RADIATION AND MAXWELL STRESS STABILIZATION OF LIQUID BRIDGES

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1 Introduction

A unique and potentially useful fluid configuration which is possible in zero gravity is the liquid cylinder. One example is floating-zone crystal growth which, in zero gravity, involves a cylindrical melt bridging two solid cylinders. In low gravity, when the length of a cylindrical liquid bridge exceeds it's circumference, the bridge becomes unstable and breaks. A liquid bridge of length L and radius R is thus characterized by the ratio $S = L/2R$, where S is termed the slenderness of the bridge. The stability limit of $S = \pi$ is known as the Rayleigh-Plateau (RP) limit. For floating-zone crystal growth one might wish to have a longer liquid bridge in order to reduce the temperature gradient at the growth front since thermal stress can lead to defects in the growing crystal. A number of groups have studied stabilization methods which rely on the electrical properties of the liquid column. For example, an axial electric field has been shown to stabilize a dielectric bridge beyond the RP limit. The stabilization methods explored in this work are of a fundamentally different nature.

The modes of oscillation for a liquid bridge in zero gravity are associated with volume-conservative surface deformations which can be characterized in terms of an axial index N and azimuthal index m. The index N is the number of half-wavelengths in the axial direction and the index m is the number of wavelengths in the azimuthal direction. Axisymmetric modes of oscillation have $m = 0$ and the $(N, m) = (2, 0)$ mode is the first mode to become unstable as S is increased beyond π in the absence of stabilization. The breaking process thus involves one end of the bridge becoming fat while the other end becomes increasingly thin, leading to rupture. Stabilization is accomplished by applying a stress to the surface of the bridge to counteract the growth of the $(2, 0)$ mode. The coupling of modulated acoustic radiation stress to both axisymmetric and non-axisymmetric modes of a cylindrical liquid bridge in a Plateau tank has been demonstrated [1]. Both acoustic radiation stress [2] and the Maxwell stress [3] have been used to stabilize liquid bridges in this work. Acoustic radiation stress arises from the time-averaged acoustic pressure at the surface of an object immersed in a sound field. Both passive and active acoustic stabilization schemes as well as an active electrostatic method have been explored.

2 Experimental results

Ground based testing of stabilization past the RP limit requires simulating low gravity. This was accomplished in a Plateau tank. The bridge was deployed in a liquid bath, where the bridge liquid and the bath were of the same density and immiscible in each other. In the Plateau tank, two methods of stabilization were explored: one using acoustic radiation stresses and the other using Maxwell stresses. Acoustic transducers for use on a liquid bridge in air were also developed, and tested both in normal gravity and in reduced gravity on NASA's low-gravity KC-135 aircraft.

2.1 Active acoustic stabilization in a Plateau tank

For these experiments, the Plateau tank had an ultrasonic transducer incorporated to produce an acoustic standing wave inside the tank. The acoustic transducer has a step machined into its surface so as to create a different liquid depth between the halves of the tank. This difference in depth causes a difference in resonance frequencies between the halves, so by adjusting the drive frequency, the acoustic energy can be shifted from side to side. In the system of interest, the bridge liquid is attracted to a pressure antinode, so by spatially shifting the pressure antinode, different areas of the bridge can be affected. To detect the shape of the bridge, a two-segment photodiode is used. The bridge is illuminated with an expanded laser beam, and the image is projected onto the photodiode. The difference between the photodiode signals provides information about the shape of the bridge. This error signal is used to adjust the drive frequency of the transducer in order to keep the bridge shape cylindrical [2].

In figure 1, images of a capillary bridge that is stabilized using active feedback in a Plateau tank are shown. In figure 1 (a), the bridge is stabilized to a length of $S = 4.1$. Shortly before (b), the acoustic drive is turned off, and the $(2, 0)$ mode subsequently grows and causes the bridge to break as shown in (c).
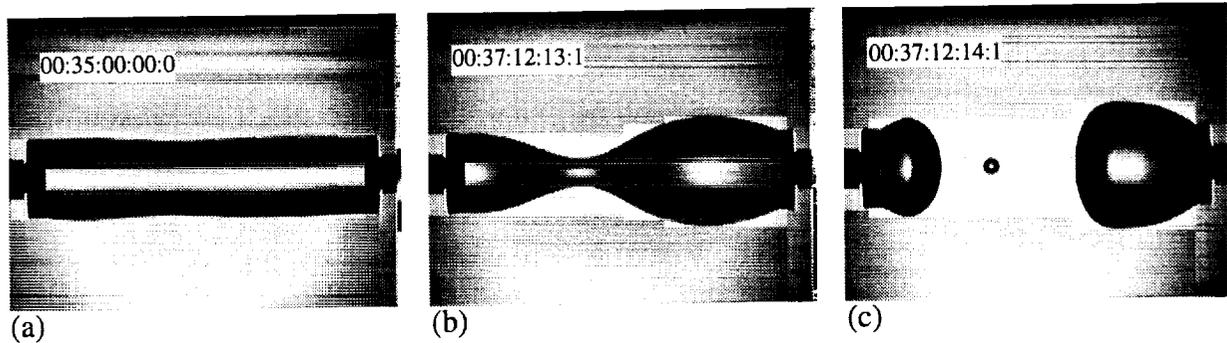
Figure 1: A capillary bridge stabilized to $S = 4.1$ using active feedback and acoustic radiation pressure. The time code shown is in the format "hours : minutes : seconds : frame : field", where there are 30 frames per second, and 2 fields per frame. In (a), the bridge has been stabilized beyond the RP limit for approximately 2.5 minutes. About 2 minutes later the acoustic drive is shut off. 470 ms after the sound is shut off, (b) shows the growth of the $(2,0)$ mode, and the bridge is shown to be completely broken in (c), which was taken 33 ms after (b).

2.2 Active stabilization in a Plateau tank using electric fields

This case is similar to that described in §2.1. The bridge is deployed in a Plateau tank; however, here the bridge liquid is electrically conducting, and the outer bath is electrically insulating. The detection system is as previously described, except that now the control uses Maxwell electrostatic stresses instead of radiation stresses [3]. Two ring electrodes placed coaxial with the bridge may be seen in figure 2. The error signal from the photodiode now controls the potential applied to each electrode via high-voltage amplifiers. Since the conducting liquid is attracted to the ring with a high voltage, when the photodiode detects one side thinning, a potential is applied to that ring, pulling the liquid back. The electrodes were designed to optimize the coupling to the $(2,0)$ mode.

In figure 2 (a), the bridge is stabilized using active feedback and Maxwell stresses to $S = 4.1$. Just before (b) the control is turned off, and in (b) and (c) the $(2,0)$ mode grows and breaks.

2.3 Soap-film tests in normal gravity

To extend the active feedback scheme to a liquid bridge in air, a transducer needed to be developed that could rapidly spatially redistribute the acoustic energy. Testing such a transducer in normal gravity is somewhat difficult using a liquid bridge, because the maximum slenderness of a liquid bridge in normal gravity is much shorter than that in low-gravity. Using a gas-filled soap-film bridge, the effect of the transducer on the bridge

can be tested with a longer bridge. The soap-film bridge also remains cylindrical, while a liquid bridge sags in normal gravity. However, the sound field does not couple well into a soap-film bridge filled with air, so a gas with different sound speed and density was needed to produce sufficient acoustic contrast to allow the sound field to couple into the bridge mode of interest. Sulfur hexafluoride (SF_6) was chosen because the speed of sound in SF_6 is about 1/3 that in air, and the density of SF_6 is about 5 times that of air.

Two transducer configurations have been designed and initial tests performed with the soap-film bridge. In one design, a standing wave is established between a dual-horn aluminum bar transducer and a cylindrical reflector. The two horns have a slightly different resonance so that a small change in the driving frequency can cause one horn or the other to radiate more intensely [4]. The spatial redistribution of radiation pressure on the bridge can then be accomplished by simply changing the drive frequency. The second design involves two tweeters which face each other with a lateral offset. Spatial redistribution of sound intensity in this case is accomplished simply by adjusting the relative drive amplitude of the two tweeters. Each of these designs was shown to be capable of exciting the $(2,0)$ mode of the soap-film bridge filled with SF_6. Figure 3 shows the $(2,0)$ mode oscillation for the soap-film bridge driven by the dual-horn transducer.

A linear model of bridge dynamics with active feedback (§ 3) suggests that a delay in applying the feedback force to the bridge can seriously limit the effectiveness of the feedback. Feedback delay is expected to be signif-
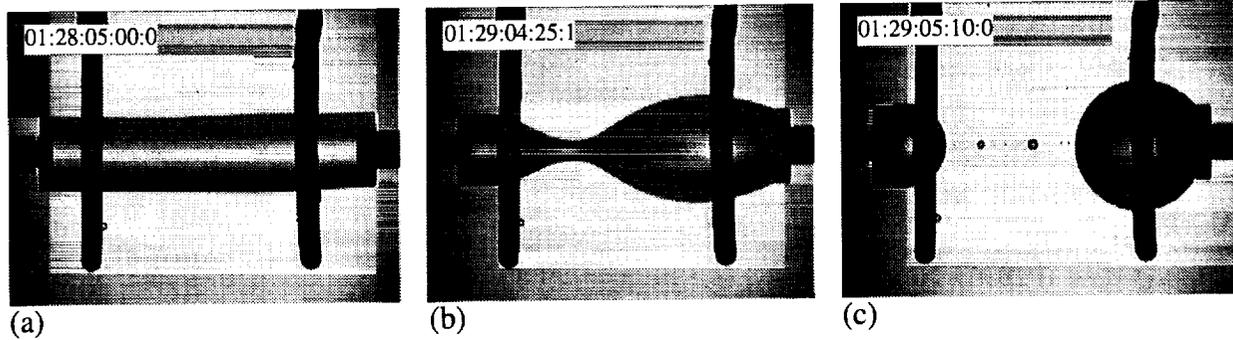
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Figure 2: A capillary bridge stabilized to $S = 4.1$ using active feedback and Maxwell stresses. About 1 minute after (a), the feedback control is turned off. 520 ms after the control is turned off, (b) shows the growth of the $(2,0)$ mode, and the bridge is shown to be completely broken in (c), which was taken 480 ms after (b).

icantly less for the dual-tweeter configuration than for the dual-horn system because the lower Q of the dual-tweeter design allows for more rapid shifting of acoustic energy from one end of the bridge to the other. A clean test of active feedback stabilization for a liquid bridge in air under low-gravity conditions has not yet been made.

2.4 Passive acoustic stabilization in low gravity

A liquid bridge in air is predicted to be stabilized when positioned at a pressure node in an acoustic standing wave with the bridge axis perpendicular to the sound propagation direction [5]. In this case, the bridge is not actively controlled as in §§ 2.1 and 2.2; the stabilization is a passive result of the interaction of the sound field with the bridge. The radius of the bridge supports and the frequency of the sound field are chosen such that the radiation stresses cause the regions of smaller local radius to be expanded, and the regions of larger local radius to be squeezed, thus stabilizing the bridge against breakup of the $(2,0)$ mode.

Passive stabilization was demonstrated in low gravity for a liquid bridge in air during parabolic flights aboard NASA's KC-135 aircraft. In figure 4 (a), a liquid bridge composed of a 20 cS water and glycerol mixture is extended in a 21 kHz acoustic standing wave to $S = 3.5$. The bridge support radius is 2.16 mm. Shortly before (b), the sound field is turned off, and in (b) and (c), the $(2,0)$ mode grows and breaks. It can be seen in figure 4 (a) that the sound field also causes a static deformation in shape. The bridge contains precisely enough liquid to form a circular cylinder, but the sound field causes the bridge to be flattened. This effect is also seen when acoustically levitating liquid drops.

Since each KC-135 parabolic maneuver results in only about 20–30 seconds of low gravity, an automated bridge deployment system was developed to facilitate drawing the bridge in such a short time. The system uses synchronized stepper motors to position the supports and inject the proper volume of liquid.

3 A linear model of bridge dynamics with feedback

A linear model of the bridge dynamics including the effects of a feedback force, which is applied with some delay and gain, has been developed [2, 6]. The model accounts for viscous dissipation both in the bridge liquid and in the surrounding fluid. The boundary conditions on the end of the bridge are only approximately satisfied. The response of the bridge is analogous to that of a driven, damped, harmonic oscillator where the $(2,0)$ mode amplitude corresponds to the oscillator's displacement. The $(2,0)$ mode surface deformation has an associated change in surface area which provides a restoring force which is linearly related to the $(2,0)$ mode amplitude. The effective spring constant for this restoring force is proportional to $[(\pi/S)^2 - 1]$. Thus the natural spring constant becomes negative when $S > \pi$ which results in instability. By adding an external force proportional to the $(2,0)$ mode amplitude with the appropriate gain, the effective spring constant can be made positive even when $S > \pi$.

A delay in applying the feedback force to the bridge arises because of the finite time required to sense the bridge deformation and adjust the stress distribution on the bridge. The model predicts that this delay has the effect of reducing the effective damping of bridge os-
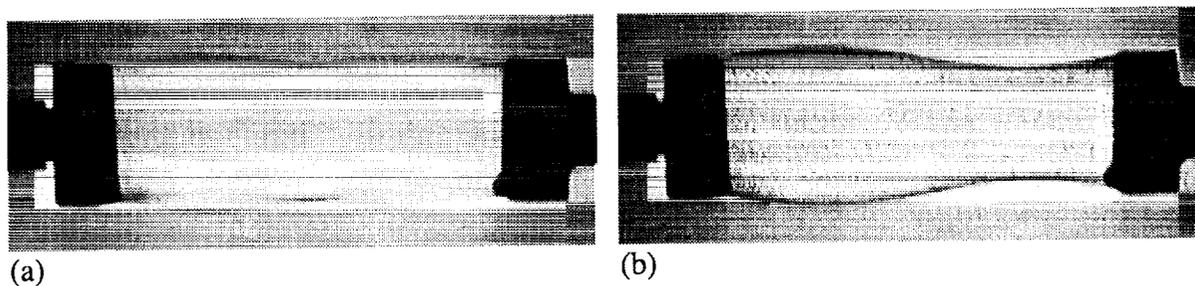
Figure 3: Images of a SF_6 filled soap-film bridge being driven in the $(2,0)$ mode. In (a), an average of several frames was taken, and (b) is a single frame.
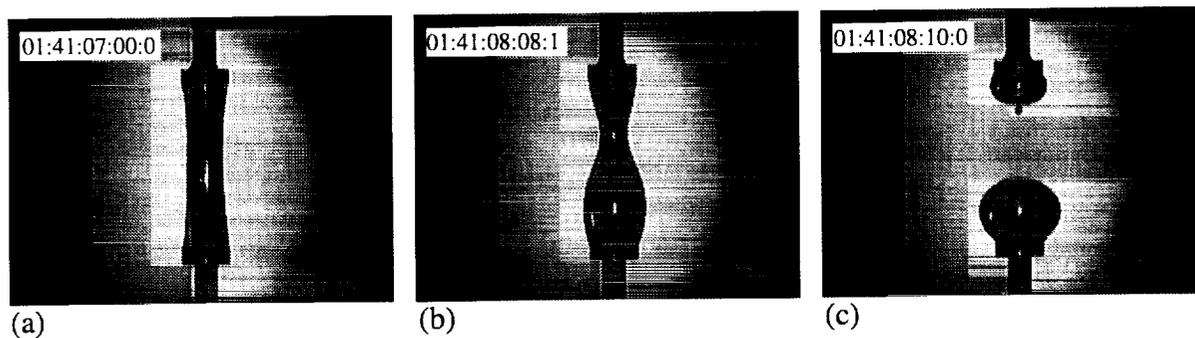

Figure 4: A liquid capillary bridge in air stabilized to $S = 3.5$ in low gravity using acoustic radiation pressure. About 1 second after (a), the sound is turned off. In (b), the growth of the $(2,0)$ mode is shown 330 ms after the sound is turned off. 50 ms after (b), the bridge has broken as shown in (c).

cillations, therefore making the bridge less stable. The model indicates that for a bridge beyond the RP limit, stability is expected for a range of gain which depends on the feedback delay. As feedback delay increases, the range of gain for stability decreases until stabilization becomes impossible. Thus the feedback delay must be less than a critical value in order for stabilization to be possible. The critical feedback delay decreases for increasing values of slenderness. Critical feedback delay also decreases for decreasing fluid viscosities.

Acknowledgment

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