OPTIMIZATION OF A GO_2/GH_2 IMPINGING INJECTOR ELEMENT

P. Kevin Tucker
NASA
Marshall Space Flight Center, AL 35812

Wei Shyy and Rajkumar Vaidyanathan
Department of Aerospace Engineering, Mechanics & Engineering Science
University of Florida
Gainesville, Florida 32611

ABSTRACT

An injector optimization methodology, *method i*, is used to investigate optimal design points for a gaseous oxygen/gaseous hydrogen (GO_2/GH_2) impinging injector element. The unlike impinging element, a fuel-oxidizer-fuel (F-O-F) triplet, is optimized in terms of design variables such as fuel pressure drop, ΔP_f, oxidizer pressure drop, ΔP_o, combustor length, L_{comb}, and impingement half-angle, α, for a given mixture ratio and chamber pressure. Dependent variables such as energy release efficiency, *ERE*, wall heat flux, Q_w, injector heat flux, Q_{inj}, relative combustor weight, W_{rel}, and relative injector cost, C_{rel}, are calculated and then correlated with the design variables. An empirical design methodology is used to generate these responses for 163 combinations of input variables. *Method i* is then used to generate response surfaces for each dependent variable. Desirabilitiy functions based on dependent variable constraints are created and used to facilitate development of composite response surfaces representing some, or all, of the five dependent variables in terms of the input variables. Three examples illustrating the utility and flexibility of *method i* are discussed in detail. First, joint response surfaces are constructed by sequentially adding dependent variables. Optimum designs are identified after addition of each variable and the effect each variable has on the design is shown. This stepwise demonstration also highlights the importance of including variables such as weight and cost early in the design process. Secondly, using the composite response surface which includes all five dependent variables, unequal weights are assigned to emphasize certain variables relative to others. Here, *method i* is used to enable objective trade studies on design issues such as component life and thrust to weight ratio. Finally, specific variable weights are further increased to illustrate the high marginal cost of realizing the last increment of injector performance and thruster weight.

NOMENCLATURE

Symbols

A	minimum acceptable value
B	target value
C	target value
D	composite desirability (joint response)
E	maximum acceptable value
ERE	energy release efficiency
H	height

NOMENCLATURE (Con't)

L	length
MR	momentum ratio
O/F	oxidizer to fuel mass ratio
P	pressure
d	diameter or desirability
m	mass flow rate
u	velocity
α	impingement half-angle
ΔP	pressure drop

Subscripts

c	chamber
comb	combustor
f	fuel
fs	freestream
impinge	impingement
inj	injector
ni	normalized injection
o	oxidizer
rel	relative
w	wall

Superscripts

s	desirability function weight
t	desirability function weight

INTRODUCTION

In order to meet future launch program goals, the Spaceliner 100 Technology Roadmap[1] specifies very aggressive system goals for safety, life and cost per pound of payload launched into Earth orbit. Spaceliner 100 safety goals would decrease catastrophic events from the current 1 in 200 to 1 in 1,000,000 in 15 years. The life goal would be increased from the current 200 manned missions per year to 2000-5000 per year over the same time period. Concurrently, the cost goal aims to reduce the cost of delivering payloads to Earth orbit from the current $10,000 per pound to $1000 per pound in 10 years and to $100 per pound in 15 years and ultimately to $10 per pound.

NEED FOR IMPROVED INJECTOR DESIGN METHODOLIGIES

Design and development of advanced propulsion systems will be crucial to meeting these goals. Propulsion systems which meet these requirements must not only have high thrust to weight ratios, but also achieve higher operability and maintainability standards than in previous or current programs. Combustor designs, and injector designs in particular, will be key issues in meeting these goals. The injector design determines performance and stability, and is, therefore, the key factor governing injector face and chamber wall heat transfer/compatibility issues. Injector design also affects engine weight, cost, operability and maintainability.

The injector design methodologies used successfully in previous programs were typically based on large subscale databases and the empirical design tools derived from them[2,3,4,5,6]. These methodologies were often guided by extensive sub-and full-scale hot-fire test programs. Current and planned launch vehicle programs have relatively low budgets and aggressive schedules; neither of which is conducive to the large test programs of the past. Also, new requirements for operability and maintainability require that the injector design be robust. Hence, variables not previously included in the injector design now merit consideration for inclusion in the design process. These new programs with compressed schedules, lower budgets and more stringent requirements make the development of broader and more efficient injector design methodologies an worthy goal.

METHOD I

This work demonstrates a new design methodology called *method i* [7,8](Methodology for Optimizing the Design of Injectors) which seeks to address the above issues in the context of injector design. Simply put, *method i* is used to generate appropriate design data and then guide the designer through the information toward an optimum design subject to his specified constraints. Since the information generated by *method i* is not linked to any information type or source, it potentially affords the designer the ability to consider any relevant combination of design variables for a wide variety of injector types and propellant combinations. This generality also allows *method i* to use information at varying levels of breadth (i.e., scope of design variables) and depth (i.e., detail of design variables). Hence, *method i* could be useful for both element selection and the preliminary design phase. Once injector selection and preliminary designs are accomplished, *method i* can be used to optimize the injector design. Since *method i* is structured so that any pertinent information source can be used, design data can be obtained from existing databases and empirical design methodologies. If required, new data can be generated with modern experimental techniques or appropriate CFD models.

As implied above, *method i* is comprised of two discrete entities. The first element is the tool used to generate the design data—in this work, an empirical design methodology for GO_2/GH_2 injectors. Injector designs using GO_2/GH_2 propellants serve as a good point for the initial evaluation of *method i* for a number of reasons. First, the physics of the system are relatively simple. Atomization and vaporization do not complicate matters as they do when a liquid propellant is present. Also, an experimental database developed by Calhoon et al.[9] exists along with an empirical design methodology[10] derived from the data. Finally, should additional information be required, both modern laser-based diagnostic techniques[11,12,13,14] and CFD modeling[14,15] have been successfully applied to injector elements using GO_2/GH_2 propellants.

The second entity in *method i* is a group of optimization techniques. It is the optimization capability that extends *method i* beyond previous injector design methodologies. Historically, injectors have been designed, fabricated and tested based on experience and intuition. As the hardware was tested, designers proposed modifications aimed at obtaining an improved design. Despite their experience and skill, these efforts were unlikely to produce the optimal design in a short time frame. Also, as more design variables are considered, the design process becomes increasingly complex and it is more difficult to foresee the effect of the modification of one variable on other variables. Use of an optimization approach to guide the design addresses both of these issues. The optimization scheme allows large amounts of inter-related information to be managed in such a way that the extent to which variables influence each other can be objectively evaluated and optimal design points can be identified with confidence. *Method i* currently uses the Response Surface Method (RSM)[16] to facilitate the optimization. The RSM approach is to conduct a series of well-chosen experiments (i. e., numerical, physical, or both) and use the resulting function values to construct a global approximation (i. e., response surface) of the measured quantity (i. e., response) over the design space. A standard constrained optimization algorithm is then used to interrogate the response surface for an optimum design.

DEVELOPMENT APPROACH AND STATUS

The approach used to develop and demonstrate this new methodology can be divided into three main tasks. Task 1 can be viewed as a proof of concept where the basic methodology is developed and demonstrated on single element injectors. This task involves demonstration of *method i* in the element selection/preliminary design process. Design data from empirical methodologies is to be generated for three major element types—shear coaxial, swirl coaxial and impinging elements. In addition to the typical design output variable such as performance and heat flux, a goal is to enable the inclusion of additional parameters such as cost and weight early in the design process. This work for the shear coaxial element is essentially complete[7] and the work for the impinging element is presented below. Generation of design data for the swirl coaxial element will finish the empirical database for Task 1. Then a swirl coaxial element will be optimized in a process similar to what has been done with the other two elements. Finally, to complete Task 1, all the design data, along with the optimization techniques developed to date, will be demonstrated in an element selection/preliminary design process.

Also, any potential "show stoppers" are to be identified and addressed in Task 1. Empirical design methodologies, such as found in Calhoon et al, may allow the designer to generate large quantities of data within a design space. However, due to their empiricism, these methodologies are often sufficiently accurate only over the range of variables for which test data was taken to develop the methodology. For some injector types, propellant combinations or design conditions, this limitation may require that more relevant data be generated to ensure confidence in the design. Historically, this data has been generated in sub- and full-scale test programs. More recently CFD analysis from validated models has been used to augment the test data. The data from test programs and CFD analysis are expensive and time consuming to obtain. Recognition of this fact has direct implications for the usefulness of optimization techniques in injector design methodologies. Although the optimization scheme must be capable of efficiently organizing large amounts of design information generated from empirical design methodologies, it must also be able to make effective use of the relatively small amounts of data available in some cases. An optimization scheme that requires large amounts of data to generate meaningful results will be marginally useful, if at all, when only small amounts of data are available for use. This potential shortcoming was addressed by using Neural Networks to augment the design optimization process[8]. In a process that simulated a case where only a limited amount of design data was available, a radial basis neural network was trained on the available data and then used to generate additional design data. The accuracy of the new data proved to be sufficient to allow it to be used reliably in the design optimization process.

Task 2 involves replacing/augmenting the empirical data with data from physical and numerical experiments (i.e., test data and validated CFD analyses). CFD models will be further validated and applied to selected cases already represented by data from the empirical methodology. Allowance in the optimization process will be made for the differences in depth and breadth of the different types of information since data from physical and numerical experiments are multi-dimensional and allow more design variables to be examined and included in the process. Also, in general, the numerical and physical experiments should be more accurate than the empirical data used to date. The different levels of accuracy must therefore be addressed in Task 2.

Task 3 involves using CFD analyses and empirical methods to design a multi-element injector consisting of 7-12 elements. Optimization will be done in the context of single element variables plus element pattern, element spacing, film cooling, etc.

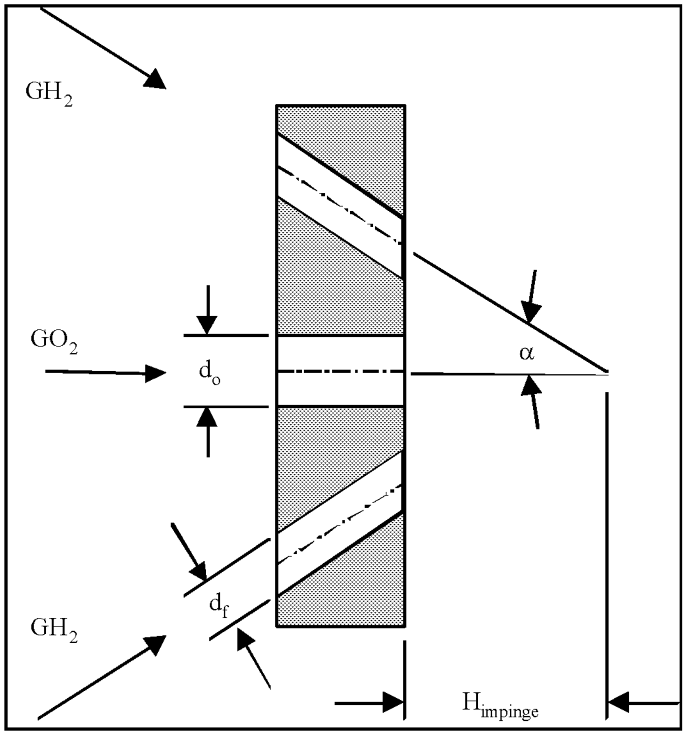
SCOPE OF CURRENT EFFORT

This paper presents the design optimization of a impinging injector—the second element to be evaluated in Task 1. The first element to be evaluated in Task 1 was a shear coaxial GO_2/GH_2 element. Here, an F-O-F triplet element is chosen for the demonstration. This element type is widely used and is capable of operating at high efficiency levels. A schematic of an F-O-F element is shown in Fig. 1.

The empirical design methodology of Calhoon et al uses the oxidizer pressure drop, ΔP_o, fuel pressure drop, ΔP_f, combustor length, L_{comb}, and the impingement half-angle, α as independent variables. For this injector design, the pressure drop range is set to 10-20% of the chamber pressure due to stability considerations. The combustor length, defined as the distance from the injector to the end of the barrel portion of the chamber ranges from 2-8 inches. The impingement half angle is allow to vary from 15-50°.

Dependent variables include *ERE* (a measure of element performance), wall heat flux, Q_w, injector heat flux, Q_{inj}, relative combustor weight, W_{rel}, and relative injector cost, C_{rel}.

Figure 1. Schematic of F-O-F Injector Element



In the following sections, the injector model and the generation of design data are discussed in some detail. Response surfaces for each of the dependent variables are generated and then combined into a joint surface to facilitate the optimization process. Optimization of the element is then demonstrated by applying equal weights for all dependent variables as they are added to the joint response surface one at a time, by applying unequal weights that might reflect specific design priorities and trades, and finally, over a modified constraint range, by examining the extraction of the last increments of certain variables and the high marginal cost this process levies on other variables.

F-O-F INJECTOR MODEL

This section details the models used to generate the design data for the dependent variables noted above. The process for generating the design data is described and sample results are also presented. The conditions selected for this example are:

$$P_c = 1000\,psi$$

$$MR = 6$$

$$m_{GO_2} = 0.25 lb_m / \sec$$

$$m_{GH_2} = 0.042 lb_m \sec$$

The gaseous propellants are injected at a temperature of 540 R.

MODELS FOR DEPENDENT VARIABLES

As noted above, the empirical design methodology used to characterize the ERE and Q_w was developed by Calhoon et al. This methodology uses a quantity called the normalized injection momentum ratio to correlate the mixing at the different design points for the triplet element. They define this quantity as

$$MR_{ni} = \frac{2.3 m_o u_o}{m_f u_f \sin\alpha} \qquad (1)$$

The maximum mixing, and thus maximum ERE, occurs at an MR_{ni} of 2.0. Since the propellant mass flowrates are fixed, only the propellant velocities and the impingement half-angle influence the normalized injection momentum ratio. The velocities are proportional to the square root of the respective pressure drops across the injector, ΔP_o and ΔP_f. For the flow conditions and variable ranges considered in this problem, MR_{ni} ranges from 3.2 to 17.8. Accordingly, lowering ΔP_o, raising ΔP_f, increasing α, or some combination of these actions will increase ERE.

The wall heat flux is correlated with the propellant momentum ratio as defined by

$$MR = \frac{m_o u_o}{m_f u_f} \qquad (2)$$

For the F-O-F triplet element, the maximum wall heat flux occurs at a momentum ratio of approximately 0.4. High heat flux is the result of over-penetration of the fuel jet which produces a high O/F in the wall region. For the flow conditions and variable ranges considered in this effort, MR ranges from 1.06 to 2.11. Hence, increasing the value of this ratio by either increasing ΔP_o or decreasing ΔP_f lowers the wall heat flux.

The heat flux seen by the injector face, Q_{inj}, is qualitatively modeled by the impingement height, $H_{impinge}$. The notion being that, as the impingement height decreases, the combustion occurs closer to the injector face, causing a proportional increase in Q_{inj}. Thus, for the purposes of this exercise, Q_{inj} is modeled as the reciprocal of the $H_{impinge}$. Impingement height is a function of α and ΔP_f. Reference to Fig. 1 shows that as α is increased, $H_{impinge}$ is shortened. The dependence of $H_{impinge}$ on the fuel orifice diameter, d_f, and thus, ΔP_f, results from making the freestream length of the fuel jet, L_{fs}, a function of d_f[17]. For each ΔP_f, L_{fs} was set to six times d_f for an impingement half-angle of 30°. So, as d_f increases (corresponding to decreasing ΔP_f), L_{fs} increases, as does $H_{impinge}$.

The models for W_{rel} and C_{rel} are simple but represent the correct trends. W_{rel} is a function only of L_{comb}, the combustor length from injector face to the end of the chamber barrel section. The dimensions of the rest of the thrust chamber assembly are assumed to be fixed. So, as L_{comb} increases, W_{rel} increases accordingly. The model for C_{rel} is based on the notion that smaller orifices are more expensive to machine. Therefore, C_{rel} is a function of both propellant pressure drops. As the ΔP's increase, the propellant velocity through the injector increases and the orifice area decreases. So, as either, or both, ΔP_o and ΔP_f increase, C_{rel} increases.

GENERATION OF DESIGN DATA

The system variables given above and independent variables (constrained to the previously noted ranges) are used to generate the design data for element optimization studies. Since propellant momentum ratio is an important variable in the empirical design methodology, a matrix of momentum ratios was developed over the 100-200 psi propellant pressure drop range. The matrix of 49 combinations of fuel and oxidizer pressure drops is shown in Table 1 where momentum ratios range from 1.06 to 2.11. Nine pressure drop combinations, eight around the border and one in the middle, were selected for use in populating the design data base. These nine points are highlighted in Table 1 in bold type.

Table 1. Propellant Momentum Ratio as a Function of Propellant Pressure Drops.

	ΔP_o						
ΔP_f	200	180	160	150	140	120	100
200	**1.49**	1.42	1.33	**1.30**	1.25	1.16	**1.06**
180	1.57	1.50	1.41	1.37	1.32	1.22	1.11
160	1.67	1.59	1.50	1.45	1.40	1.30	1.18
150	**1.73**	1.64	1.54	**1.49**	1.44	1.34	**1.22**
140	1.79	1.70	1.60	1.55	1.50	1.39	1.27
120	1.93	1.83	1.72	1.67	1.61	1.50	1.37
100	**2.11**	2.00	1.89	**1.83**	1.77	1.64	**1.49**

Detailed design results for the case with both ΔP_o and ΔP_f at 200 psi are shown in Table 2. Similar data was generated for the other eight pressure drop combinations. There are 20 combinations of L_{comb} and α for each ΔP combination, making a total of 180 design points selected. Seventeen of these were outside the database embodied by the empirical design methodology, resulting in 163 design points actually being evaluated. The data trends are as expected. *ERE*, for a given ΔP combination, increases with increasing L_{comb} and α. The increased L_{comb} provides more residence time for the propellants to mix and burn. Increasing α increases the radial component of the injected fuel, thus providing better mixing. The wall heat flux is constant for a given ΔP combination. Impingement height increases with increasing α. Relative combustor cost increases with increasing L_{comb} and the relative injector cost is constant for a given ΔP combination.

Table 2. Design Data for ΔP_o and $\Delta P_f = 200$ psi.

ΔP_o	ΔP_f	L_{comb}	α	ERE	Q_w	$H_{impinge}$	W_{rel}	C_{rel}
200	200	2	15	NA	0.85	0.84	0.923	1.083
200	200	2	20	85	0.85	0.62	0.923	1.083
200	200	2	30	92.8	0.85	0.39	0.923	1.083
200	200	2	45	95.4	0.85	0.23	0.923	1.083
200	200	2	50	95.8	0.85	0.19	0.923	1.083
200	200	4	15	91	0.85	0.84	1	1.083
200	200	4	20	95.2	0.85	0.62	1	1.083
200	200	4	30	96.8	0.85	0.39	1	1.083
200	200	4	45	98.1	0.85	0.23	1	1.083
200	200	4	50	98.4	0.85	0.19	1	1.083
200	200	6	15	95.6	0.85	0.84	1.077	1.083
200	200	6	20	97.8	0.85	0.62	1.077	1.083
200	200	6	30	98.5	0.85	0.39	1.077	1.083
200	200	6	45	99.2	0.85	0.23	1.077	1.083
200	200	6	50	99.4	0.85	0.19	1.077	1.083
200	200	8	15	98.3	0.85	0.84	1.154	1.083
200	200	8	20	99.1	0.85	0.62	1.154	1.083
200	200	8	30	99.4	0.85	0.39	1.154	1.083
200	200	8	45	99.6	0.85	0.23	1.154	1.083
200	200	8	50	99.7	0.85	0.19	1.154	1.083

RESPONSE SURFACE GENERATION

In this effort, *method i* uses the Response Surface Method (RSM) to find optimal values of ERE, Q_w, Q_{inj}, W_{rel} and C_{rel} for acceptable values of ΔP_o, ΔP_f, L_{comb} and α. The approach of RSM is to perform a series of experiments, or numerical analyses, for a prescribed set of design points, and to construct a response surface of the measured quantity over the design space. In the present context, the five responses of interest are ERE, Q_w, Q_{inj}, W_{rel} and C_{rel}. The design space consists of the set of relevant design variables ΔP_o, ΔP_f, L_{comb} and α. The response surfaces are fit by standard least-squares regression with a quadratic polynomial using the JMP[18] statistical analysis software. JMP is an interactive, spreadsheet-based program which provides a variety of statistical analysis functions. A backward elimination procedure based on t-statistics is used to discard terms and improve the prediction accuracy[19].

INDIVIDUAL RESPONSE SURFACES

When the JMP software is used to analyze the 163 design points, five individual full response surfaces for the variables in the design space are approximated by quadratic polynomials that contain 15 terms each. Using the t-statistics approach noted above and detailed in Tucker et al[7], unnecessary terms in each equation can be eliminated to give the reduced surfaces shown below in equations 3-7.

$$ERE = 0.0028 L_{comb}\Delta P_o - 0.0043 L_{comb}\Delta P_f - 0.2248 L_{comb}^2 + 0.00024\Delta P_o\alpha - 0.000511\Delta P_f\alpha - 0.0445 L_{comb}\alpha \quad (3)$$
$$-0.006\alpha^2 - 0.0311\Delta P_o + 0.0547\Delta P_f + 5.268 L_{comb} + 0.814\alpha + 63.344$$

$$Q_w = 0.000017\Delta P_o^2 - 0.0000211\Delta P_o\Delta P_f + 0.0000075\Delta P_f^2 - 0.0043\Delta P_o + 0.0029\Delta P_f + 0.959 \quad (4)$$

$$H_{impinge} = 0.0000034\Delta P_f^2 + 0.000028\Delta P_f\alpha + 0.00058\alpha^2 - 0.0027\Delta P_f - 0.061\alpha + 1.924 \quad (5)$$

$$W_{rel} = 0.0385 L_{comb} + 0.846 \quad (6)$$

$$C_{rel} = -0.0000035\Delta P_o^2 + 0.0000065\Delta P_f^2 - 0.0043\Delta P_o - 0.00096\Delta P_f + 0.845 \quad (7)$$

A survey of the reduced response surfaces indicates that the equations reflect the functionality used to construct the models for the dependent variables.

JOINT RESPONSE SURFACES

In the current study, it is desirable to attempt to maximize ERE and while simultaneously minimizing Q_w, Q_{inj}, W_{rel} and C_{rel}. One method of optimizing multiple responses simultaneously is to build from the individual responses a composite response known as the desirability function. The method allows for a designer's own priorities for the response values to be built into the optimization procedure. The first step in the method is to develop a desirability, d, for each response. In the case where a response should be maximized, such as ERE, the desirability takes the form:

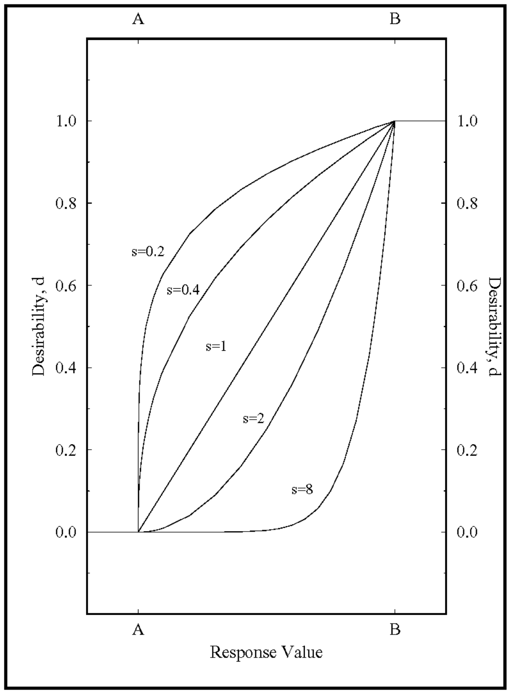
$$d_1 = \left(\frac{ERE - A}{B - A}\right)^s \quad (8)$$

where B is the target value and A is the lowest acceptable value such that $d = 1$ for any $ERE > B$ and $d = 0$ for $ERE < A$. The power value s is set according to one's subjective impression about the role of the response in the total desirability of the product. In the case where a response is to be minimized, such as Q_w, the desirability takes on the form:

$$d_2 = \left(\frac{Q - E}{C - E}\right)^t \quad (9)$$

where C is the target value and E is the highest acceptable value such that $d = 1$ for any $Q_w < C$ and $d = 0$ for $Q_w > E$. Choices for A, B, C, and E are chosen according to the designer's priorities or, as in the present study, simply as the boundary values of the domain of ERE and Q_w .

Figure 2. Desirability Function for Various Weight Factors, *s*.



Choices for *s* and *t* are more difficult, but plots such as Figure 2 can be instructive. Figure 2 shows the appearance of the desirability function for the case of maximizing a response. Desirabilities with $s<<1$ imply that a product need not be close to the response target value, B, to be quite acceptable. But $s = 8$, say, implies that the product is nearly unacceptable unless the response is close to B.

A single composite response is developed which is the geometric mean of the desirabilities of the individual responses. The composite response is defined as:

$$D = \left(d_1 \cdot d_2 \cdot d_3 \ldots d_m\right)^{1/m} \tag{10}$$

The complete joint response surface for the present case is given by:

$$D = \left(d_{ERE} d_{Q_w} d_{Q_{inj}} d_{W_{rel}} d_{C_{rel}}\right)^{1/5} \tag{11}$$

OPTIMIZATION RESULTS & DISCUSSION

Three set of results are presented below to demonstrate the capability of *method i* for the current injector design. These three examples illustrate the effect of each variable on the optimum design, the trade-offs between life and performance issues, and the effect on the design of extracting the last increment of performance.

EFFECT OF EACH VARIABLE ON THE DESIGN USING ORIGINAL CONSTRAINTS & EQUAL WEIGHTS

The results in this section were obtained by building the joint response surface with the addition of one dependent variable at a time. The results are shown in Table 3. Since current non-optimizer based design methods yield high-performing injector elements, simply maximizing the *ERE* is not a challenge. Accordingly, the initial results (Case 1) are obtained with a joint *ERE* and Q_w response surface. The results in Case 2 have the impingement height added, Case 3 adds the relative chamber weight and the relative cost is added in Case 4. All results are obtained using the original independent variable constraints and all dependent variables have equal weights of one. The results for Case 1 show that *ERE* is at its maximum and Q_w is very near its minimum desirability limit. Minimizing Q_w requires a small ΔP_f relative to ΔP_o as evidenced by the values of 100 psi and 183 psi, respectively. Maximum *ERE* values are found at the longest chamber length, L_{comb}=8 inches. Even with the relatively high value of 183 psi for ΔP_o and low value of ΔP_f of 100 psi, *ERE* is maximized to 99.9% with an impingement half-angle of 33.1°.

Table 3. Effect of Each Variable on the Design--Optimal Designs for Original Constraints & Equal Weights

Independent Variable	Constraints	Results Case 1	Results Case 2	Results Case 3	Results Case 4
ΔP_o	100-200	183	183	179	100
ΔP_f	100-200	100	132	149	100
L_{comb}	2-8	8.0	8.0	6.6	6.5
α	15-50	33.1	18.9	22.3	24.0
Dependent Variable	**Desirability Limits**	**ERE & Q_w**	**ERE, Q_w, $H_{impinge}$**	**ERE, Q_w, $H_{impinge}$, W_{rel}**	**ERE, Q_w, $H_{impinge}$, W_{rel}, C_{rel}**
ERE	95.0-99.9	99.9	98.3	98.0	98.0
Q_w	0.7-1.3	0.74	0.76	0.79	0.86
$H_{impinge}$	0.2-1.0	—	0.75	0.61	0.63
W_{rel}	0.9-1.2	—	—	1.1	1.1
C_{rel}	0.7-1.1	—	—	—	0.93

Addition of the impingement height to Case 2 to model the injector face heat flux, Q_{inj}, forces α lower to increase $H_{impinge}$ and decrease Q_{inj}. This decrease in the radial component of the fuel momentum has an adverse affect on *ERE*. This effect is mitigated to a degree by increasing the ΔP_f by 32 psi to 132 psi. *ERE* is still reduced by 1.6%. Also, the increase in ΔP_f causes increased penetration of the fuel jet which results in a slightly higher Q_w.

Case 3 adds the relative combustor weight to the list of dependent variables modeled. Since W_{rel} is only a function of L_{comb}, minimizing W_{rel} shortens the combustor length from 8 to 6.6 inches. The shorter L_{comb} tends to lower *ERE*. This effect is offset to a large degree by increases in ΔP_f and α, both of which increase the radial component of the fuel momentum. The increase in ΔP_f also causes a slight increase in Q_w. The increase in α causes a significant decrease in $H_{impinge}$ which increases the injector face heat flux.

Finally, the relative cost of the injector is added in Case 5. Since C_{rel} is only a function of propellant pressure drops, both ΔP_o and ΔP_f are driven to their respective minimum values. This and a slight increase in α allow *ERE* to be maintained at 98%, even with a slight decrease in L_{comb}. The largest effect of this fairly dramatic decrease in propellant pressure drops is on Q_w. Even though the values for ΔP_o and ΔP_f fell, ΔP_f increased relative to ΔP_o causing Q_w to increase by almost 9%. Impingement height and relative combustor weight are essentially unchanged.

Although several of the variables included in this exercise are qualitative, an important conclusion can still be drawn. The sequential addition of dependent variables to an existing design results in changes to both the independent and dependent variables in the existing design. The direction and magnitude of these changes depends on the sensitivity of the variables, but the changes may well be significant. The design in Case 4 is quite different that the one in Case 1. Consideration of a larger design space results in a different design—the sooner the additional variables are considered, the more robust the final design will be.

EMPHASIS ON LIFE & PERFORMANCE ISSUES USING ORIGINAL CONSTRAINTS & UNEQUAL WEIGHTS

The purpose of this section is to illustrate the effect of emphasizing certain aspects of the design during the optimization process. *Method i* allows this emphasis via the weights applied to the desirability functions in the joint response surface. The set of results shown in Table 4 facilitate the illustration. The Case 1 (baseline) results are repeated from Case 4 in Table 3 where the entire design space is considered with the original constraints and equal weights for the dependent variables. The results in the Case 2 column are obtained by emphasizing the minimization of the wall and injector face heat fluxes. Desirability functions for both of these variables are given a weight of five. Since lower heat fluxes tend to increase component life, weighting these two variables is equivalent to emphasizing a life-type issue in the design. As expected, α is decreased to increase $H_{impinge}$, thus decreasing Q_{inj}. Since the fuel pressure drop is already at the minimum, the oxidizer pressure drop is increased by 58% to decrease Q_w. Both of these changes tend to decrease *ERE*. While *ERE* does decrease, the effect is somewhat mitigated by an increase in L_{comb}. The increases in L_{comb} and ΔP_o cause increases in W_{rel} and C_{rel}, respectively. The emphasis on life extracts the expected penalty on performance. Additionally, for the current model, there are also weight and cost penalties.

Table 4. Effect of Emphasizing & Life & Performance Issues—Optimal Designs for Original Constraints and Modified Weights

Independent Variable	Constraints	Results Case1	Constraints	Results Case 2	Constraints	Results Case 3
ΔP_o	100-200	100	100-200	158	100-200	100
ΔP_f	100-200	100	100-200	100	100-200	137
L_{comb}	2-8	6.5	2-8	7.7	2-8	5.2
α	15-50	24.0	15-50	15.0	15-50	36.0
Dependent Variable	***Baseline* Variable Weight**		***Life* Variable Weight**		***Thrust/Weight* Variable Weight**	
ERE	1	98.0	1	96.7	5	99.1
Q_w	1	0.86	5	0.75	1	0.95
$H_{impinge}$	1	0.63	5	0.94	1	0.32
W_{rel}	1	1.10	1	1.14	5	1.05
C_{rel}	1	0.93	1	0.97	1	0.95

The results for Case 3 are obtained by emphasizing maximization of *ERE* and minimization of W_{rel} with desirability weightings of five. Increased weighting for these two variables is equivalent to emphasizing a thrust to weight goal for the injector/chamber. The relative chamber length is shortened to lower W_{rel}. *ERE* is maximized by increasing the radial momentum of the fuel jet. Both ΔP_f and α are increased to accomplish *ERE* maximization. As noted earlier, increasing ΔP_f and α lead to increased wall and injector heat fluxes, respectively. Reference to Table 4 indicates that to be the case here. For this case, emphasis on thrust and weight tend to have an adverse affect on both Q_w and Q_{inj}. Relative cost, for the current model, is not significantly affected.

EXTRACTION OF LAST PERFORMANCE & WEIGHT INCREMENTS (MODIFIED CONSTRAINTS & UNEQUAL WEIGHTS)

Here, the high marginal cost of realizing the last increment of thrust to weight is shown. This section illustrates the capability to modify the constraints on the independent variables and use unequal weights on the dependent variables at the same time. The results for Case 3 in Table 4 are carried over to Case 1 in Table 5 as the baseline for this example. Here the original constraints are used but increased weights have been applied to emphasize *ERE* and W_{rel}. Cases 2 and 3 modify the constraints on the propellant pressure drops, raising the minimum pressure drop from 100 psi to 150 psi. For Case 2, both ΔP_o and ΔP_f are now at the minimum level for the modified constraints. L_{comb} is increased slightly to maintain *ERE*. The decrease of ΔP_f relative to ΔP_o causes a decrease in Q_w. The slightly higher pressure drops also cause C_{rel} to increase somewhat. Other variables are not changed appreciably.

Table 5. Effects of Realizing the Last Increments of Performance & Weight—Optimum Designs for Modified Constraints and Unequal Weights

Independent Variable	Original Constraints	Results Case1	Modified ΔP Constraints	Results Case 2	Modified ΔP Constraints	Results Case 3
ΔP_o	100-200	100	150-200	150	150-200	150
ΔP_f	100-200	137	150-200	150	150-200	200
L_{comb}	2-8	5.2	2-8	5.4	2-8	4.4
α	15-50	36.0	15-50	35.6	15-50	44.8
Dependent Variable	**Variable Weight (5:1)**		**Variable Weight (5:1)**		**Variable Weight (100:1)**	
ERE	5	99.1	5	99.0	10	99.1
Q_w	1	0.95	1	0.84	0.1	0.95
$H_{impinge}$	1	0.32	1	0.31	0.1	0.21
W_{rel}	5	1.05	5	1.05	10	1.01
C_{rel}	1	0.95	1	1.00	0.1	1.07

For Case 3, *ERE* and W_{rel} are further emphasized by increasing their desirability weights to 10 while decreasing the other weights to 0.1. L_{comb} is shortened to respond to the increased emphasis on weight minimization. Maintaining the high level of *ERE* requires large increases in ΔP_f and α to increase the radial component of the fuel jet momentum. The increase in ΔP_f causes over-penetration of the fuel jet which results in an increase in wall heat flux. The large increase in α yields the expected decrease in $H_{impinge}$ which increases the injector face heat flux. The additional emphasis on *ERE* and C_{rel} yields essentially no increase in *ERE* in this range of ΔP*'s*, although a small weight savings is seen. These marginal improvements are offset by fairly large increases in C_{rel} and Q_{inj}.

SUMMARY

An unlike impinging GO_2/GH_2 injector element design has been employed to facilitate optimization studies. Starting with propellant pressure drops, combustor length, and impingement half-angle, an empirical design methodology was used to calculate the dependent variables for 163 design points. The dependent variables were energy release efficiency, chamber wall and injector face heat fluxes, relative chamber weight, and relative injector cost. The response surface methodology was used to fit the results with quadratic polynomials. Desirability functions were used to create joint response surfaces which were used in the optimization studies.

Three sets of results were generated to illustrate the capability of *method i* in the context of injector design and optimization. The first set of results started with a design optimized for *ERE* and Q_w, then added the other three dependent variables to the design one at a time. Each sequential optimal design was different than previous designs with the final design being quite different than the initial design. The result qualitatively showed the importance of including as many variables as possible early in the design. The optimization techniques embodied in *method i* facilitate this early inclusion by allowing efficient management of large amounts of data.

The second set of results focuses on the inherent design trade-offs between performance and component life issues. Different weights were applied to emphasize variables related to performance (*ERE* and W_{rel}). While the thrust to weight ratio was improved, the adverse affect on variables related to component life (Q_w and Q_{inj}) were clearly shown. Conversely, when Q_w and Q_{inj} were emphasized, the toll on the performance variables was clear. These techniques can be used to identify both qualitative trends and to examine the quantitative trade-offs present in this and other design processes.

Finally, a third set of results was used to illustrate the effect on the over all design of different degrees of emphasis on certain variables. Over a narrower range of some of the independent variables, *ERE* and W_{rel} were weighted over the other variables by a factor of 5 and then by a factor of 100 in the composite desirability function. As the emphasis on *ERE* and W_{rel} was increased, the resulting marginal improvements were shown to be offset by the fairly large adverse effects on the other variables. *Method i* allows the designer to objectively evaluate these adverse effects as he seeks to improve the design.

The flexibility and utility of *method i* have been demonstrated in this effort. Use of *method i* can allow an injector designer to confidently and efficiently manage large amounts of data to conduct a range of design optimization studies. Constraints on independent variables can be modified to allow optimum designs to be sought in specific portions of the parameter space. Also, individual or specific groups of dependent variables can be emphasized to reflect a designer's priorities in the design optimization process.

A similar study will be conducted for a GO_2/GH_2 swirl coaxial injector element. Then, the data and response surfaces generated for the shear coaxial, swirl coaxial, and impinging elements will be used to demonstrate the ability of *method i* to select an optimum element type based on a range of constraints and design priorities.

REFERENCES

1. Personal Communication with Garry Lyles.
2. Rupe, J. H., "An Experimental Correlation of the Nonreactive Properties of Injection Schemes and Combustion Effects in a Liquid Rocket Engine," NASA TR 32-255, 1965.
3. Pieper, J. L., "Oxygen/Hydrocarbon Injector Characterization," PL-TR 91-3029, October, 1991.
4. Nurick, J. H., "DROPMIX - A PC Based Program for Rocket Engine Injector Design," JANNAF Propulsion Conference, Cheyenne, Wyoming, 1990.
5. Dickerson, R., Tate, K., Nurick, W., "Correlation of Spray Injector Parameters with Rocket Engine Performance," AFRPL-TR-68-11, January, 1968.
6. Pavli, A. L., "Design and Evaluation of High Performance Rocket Engine Injectors for Use with Hydrocarbon Fuels," NASA TM 79319, 1979.
7. Tucker, P. K., Shyy, W, Sloan, J. G., "An Integrated Design/Optimization for Rocket Engine Injectors," 34th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, July 13-15, 1998, AIAA 98-3513, Cleveland, OH.
8.Shyy, W, Tucker, P. K. and Vaidyanathan, R., "Response Surface and Neural Network Techniques for Rocket Engine Injector Optimization," 35th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit, June 20-23, 1999, AIAA 99-2455, Los Angeles, CA.
9. Calhoon, D., Ito, J. and Kors, D., "Investigation of Gaseous Propellant Combustion and Associated Injector-Chamber Design Guidelines," Aerojet Liquid Rocket Company, NASA CR-121234, Contract NAS3-13379, July 1973.
10. Calhoon, D., Ito, J. and Kors, D., "Handbook for Design of Gaseous Propellant Injectors and Combustion Chambers," NASA CR-121234, Contract NAS3-13379, July 1973

11. Foust, M. J., Pal S., and Santoro, R. J. "Gaseous Propellant Rocket Studies Using Raman Spectroscopy," AIAA 96-2766, 32nd AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 1-3, 1996, Lake Buena Vista, FL.
12. Moser, M. D. Pal, S. and Santoro R. J., "Laser Light Scattering Measurements in a GO_2/GH_2 Uni-Element Rocket Chamber," AIAA 95-0137, 33rd Aerospace Sciences Meeting and Exhibit, January 9-12, 1995, Reno NV.
13. Santoro, R. J., "An Experimental Study of Characteristic Combustion-Driven Flow for CFD Validation," Final Report for NASA Contract NAS8-38862, October, 1997.
14. Tucker, P. K., Klem, M. D., Smith, T. D., Farhangi, S., Fisher, S. C., and Santoro, R. J., "Design of Efficient GO_2/GH_2 Injectors: A NASA, Industry and University Effort," AIAA 97-3350, 33rd AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 6-9, 1997, Seattle, WA.
15. Schley, C. A., Hagemann, G., Tucker, P. K., Venkateswaran, S. and Merkle, C. L., "Comparison of Computational Codes for Modeling Hydrogen-Oxygen Injectors," AIAA 97-3302, 33rd AIAA/ASME/SAE/ASEE Joint Propulsion Conference, July 6-9, 1997, Seattle, WA.
16. Myers, R.H. and Montgomery, D. C., Response Surface Methodology-Process and Product Optimization Using Designed Experiments, John Wiley & Sons, 1995.
17. Gill, G. S., and Nurick, W. H., "Liquid Rocket Engine Injectors," NASA SP-8089, March, 1976.
18. SAS Institute Inc. (1995). JMP version 3. Cary, NC.
19. Microsoft Corporation. (1985-1996). Microsoft Excel 97