Reduced Vector Preisach Model

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***Abstract*— A new vector Preisach model, called the Reduced Vector Preisach model (RVPM), was developed for fast computations. This model, derived from the Simplified Vector Preisach model (SVPM), has individual components that like the SVPM are calculated independently using coupled selection rules for the state vector computation. However, the RVPM does not require the rotational correction. Therefore, it provides a practical alternative for computing the magnetic susceptibility using a differential approach. A vector version, using the framework of the DOK model, is implemented. Simulation results for the reduced vector Preisach model are also presented.**

***Index Terms*—Hysteresis modeling, Preisach model, vector magnetization modeling.**

I. Introduction

THE accurate characterization of magnetic processes requires a vector model of magnetic hysteresis. The classical Preisach model cannot adequately represent vector magnetic processes since it is inherently a scalar model. Several authors have modified the scalar Preisach model to include the vector features of a magnetic medium [1-3]. The Simplified Vector Preisach Model (SVPM) [1] was developed for computing the vector magnetization in response to a vector-applied field.

The SVPM is a coupled-hysteron model that exhibits both the saturation property and the loss property [1]. The vector magnetization is computed from the integration of the product of a state vector and a Preisach function and then performing the rotational correction. The state vector is computed using the selection rules determined by the applied field. The rotational correction term provides the cross-axis coupling effect of applied fields. This cross-axis coupling term makes it impossible to obtain an analytical closed form solution for the magnetic susceptibility.

This paper presents a Reduced Vector Preisach Model (RVPM) that does not require the rotational correction. The developed vector model uses modified selection rules for state vector calculation.

II. Simplified Vector Preisach Model

The SVPM computes the normalized irreversible magnetization components as the product of the rotational correction $R(I_x, I_y, I_z)$, and the basic Preisach integrals I_j,

$$m_{Ij} = R(I_x, I_y, I_z) I_j, \text{ for } j = x, y, \text{ or } z. \quad (1)$$

In two dimensions, the output from those basic Preisach integrals I_j are computed as

$$I_j = \iint_{v_j < u_j} Q_j p(u_j, v_j) du_j dv_j, \text{ for } j = x, y, \text{ or } z, \quad (2)$$

where the up and down switching fields are u_j and v_j, the normalized Preisach function p, and the state function Q_j. The state function Q_j is determined by selection rules. These selection rules are summarized in Table I for the 2-D model that can be generalized for the 3-D model. The subscript d is used for the direction in which Q_d is being computed. The subscript c is used for the cross direction.

The rotational correction is given by

$$R(I_x, I_y, I_z) = \frac{|I_x| + |I_y| + |I_z|}{\sqrt{I_x^2 + I_y^2 + I_z^2}} \quad (3)$$

It was shown [1] that for any set of values of I_j, $1 \le R \le \sqrt{3}$, the rotation correction ensures the correct magnitude of the magnetization.

The normalized irreversible magnetization is computed as the vector sum of three basic Preisach models as

$$m_I = RI. \quad (4)$$

The irreversible magnetization can be computed as the vector sum

$$M_I = M_S S m_I, \quad (5)$$

where M_s is the saturation magnetization and S is the material squareness matrix defined as

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix}. \quad (6)$$

Thus, to simulate anisotropic media, this model can allow different values for the S's along each of the axes and parameters in the basic Preisach models.

The normalized reversible magnetization components can be computed as

$$m_{Rj} = a_{j+} f(H_j) - a_{j-} f(-H_j), \text{ for } j = x, y, \text{ or } z, \quad (7)$$

where the $a_{j\pm}$ variables can be implemented with either a state-independent, magnetization-dependent, or state-dependent reversible magnetization as in the case of the scalar models. The non-linear function $f(H_j)$ is defined as

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$$f(H_j) = 1 - e^{-\xi H_j}, \quad (8)$$

where ξ is a model parameter that needs to be identified.

The reversible magnetization can be computed as a vector sum,

$$M_R = (1 - S) M_S m_R, \quad (9)$$

where S is the squareness matrix defined in (6).

Then the total magnetization can be expressed as

$$M_T = M_I + M_R. \quad (10)$$

III. Reduced Vector Preisach Model

The Reduced Vector Preisach Model computes the normalized irreversible magnetization components using the basic Preisach integrals

$$m_{Ij} = \iint_{v_j < u_j} Q_j p(u_j, v_j) du_j dv_j \text{ , for } j = x, y, \text{ or } z, \quad (11)$$

where the up and down switching fields are u_j and v_j, the normalized Preisach function p, and the state function Q_j.

The state vector is computed by new selection rules summarized in Table I for the 2-D case. These rules differ from the SVPM case only at the corners. The subscript d is used for the direction in which Q_d is being computed. The subscript c is used to indicate the cross direction. These new selection rules are defined such that no rotational correction is required for computing the magnetization [5].

Similarly, the irreversible magnetization, the reversible magnetization component and the total magnetization are computed using (5), (9) and (10) respectively.

The application of selection rules shown in Table II shows that at any point on the Preisach hyperplane, the square of the sum of the Cartesian components of the state vector obeys

$$Q_x^2 + Q_y^2 + Q_z^2 = 1. \quad (12)$$

For the ellipsoidal magnetization behavior, the major remanence path must satisfy

$$m_{Ix}^2 + m_{Iy}^2 + m_{Iz}^2 = 1. \quad (13)$$

For sufficiently strong fields, the normalized reversible magnetization satisfies

$$m_{Rx}^2 + m_{Ry}^2 + m_{Rz}^2 = 1. \quad (14)$$

Therefore, the normalized total magnetization can be expressed as

$$m_{Tx}^2 + m_{Ty}^2 + m_{Tz}^2 = 1. \quad (15)$$

The important properties of the RVPM can be summarized as:

- It is applicable to the anisotropic media as well as the isotropic media.
- It can be reduced to the scalar model if the applied field lies only along one of the principal axes and the magnetization initially lies along that axis. Also, the vector model will have all the properties of the scalar moving model.
- For both isotropic and anisotropic media, in the presence of large fields, the normalized irreversible magnetization and the normalized reversible magnetization trace out an ellipse.

IV. Simulations

The RVPM is applied to an isotropic magnetic medium. For an isotropic medium, values for σ_i and σ_k are equal, and negligible compared to the average critical field $\overline{H_k}$. The material parameters used for simulations are: $\sigma_i = \sigma_k = 165$, $\overline{H_k} = 633$, $S = 0.57$, $M_S = 0.014127$, $\alpha = 33332.81$, $\xi = 0.0009$.

Simulations were carried out for the orthogonal component H_R of the applied field of $1.1\overline{h_k}$, $1.4\overline{h_k}$ and $4\overline{h_k}$. The vector DOK model [7] is implemented based on the RVPM using the cobweb method [6] for computation speed. Figure 1 shows a plot for the magnetization angle versus applied field angle. It can be seen that the magnetization ratchets as the applied field rotates. For very large applied fields, the magnetization angle becomes equal to the applied field angle for all values.

Figure 2 shows the locus of the magnetization as the applied field is rotated. It is seen that as the field increases, the curves become rounder. The flattening of the loci close to 135 degrees and 315 degrees is a discretization error caused by the jump in the magnetization angle for the respective applied fields.

For an isotropic media, since the magnetization rotates faster than the applied field, the rotations of the magnetization need to be corrected using the correction rules defined in [8]. Applying these corrections for the applied fields as in Figure 1, the corrected magnetization angle vs. applied field angle is plotted as shown in Figure 3. Figure 4 shows the magnetization loci with corrections for an applied field rotation of 360 degrees. It is seen that the curves are more rounded.

V. Conclusion

A simpler vector model is developed that does not require the rotational correction for computation speed. The elimination of the rotational correction makes it possible to implement the differential method, a very effective way to compute the magnetization. The simulation results show that the presented model represents isotropic media accurately. Further refinement of this vector model in terms of speed and accuracy is a topic of future research.

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References

[1] I.D. Mayergoyz, Mathematical Models of Hysteresis, Springer-Verlag:New York, 1991.

[2] E. Della Torre, Magnetic Hystersis, IEEE pres s: Piscataway, NJ, 1999.
[3] E. Della Torre, "A simplified vector Preis ach model," *IEEE Trans. Magn.*, vol. 34, Mar. 1998, pp.495-501.
[4] A. Reimers and E. Della Torre, "Fas Preisach-based Vector Magnetization model," *IEEE Trans. Mag 1.*, vol. 37, Sept. 2001, pp.3349-3352.
[5] U. Patel, Fast Implementation of Scalar anc Vector Preisach Models, D.Sc. Dissertation, The George Washington University, Washington, DC, USA, May 2002.
[6] O. Alejos and E. Della Torre, "Improving numerical simulations of Preisach models for accuracy and speed," *IEEE Trans. Magn.*,vol. 36, Sept. 2000, pp.3857-3866.
[7] E. Della Torre and A. Reimers, "Energy considerations in vector magnetization models," *J. Appl. Phys.*, **89(11)**, June 2001.
[8] E. Della Torre and A. Reimers, "Isotropic Media and the Simplified Vector Preisach Model," submitted to *Physica B*. Presented at Symposium on Hysteresis and Microm gnetic Modeling, GWU, Ashburn, VA May 21-23, 2001.

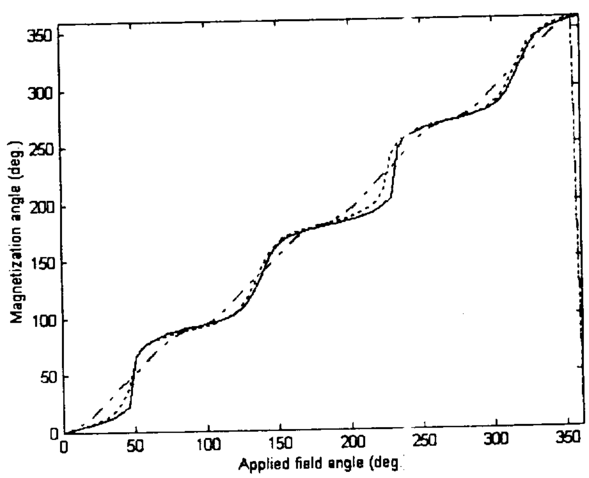

Figure 1. Magnetization angle vs. applied field angle for applied fields of $1.1\overline{h_k}$ (solid line), $1.4\overline{h_k}$ (dotted line), and $4\overline{h_k}$ (dash-dot line).

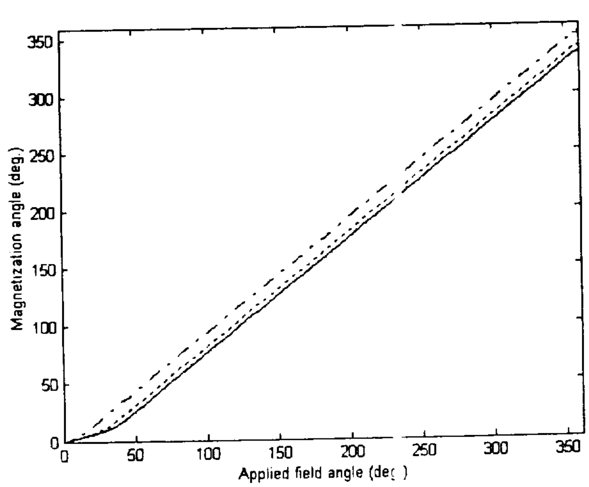

Figure 2. A plot of the corrected magnetization angle vs. applied field angle for the same conditions as in Fig. 1.

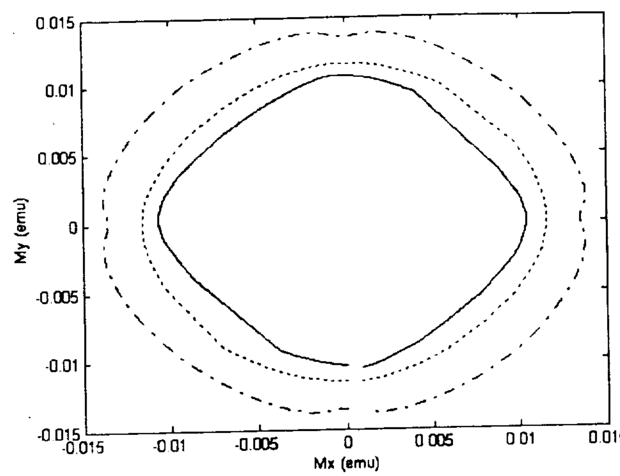

Figure 3. Locus of magnetization for the same set of applied field rotations as shown in Fig. 1.

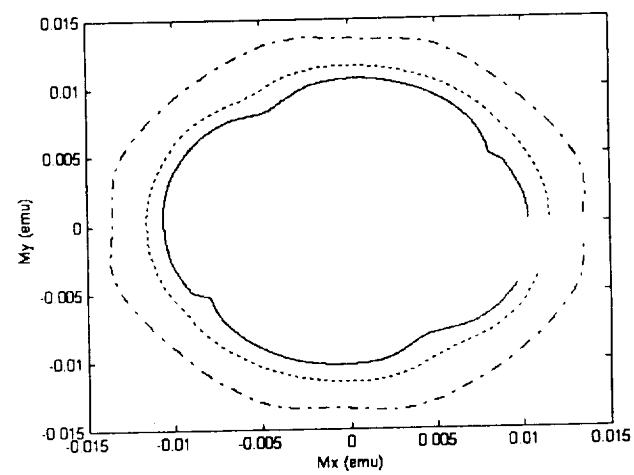

Figure 4. A plot of the corrected magnetization angle vs. applied field angle for the same conditions as in Fig. 1.

TABLE I
STATE FUNCTION VALUES IN TWO DIMENSIONS, Q_j

	$v_d > h_d$	$v_d \le h_d \le u_d$	$h_d > u_d$
$v_c > h_c$	$\frac{h_d - v_d}{\lvert h_d - v_d\rvert + \lvert h_c - v_c\rvert}$	0	$\frac{h_d - u_d}{\lvert h_d - u_d\rvert + \lvert h_c - v_c\rvert}$
$v_c \le h_c \le u_c$	-1	no change	1
$h_c > u_c$	$\frac{h_d - v_d}{\lvert h_d - v_d\rvert + \lvert h_c - u_c\rvert}$	0	$\frac{h_d - u_d}{\lvert h_d - u_d\rvert + \lvert h_c - u_c\rvert}$

TABLE II
STATE FUNCTION VALUES IN TWO DIMENSIONS, Q_d

	$v_d > h_d$	$v_d \le h_d \le u_d$	$h_d > u_d$
$v_c > h_c$	$\frac{h_d - v_d}{\sqrt{(h_d - v_d)^2 + (h_c - v_c)^2}}$	0	$\frac{h_d - u_d}{\sqrt{(h_d - u_d)^2 + (h_c - v_c)^2}}$
$v_c \le h_c \le u_c$	-1	No change	1
$h_c > u_c$	$\frac{h_d - v_d}{\sqrt{(h_d - v_d)^2 + (h_c - u_c)^2}}$	0	$\frac{h_d - u_d}{\sqrt{(h_d - u_d)^2 + (h_c - u_c)^2}}$

TABLE III
STATE FUNCTION VALUES IN THREE DIMENSIONS, Q_j

Number of violations	Violations	States
0	$v_j \le h_j \le u_j$ holds for $j = x$, y and z	no change
1	$h_j > u_j$ or $h_j < v_j$	$Q_j = 1$ or $Q_j = -1$ and $Q_i = 0$, $i \ne j$
2	Any two combinations of violations in u or v where the thresholds violated are called t_j and t_k.	$Q_j = \frac{h_j - t_j}{\sqrt{(h_j - t_j)^2 + (h_k - t_k)^2}}$, $Q_k = \frac{h_k - t_k}{\sqrt{(h_j - t_j)^2 + (h_k - t_k)^2}}$ and $Q_i = 0$, $i \ne j,k$
3	Any three combinations of violations in u or v where the thresholds violated are called t_i, t_j and t_k.	$Q_i = \frac{h_i - t_i}{\sqrt{(h_i - t_i)^2 + (h_j - t_j)^2 + (h_k - t_k)^2}}$, $Q_j = \frac{h_j - t_j}{\sqrt{(h_i - t_i)^2 + (h_j - t_j)^2 + (h_k - t_k)^2}}$ and $Q_k = \frac{h_k - t_k}{\sqrt{(h_i - t_i)^2 + (h_j - t_j)^2 + (h_k - t_k)^2}}$