}{dt}, \quad F_m = \frac{2}{3}\pi a^3 \rho \frac{d\left[a^3(U_\infty - U_b)\right]}{dt}, \quad F_\mu = 4\pi K_\mu a (U_\infty - U_b),$$

$$\frac{dx_b}{dt} = U_b.$$

Equations of Bubble Radial Pulsation

- Modified Keller-Miksis Equation:

$$\left(1-\frac{w_a}{C}\right)a\dot{w}_a+2\left(1-\frac{w_a}{4C}\right)\dot{a}w_a-\frac{1}{2}w_a^2=\frac{1}{\rho}\left(1+\frac{\dot{a}}{C}+\frac{a}{C}\frac{d}{dt}\right)\left[p_g-p_\infty(t)-\frac{2\sigma}{a}-\frac{4\mu}{a}w_a+\left(\frac{1}{\rho_{ga}}-\frac{1}{\rho}\right)\xi^2\right].$$

- Mass, Momentum, and Energy Conservation at the Interface:

$$\rho(\dot{a}-w_a)=\rho_{ga}(\dot{a}-w_{ga})=\xi,$$

$$-p_a+\tau_a^{rr}+\xi w_a=-p_g+\xi w_{ga}+2\frac{\sigma}{a},$$

$$\left(-p_a+\tau_a^{rr}\right)w_a-q_a+\frac{1}{2}\xi w_a^2=-p_g-q_{ga}+\frac{1}{2}\xi w_{ga}^2+\xi_v l_v+\xi_i l_i+\dot{\sigma}+\frac{2\sigma\dot{a}}{a},$$

$$\xi=\xi_v+\xi_i,\quad \xi_i=c_i\xi,\quad j_a=c_a\xi-\xi_i.$$

- Kinetics of Phase Transitions:

$$\xi_v=\frac{\beta_v(c_a,T_a)}{\sqrt{2\pi R_v T_a}}\left[p_s(T_a)-p_v\right],\quad \xi_i=\frac{\beta_i(c_a,T_a)}{\sqrt{2\pi R_v T_a}}\left[H(T_a)c_a-p_i\right]$$

- Heat and Mass Diffusion Fluxes:

$$q_{ga}=-\lambda_g\left.\frac{\partial T_g}{\partial r}\right|_{r=a(t)},\quad q_a=-\lambda\left.\frac{\partial T}{\partial r}\right|_{r=a(t)},\quad j_a=-\rho D\left.\frac{\partial c}{\partial r}\right|_{r=a(t)}.$$

Heat and Mass Transfer in Gas and Liquid

- Model of Gas

$$p_g = p_i(t) + p_v(t),$$

$$p_g(t) = \rho_g(r,t)R_g(t)T_g(r,t), \quad R_g(t) = c_i(t)R_i + c_v(t)R_v,$$

$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r^2}\frac{\partial (r^2 \rho_g w_g)}{\partial r} = 0, \quad \rho_g c_{pg}\left(\frac{\partial T_g}{\partial t} + w_g \frac{\partial T_g}{\partial r}\right) + \dot{p}_g = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \lambda_g \frac{\partial T_g}{\partial r}\right),$$

$$\text{at } r = a: \quad w_g = w_{ga}, \quad T_g = T_a.$$

- Model of Liquid

$$\rho c_l \left(\frac{\partial T}{\partial t} + \frac{a^2 w_a}{r^2}\frac{\partial T}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \lambda \frac{\partial T}{\partial r}\right) + \frac{12\mu w_a^2 a^4}{r^6},$$

$$\frac{\partial c}{\partial t} + \frac{a^2 w_a}{r^2}\frac{\partial c}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D \frac{\partial T}{\partial r}\right),$$

$$\text{at } r = a: \quad T = T_a, \quad c = c_a, \quad \text{at } r = \infty: \quad T = T_\infty, \quad c = c_\infty.$$

Methods of Solution

- Asymptotic Multiscale Technique
- Direct Numerical Simulations

Multiscale Technnique:

1. Space - time transformation: $(r,t) \to (\eta,t), \quad \eta = r/a(t);$
2. Introduce mutiple scales: $t \to \{t_0, t_1, \ldots\}, \quad t_n = \varepsilon^n t;$
3. Expand derivatives: $\dfrac{d}{dt} \to \dfrac{\partial}{\partial t_0} + \varepsilon \dfrac{\partial}{\partial t_1} + \varepsilon^2 \dfrac{\partial}{\partial t_2} \ldots;$
4. Expand unknowns: $a(t) \to \langle a \rangle (t_1, t_2, \ldots)[1 + \varepsilon a_1 + \ldots];$
5. Search for periodic solutions with respect to the 'fast' time, t_0;
6. Solve diffusion problems for complex amplitudes and find complex amplitudes of the fluxes;
7. Obtain right hand side vector in the matrix equation for complex amplitudes at the m – th order of approximation:

   $$\mathbf{L}_n \mathbf{X}^0_{mn} = \mathbf{F}^0_{mn}, \quad n = 0,1,2, \quad m = 1,2,$$

8. Derive equations for the average bubble size/position from the solvability conditions:

   $$\frac{\partial \langle a \rangle}{\partial t_2} = F(\langle a \rangle) \sin^2 k \langle x_b \rangle - G(\langle a \rangle), \quad \frac{\partial^2 \langle x_b \rangle}{\partial t_2^2} = E\left(\langle a \rangle, \langle x_b \rangle, \frac{\partial \langle x_b \rangle}{\partial t_2} \right),$$

9. Investigate the influence of β on solution of these equation.