HYDRODYNAMIC STABILITY OF LIQUID-PROPELLANT COMBUSTION: LANDAU'S PROBLEM REVISITED

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EXTENDED ABSTRACT

Hydrodynamic, or Landau, instability in combustion is typically associated with the onset of wrinkling of a flame surface, corresponding to the formation of steady cellular structures as the stability threshold is crossed. As its name suggests, it stems from hydrodynamic effects connected with thermal expansion across the reaction region. In the context of liquid-propellant combustion, the classical models that originally predicted this phenomenon have been extended to include the important effects that arise from a dynamic dependence of the burning rate on the local pressure and temperature fields. Thus, the onset of Landau instability has now been shown to occur for sufficiently small negative values of the pressure sensitivity of the burning rate, significantly generalizing previous classical results for this problem that assumed a constant normal burning rate. It has also been shown that the onset of instability occurs for decreasing values of the disturbance wavenumber as the gravitational-acceleration parameter decreases. Consequently, in an appropriate weak-gravity limit, Landau instability becomes a long-wave phenomena associated with the formation of large cells on the liquid-propellant surface. Additionally, a pulsating form of hydrodynamic instability has been shown to occur as well, corresponding to the onset of temporal oscillations in the location of the liquid/gas interface. This instability occurs for sufficiently large negative values of the pressure sensitivity, and is enhanced by increasing values of the burning-rate temperature sensitivity. It is further shown that for sufficiently small values of this parameter, there exists a stable range of pressure sensitivities for steady, planar burning such that the classical cellular form of hydrodynamic instability and the more recent pulsating form of hydrodynamic instability can each occur as the corresponding stability threshold is crossed. For larger thermal sensitivities, however, the pulsating stability boundary evolves into a C-shaped curve in the (disturbance-wavenumber, pressure-sensitivity) plane, indicating loss of stability to pulsating perturbations for all sufficiently large disturbance wavelengths. It is thus concluded, based on characteristic parameter values, that an equally likely form of hydrodynamic instability in liquid-propellant combustion is of a nonsteady, long-wave nature, distinct from the steady, cellular form originally predicted by Landau.

The notion of hydrodynamic instability in combustion originated with Landau's seminal study of premixed flame propagation.[1] In that work, it was postulated that a flame could be represented by a surface of discontinuity propagating normal to itself with constant speed. It was then determined that a premixed gaseous flame was intrinsically unstable to steady (cellular) disturbances. This specific form of hydrodynamic instability, generally referred to as Landau instability, also occurs in the combustion of liquids, and was briefly addressed at the end of Landau's original study. In that problem, the unburned mixture is a liquid propellant and the burned region consists of gaseous products. The physical existence of a liquid/gas interface led to the inclusion of additional physics in the model, namely surface tension and gravitational acceleration (downward propagation was assumed). Consequently, a stability criterion was derived such that the liquid/gas interface was either hydrodynamically stable or unstable in the Landau (cellular) sense, depending on the product of gravitational acceleration and surface-tension coefficient being greater or less than a critical value. This result was later extended by Levich,[2] who considered the effects of (liquid) viscosity in lieu of surface tension and obtained a similar result. Thus, in the limit of sufficiently reduced gravity, these results suggest that liquid-propellant deflagration is intrinsically unstable to hydrodynamic perturbations. However, while the notion of a thin reaction region or sheet is often a valid asymptotic limit, the assumption of a constant normal burning rate is now generally regarded as an oversimplification when applied to the problem of combustion instability.

Early attempts at modifying the assumption of a constant burning rate consisted of postulating a linear relationship between burning rate and flame curvature,[3] while more recent approaches have employed asymptotic methods to analyze the flame structure and derive locally-dependent expressions for the burning rate.[4–7] In propellant combustion, on the other hand, it has long been customary to experimentally measure the pressure response, or pressure sensitivity, of the burning rate, as well as (to a lesser extent) its temperature sensitivity. Although asymptotic models that resolve the combustion-wave structure can be developed,[8–10] the representation of combustion as a surface that propagates according to a prescribed locally-dependent burning-rate law allows one to circumvent the intricacies of the combustion region and to impose fewer restrictions on the hydrodynamic model. Thus, for liquid-propellant combustion, the Landau/Levich hydrodynamic models have been combined and extended to account for a dynamic dependence of the burning rate on the local pressure and temperature fields.[11,12] Analysis of these extended models is greatly facilitated by exploiting the realistic smallness of the gas-to-liquid density ratio ρ, the ratio of the gas-to-liquid viscosity ratio μ, and, in the reduced-gravity limit, the smallness of the inverse Froude number Fr^{-1}. Assuming only pressure-coupling effects (in which case the burning rate is functionally dependent only on the local pressure field), an asymptotic expression may be derived for the cellular stability boundary $A_p(k)$, where A_p is the pressure sensitivity of the burning rate and k is the disturbance wavenumber.[13] In particular, we obtain the result

$$A_p \sim -\rho + \frac{2\rho\mu P\left[1 + k(\rho\gamma + 2\mu P + 2\rho P)\right]}{4\mu P(1+k\rho P) - (\rho\gamma + 2\mu P)\left[1 - (1 + 4\mu^2 P^2 k^2)^{1/2}\right]} + \frac{\rho^2}{2k} Fr^{-1}, \tag{1}$$

where P is the liquid Prandtl number and γ is the surface-tension coefficient. The cellular stability boundaries given by Eq. (1) are shown in Fig. 1 in terms of scaled quantities $A_p^* = \epsilon^{-1} A_p$, $\rho^* = \epsilon^{-1}\rho$ and $\mu^* = \epsilon^{-1}\mu$, where $\epsilon \sim \rho$ is the small expansion parameter (typical values of ρ are on the order of 10^{-3} or 10^{-4}).

Since Eq. (1) is in fact a uniform composite approximation that is valid over the wavenumber regime $\epsilon^2 \lesssim k \lesssim \epsilon^{-1}$, it is clear from both the analysis and Fig. 1 that surface tension and viscosity are stabilizing for large-wavenumber disturbances, whereas gravity is stabilizing only at small wavenumbers. Since the minimum in the stability boundary occurs for negative values of A_p, it is also apparent that in the realistic case $\rho \ll 1$, O(ϵ) negative pressure sensitivities, which often occur in a number of HAN-based formulations over ceretain pressure ranges, are required for hydrodynamic stability in the Landau (cellular) sense. The two distinct sets of curves that are exhibited in Fig. 1 correspond to $Fr^{-1} \sim$ O(1) (normal gravity) and $Fr^{-1} \sim$ O(ϵ) (reduced gravity). Thus, it is readily seen that the essential qualitative difference between the normal and reduced-gravity curves is the location of the critical wavenumber for instability. Specifically, the minimum in the neutral stability boundaries occurs for O(1) values of k under normal gravity, and at $k \sim$ O($\epsilon^{1/2}$) in the reduced-gravity limit considered here. It is also clear from Fig. 3 that increasing the values of any of the parameters P, μ or γ serves to shrink the size of the unstable domain through damping of short-wave perturbations. The non-negligible effects of gas-phase viscosity represents, aside from the incorporation of a local pressure dependence on the burning rate, an important correction to Levich's original treatment [3] in which these effects were simply assumed to be small. The present formulation thus synthesizes and significantly extends the classical Landau/Levich results [1,3], not only in allowing for a dynamic dependence of the burning rate on local conditions in the vicinity of the liquid/gas interface, but also in its formal treatment of those processes (surface tension, liquid and gas-phase viscosity) that affect damping of large-wavenumber disturbances. For $A_p = 0$, the Landau/Levich results are recovered in appropriate limiting cases, although as discussed above, this generally corresponds to a hydrodynamically unstable parameter regime for $\rho \ll 1$.[1,2,13]

Aside from the classical cellular form of hydrodynamic instability, there also exists a pulsating form of instability corresponding to the loss of stability of steady, planar burning to time-dependent perturbations.[14] This occurs for negative values of the parameter A_p that lie below the boundaries shown in Fig. 1, and is thus absent from the original Landau/Levich models. Consequently, in the extended model discussed thus far, there exists a stable band of negative pressure sensitivities bounded above by the Landau type of instability, and below by this pulsating form of hydrodynamic instability, where the latter is given explicitly by $A_p^* \sim -\rho^*(1+2Pk)^{1/2}$. Indeed, nonsteady modes of combustion have been observed at low pressures in hydroxylammonium nitrate (HAN)-based liquid propellants, which often exhibit negative pressure sensitivities.[15] While nonsteady combustion may correspond to secondary and higher-order bifurcations above the cellular boundary,[16] it may also be a manifestation of this pulsating type of hydrodynamic instability.

In addition to the pressure-coupling parameter A_p introduced thus far, the present generalized model has been extended further by incorporating a nonzero temperature sensitivity into the asymptotic analysis.[13,14,17,18] This entails a coupling of the energy equation to the previous purely hydrodynamic problem, and leads to a significant modification of the pulsating boundary such that, for sufficiently large values of a temperature-sensitivity parameter, liquid-propellant combustion can become intrinsically unstable to this alternative form of hydrodynamic instability. Thus, in addition to the pressure-coupling parameter A_p, we also introduce, in an analogous fashion, the temperature-sensitivity parameter A_Θ. In the parameter regime of interest here, the appropriate scale for A_Θ that describes the fully developed effects of thermal coupling is $A_\Theta = A_\Theta^* \epsilon^{1/4}$, although the first effects that lead to the development of the C-shaped stability boundary exhibited in Fig. 2 below occur on the smaller scale $A_\Theta \sim O(\epsilon^{1/2})$. Thus the first effects of thermal sensitivity occur for $A_\Theta/A_p \sim O(\epsilon^{-1/2}) \gtrsim 30$, which is a typical nondimensional value of the overall Arrhenius activation energy.

Referring to Figure 2, it is seen that for $A_\Theta^* > 0$, the pulsating boundary becomes C-shaped, the upper branch approaching the cellular boundary as $k \to \infty$, and the lower branch approaching the original ($A_\Theta^* = 0$) pulsating boundary. The region within the C-shaped curve is stable, and thus not only is steady, planar burning intrinsically unstable for sufficiently small wavenumbers, but also, for larger k, *any* crossing of the C-shaped boundary from the stable to the unstable region corresponds to the onset of a pulsating instability. As A_Θ^* increases, the turning point of the pulsating boundary shifts to the right; as A_Θ^* becomes small, the turning point shifts to small values of k that ultimately lie outside the O(1) wavenumber region. Thus, in the outer wavenumber regime corresponding to sufficiently large wavenumbers, the original pulsating and cellular boundaries are recovered as A_Θ^* decreases, but for A_Θ^* sufficiently large, the original cellular boundary lies within the unstable region and the basic solution becomes intrinsically unstable to oscillatory disturbances. Thus, the lower branch of the composite boundary is a pulsating boundary for all wavenumbers, whereas the upper branch transitions from a pulsating boundary for O(1) wavenumbers to a cellular boundary for $O(\epsilon^{-1})$ wavenumbers. The nature of the evolution, as A_Θ^* decreases, of the pulsating stability boundary depicted in Fig. 2 to the separate pulsating and cellular boundaries described above in the limit $A_\Theta = 0$ may be determined by analyzing the dispersion relation for smaller order-of-magnitude wavenumbers and appropriately rescaled values of A_Θ. In particular, it turns out that this transition occurs as A_Θ decreases through $O(\epsilon^{1/2})$ values,[17] thereby recovering a region of stability in which the classical form of Landau (cellular) instability occurs as A_p^* increases through small negative values.

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REFERENCES

[1] Landau, L. D., *Acta Physicochimica URSS* 19:77–85 (1944).
[2] Levich, V. G., *Dokl. Akad. Nauk SSSR* 109:975–978 (1956).
[3] Markstein, G. H., *Non-Steady Flame Propagation*, Pergamon Press, Oxford, 1964.
[4] Sivashinsky, G. I., *Combust. Sci. Tech.* 15:137–146 (1977).
[5] Pelcé, P., & Clavin, P., *J. Fluid Mech.* 124:219–238 (1982).
[6] Matalon, M., & Matkowsky, B. J., *J. Fluid Mech.* 124:239–259 (1982).
[7] Margolis, S. B., *Prog. Energy Combust. Sci.* 17:135–162 (1991).
[8] Margolis, S. B., & Williams, F. A., *Combust. Sci. Tech.* 59:27–84 (1988).
[9] Margolis, S. B., & Williams, F. A., *SIAM J. Appl. Math.* 49:1390–1420 (1989).
[10] Bechtold, J. K., & Margolis, S. B., *Combust. Sci. Tech.* 68:49-84 (1989).
[11] Armstrong, R., & Margolis, S., *22nd Symp. (Int'l) on Combustion*, 1807–1815 (1989).
[12] Armstrong, R. C., & Margolis, S. B., *Combust. Flame* 77:123–138, (1989).
[13] Margolis, S. B., *Combust. Flame* 113:406–423 (1998).
[14] Margolis, S. B., *Twenty-Seventh Symposium (Int'l) on Combustion*, 2375–2386 (1998).
[15] Vosen, S. R., *Twenty-Second Symposium (Int'l) on Combustion*, 1817–1825 (1989).
[16] Bechtold, J. K., & Margolis, S. B., *SIAM J. Appl. Math.* 51:1356–1379 (1991).
[17] Margolis, S. B., *Combust. Theory Modelling*, 613–635 (1999).
[18] Margolis, S. B., *AIAA Journal*, to appear (2001).

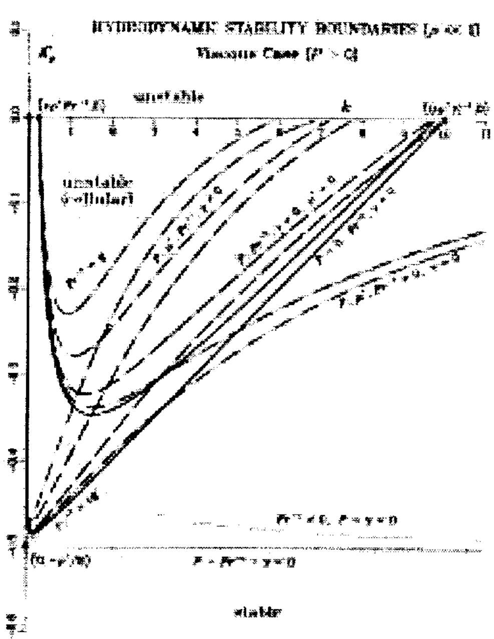

Figure 1

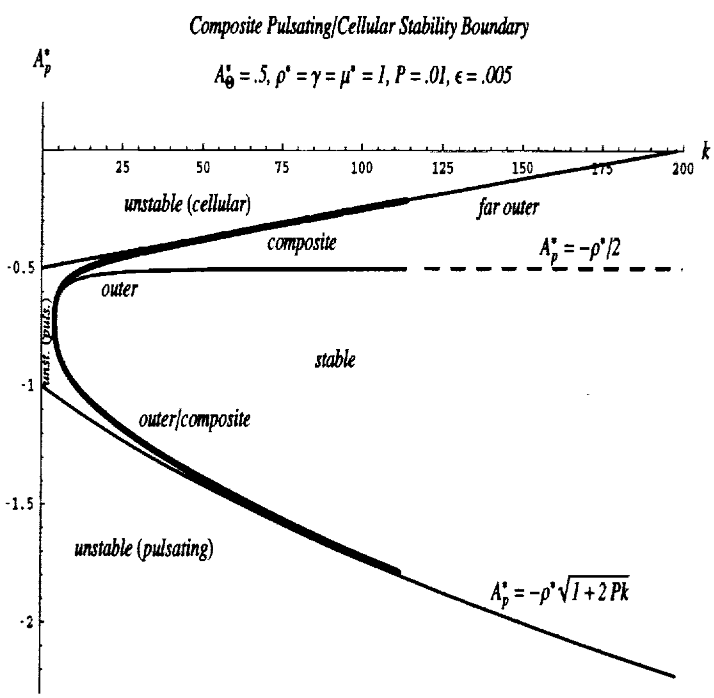

Figure 2