

# **AERODYNAMIC SHAPE OPTIMIZATION USING AN EVOLUTIONARY ALGORITHM**

Terry L. Holst and Thomas H. Pulliam  
NASA Ames Research Center  
NASA Advanced Supercomputing (NAS) Division  
Applications Branch  
Moffett Field, CA 94035

## **Abstract**

A method for aerodynamic shape optimization based on an evolutionary algorithm approach is presented and demonstrated. Results are presented for a number of model problems to assess the effect of algorithm parameters on convergence efficiency and reliability. A transonic viscous airfoil optimization problem—both single and two-objective variations—is used as the basis for a preliminary comparison with an adjoint-gradient optimizer. The evolutionary algorithm is coupled with a transonic full potential flow solver and is used to optimize the inviscid flow about transonic wings including multi-objective and multi-discipline solutions that lead to the generation of pareto fronts. The results indicate that the evolutionary algorithm approach is easy to implement, flexible in application and extremely reliable.



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USING EVOLUTIONARY ALGORITHMS**

**Seminar**  
**California Institute of Technology**  
Department of Aeronautics  
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**TERRY L. HOLST**  
**TOM PULLIAM**  
**NASA Advanced Supercomputing (NAS) Division**  
**NASA Ames Research Center**



# PRESENTATION OUTLINE

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- ▶ **EVOLUTIONARY ALGORITHMS--GENERAL**
- ▶ **SINGLE OBJECTIVE RESULTS**
- ▶ **MULTI-OBJECTIVE ALGORITHM CHARACTERISTICS--PARETO FRONTS**
- ▶ **COMPARISON OF RESULTS FROM AN EVOLUTIONARY ALGORITHM AND AN ADJOINT GRADIENT BASED ALGORITHM**
- ▶ **ADDITIONAL COMPUTATIONAL RESULTS**
- ▶ **CONCLUSIONS**



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# **GENERAL CHARACTERISTICS: SINGLE-OBJECTIVE EVOLUTIONARY ALGORITHMS**



# EVOLUTIONARY ALGORITHMS—GENERAL

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- ▶ **EVOLUTIONARY ALGORITHMS (EA)** are search algorithms based on natural selection. *“They combine survival of the fittest with structured yet randomized information exchange...”* GOLDBERG (1989)
  
- ▶ **EA optimization has many advantages:**
  - ▶ **Simplicity**
  - ▶ **Robustness**
  - ▶ **Wide applicability**
  - ▶ **Embarrassingly parallel implementation**
  
- ▶ **EA optimization works for design spaces that are**
  - ▶ **Function discontinuous**
  - ▶ **Derivative discontinuous**
  - ▶ **Multi-modal**
  - ▶ **Multi-objective**
  
- ▶ **EAs typically require more function evaluations than other methods especially gradient-based methods**



## EVOLUTIONARY ALGORITHM CHARACTERISTICS

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### ▶ ENCODING (DESIGN SPACE PARAMETERIZATION)

- ▶ Each problem being optimized must be *representable* as a set of parameters called GENES, e.g., geometric parameters used in aerodynamic shape optimization. One set of genes is called a CHROMOSOME.
- ▶ Chromosomes are constructed in one of two ways:
  - ▶ Bit strings
  - ▶ Real number strings

### ▶ FITNESS

- ▶ A FITNESS FUNCTION is used to evaluate figure of merit for each chromosome, e.g., pressure integration to obtain lift

### ▶ SELECTION

- ▶ SELECTION operation is used to determine which chromosomes will be carried forward to the next generation
- ▶ More fit individuals are always favored in the selection process

## ► TWO SELECTION ALGORITHMS HAVE BEEN STUDIED

### ● Multiple pass selection (“greedy selection”)

- **FIRST PASS:** Select all chromosomes ranked 1
- **SECOND PASS:** Select all chromosomes ranked 1 and 2
- **THIRD PASS:** Select all chromosomes ranked 1, 2 and 3
- And so on until NC chromosomes have been selected

### ● Tournament selection

- Select the NOB chromosomes with the highest fitness in each objective
- Select three chromosomes at random and compare rankings
- Retain the highest ranking (in case of ties, retain the first selected)
- Repeat until NC chromosomes have been selected



## EVOLUTIONARY ALGORITHM CHARACTERISTICS—CONT.



### ▶ New Generation is Finalized Using Various Modification Operators

#### ▶ PASSTHROUGH (Controlled by $P_1$ )

- ▶ Small number of chromosomes with highest rankings included without modification (ELITISM)

#### ▶ CROSSOVER (Controlled by $P_2$ )

- ▶ Two chromosomes (PARENTS) are chosen at random from new generation
- ▶ Genes are combined using an averaging operator to produce a CHILD with shared characteristics from each PARENT

#### ▶ MUTATION

- ▶ Random gene chosen from random chromosome in new generation
- ▶ Using a small probability the chosen gene is randomly modified
- ▶ Two types of mutation used

- ▶ PERTURBATION MUTATION: Changes are small (Controlled by  $P_3$ )

- ▶ Standard MUTATION: Changes are large (Controlled by  $P_4$ )

#### ▶ MODIFICATION OPERATOR USAGE CONTROLLED BY P-VECTOR-- $\sum P_i = 1.0$

❖ CROSSOVER is generally viewed as most important operation for producing a rapid search or exploration.

❖ MUTATION adds randomness, ensuring that no part of design space is neglected.





# SAMPLE RESULTS--SINGLE OBJECTIVE

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## ▶ HILL CLIMBING PROBLEM

- ▶ TWO GENES
- ▶ MULTI-MODAL (MULTIPLE HILLS AND VALLEYS)

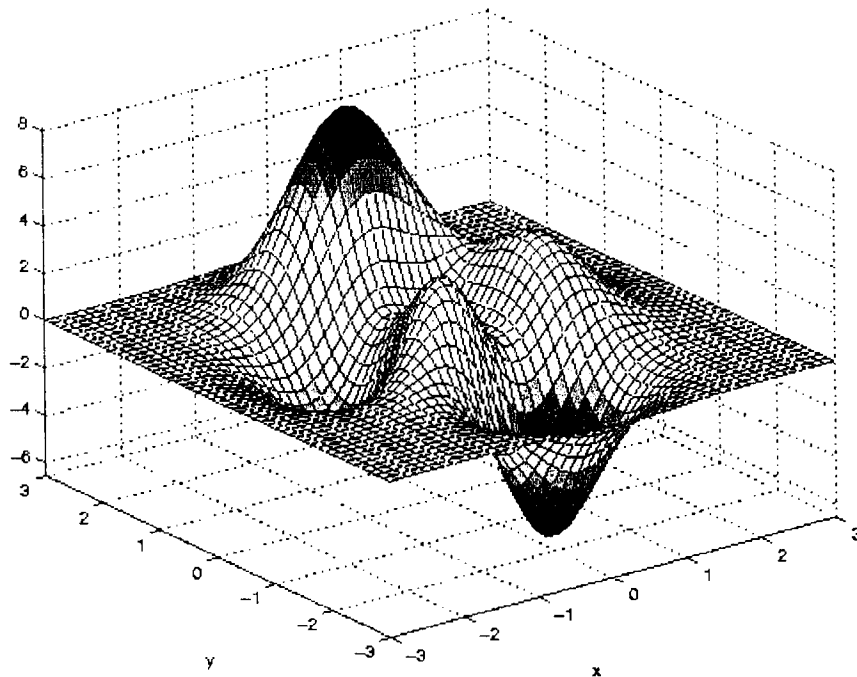
## ▶ TRANSONIC WING OPTIMIZATION

- ▶ LIFT-TO-DRAG MAXIMIZATION
- ▶ AERODYNAMIC FUNCTION EVALUATIONS
  - ▶ TRANSONIC OVERSET POTENTIAL SOLVER (TOPS)
  - ▶ CHIMERA ZONAL GRID APPROACH
  - ▶ HYPGEN USED FOR WING VOLUME GRID GENERATION
- ▶ WING PARAMETERIZATION
  - ▶ HICKS-HENNE BUMP FUNCTIONS USED (UPPER SURFACE ONLY)
  - ▶ LEADING EDGE, TRAILING EDGE AND LOWER SURFACE FIXED
  - ▶ FOUR BUMPS AT TWO STATIONS (ROOT AND TIP) + TWIST >> TEN GENES (GEOMETRIC DECISION VARIABLES)
  - ▶ LINEAR LOFTING BETWEEN ROOT AND TIP
  - ▶ FIXED PLANFORM

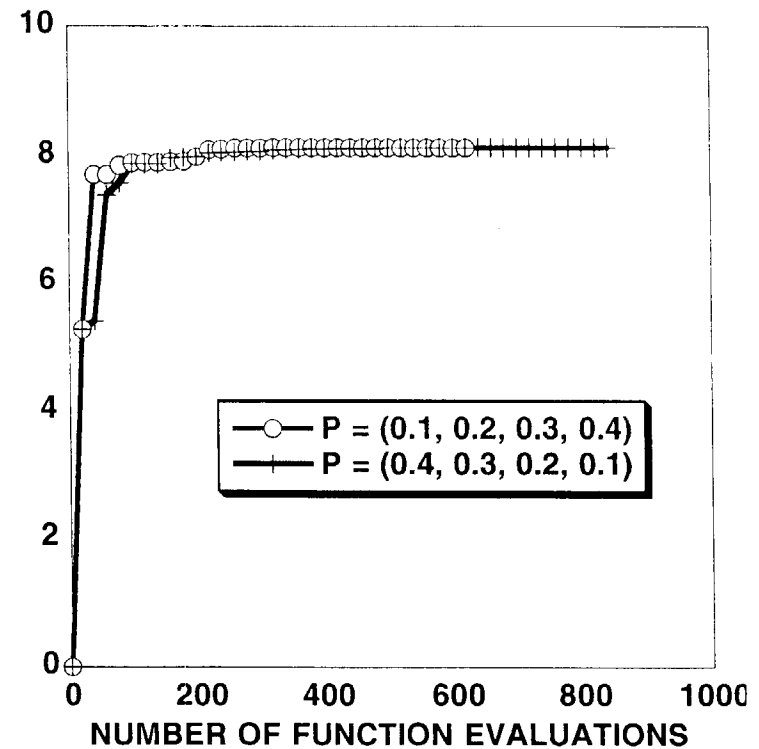
# HILL CLIMBING PROBLEM



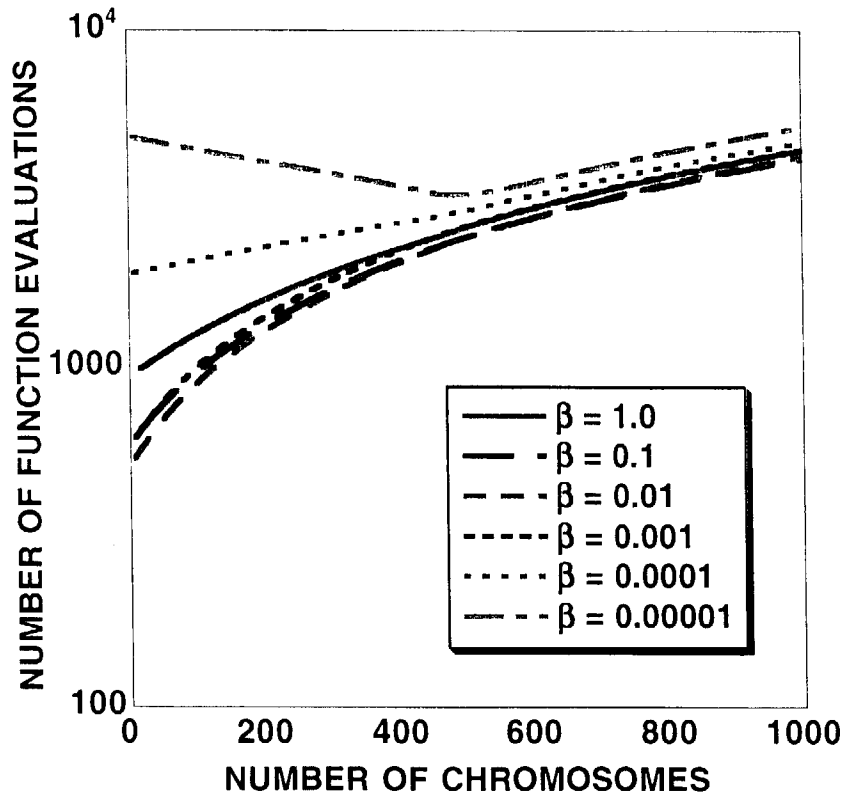
ISOMETRIC VIEW OF FUNCTION  
USED IN HILL CLIMBING PROBLEM



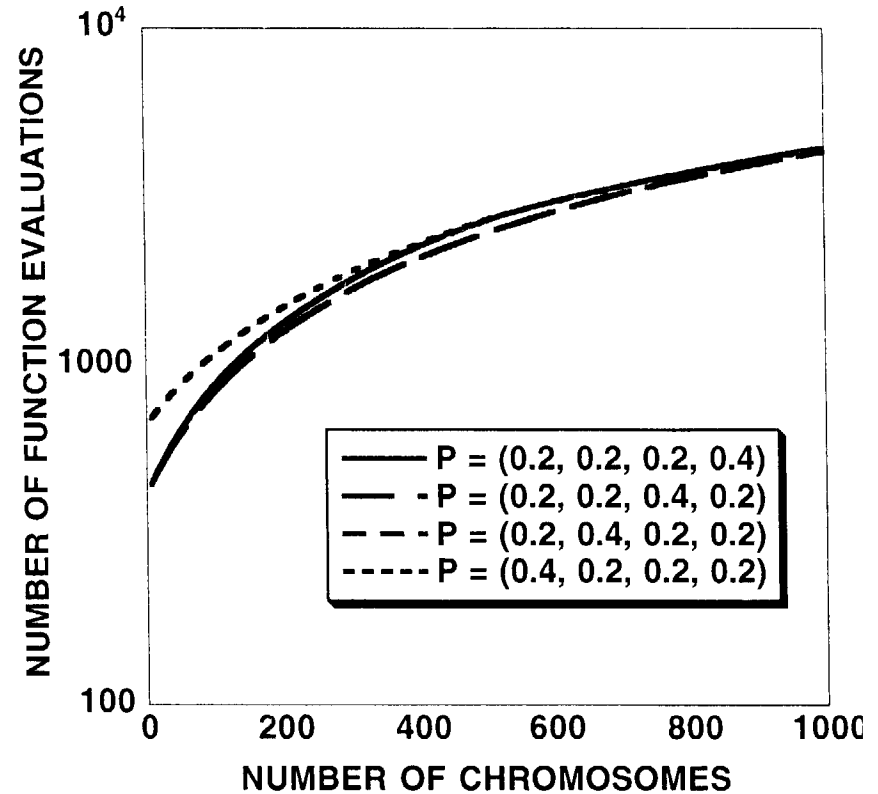
SAMPLE EA CONVERGENCE  
 $\beta=0.01$ ,  $CONV=10^{-5}$ ,  $NC=20$



**EFFECT OF  $\beta$  ON CONVERGENCE**  
 $CONV = 10^{-5}$ ,  $P = (0.1, 0.2, 0.3, 0.4)$

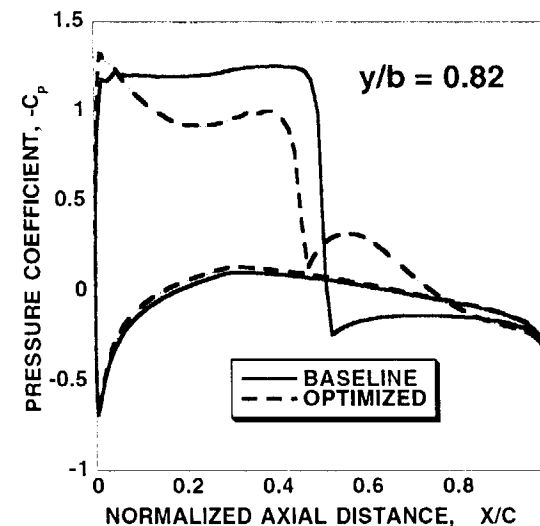
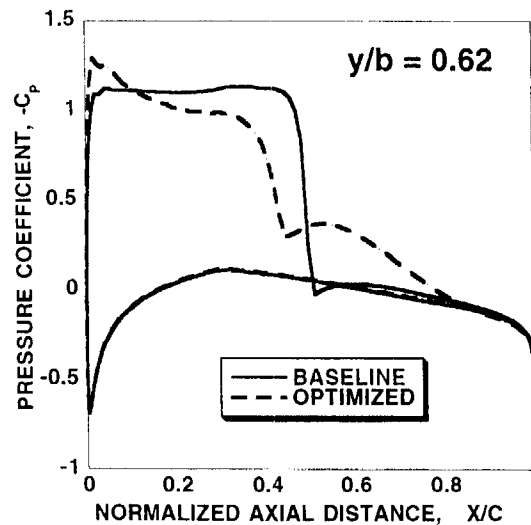
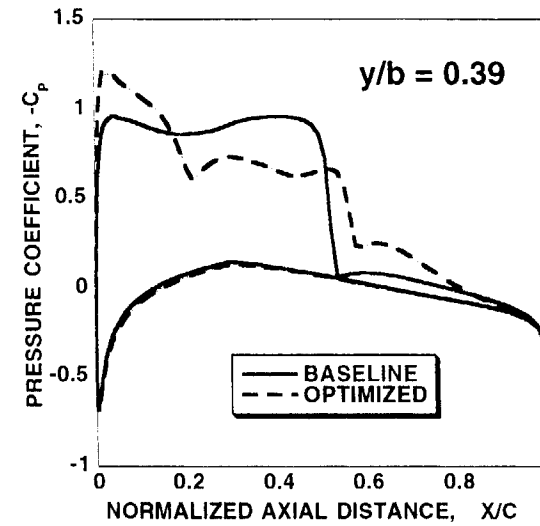
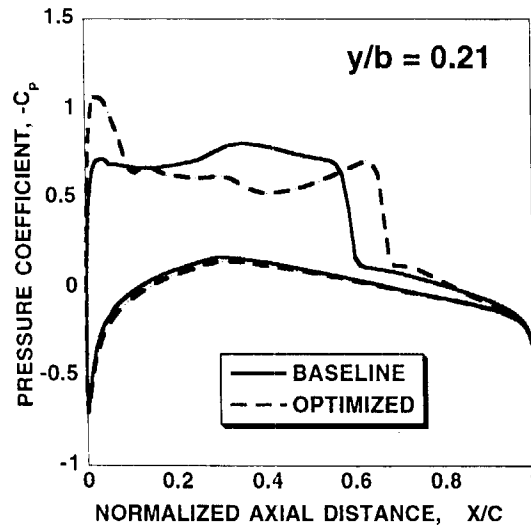


**EFFECT OF P ON CONVERGENCE**  
 $CONV = 10^{-5}$ ,  $\beta = 0.01$



$M_\infty = 0.84$   
 $\alpha = 4^\circ$   
 $TR = 0.333$   
 $AR = 6.0$   
 $\Delta_{LE} = 36.65^\circ$   
 $RMAX < 10^{-6}$   
 $NG = 10$   
 $NC = 20$   
 $\beta = 0.3$   
 $P = (0.1, 0.2, 0.3, 0.4)$

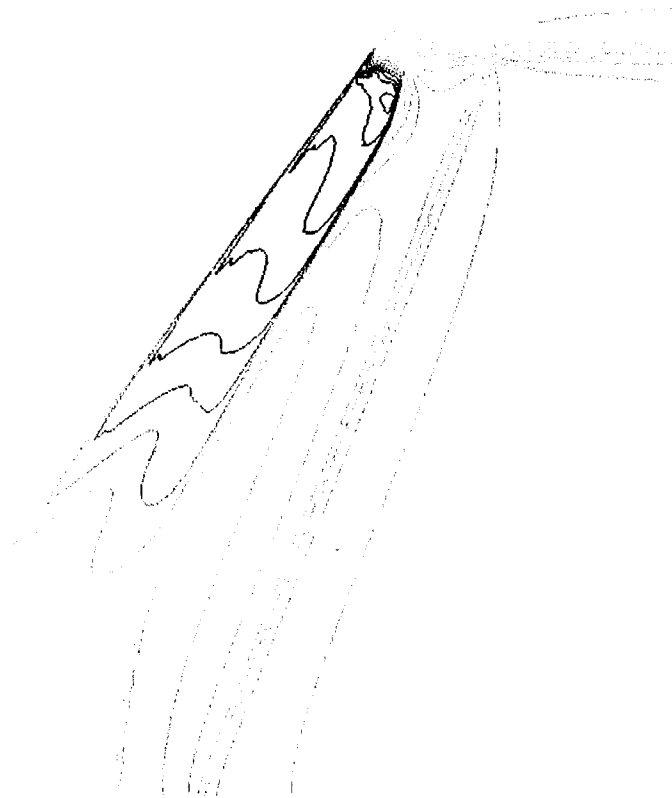
**OBJ =**  
 $1/(C_D/C_L + (C_L - 0.45)^2)$



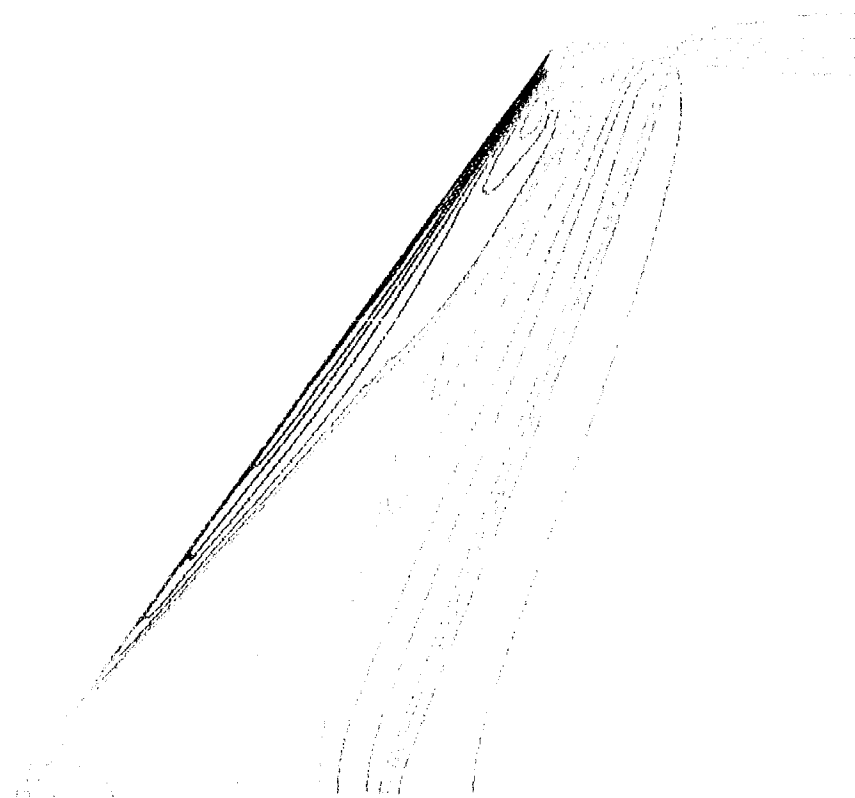
# MACH NUMBER CONTOURS—WING OPTIMIZATION

$M_\infty = 0.84$ ,  $\alpha = 4^\circ$ ,  $R_{MAX} < 10^{-6}$ ,  $NG = 10$ ,  $\beta = 0.3$ ,  $P = (0.1, 0.2, 0.3, 0.4)$

**BASELINE SOLUTION**

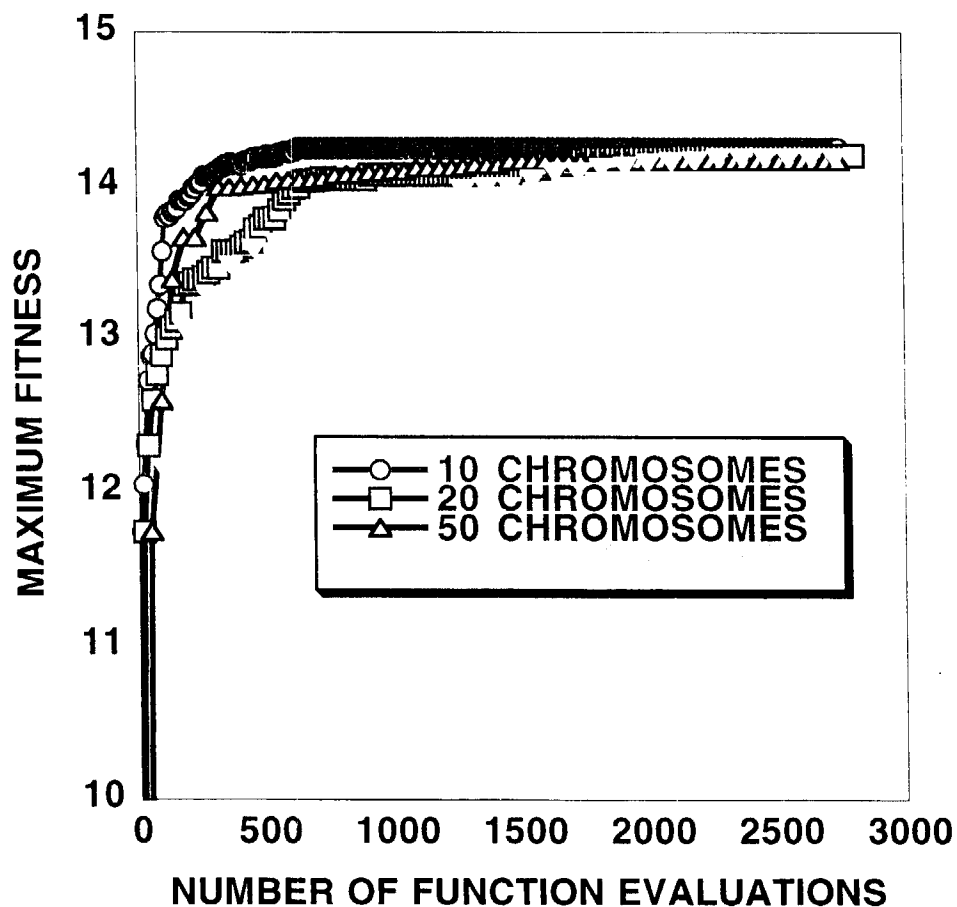


**OPTIMIZED SOLUTION**



## EFFECT OF POPULATION SIZE ON GA CONVERGENCE

$M_{\infty} = 0.82$ ,  $\alpha = 4^{\circ}$ ,  $RMAX < 10^{-6}$ ,  $NG = 55$ ,  $\beta = 0.3$ ,  $P = (0.1, 0.3, 0.4, 0.2)$

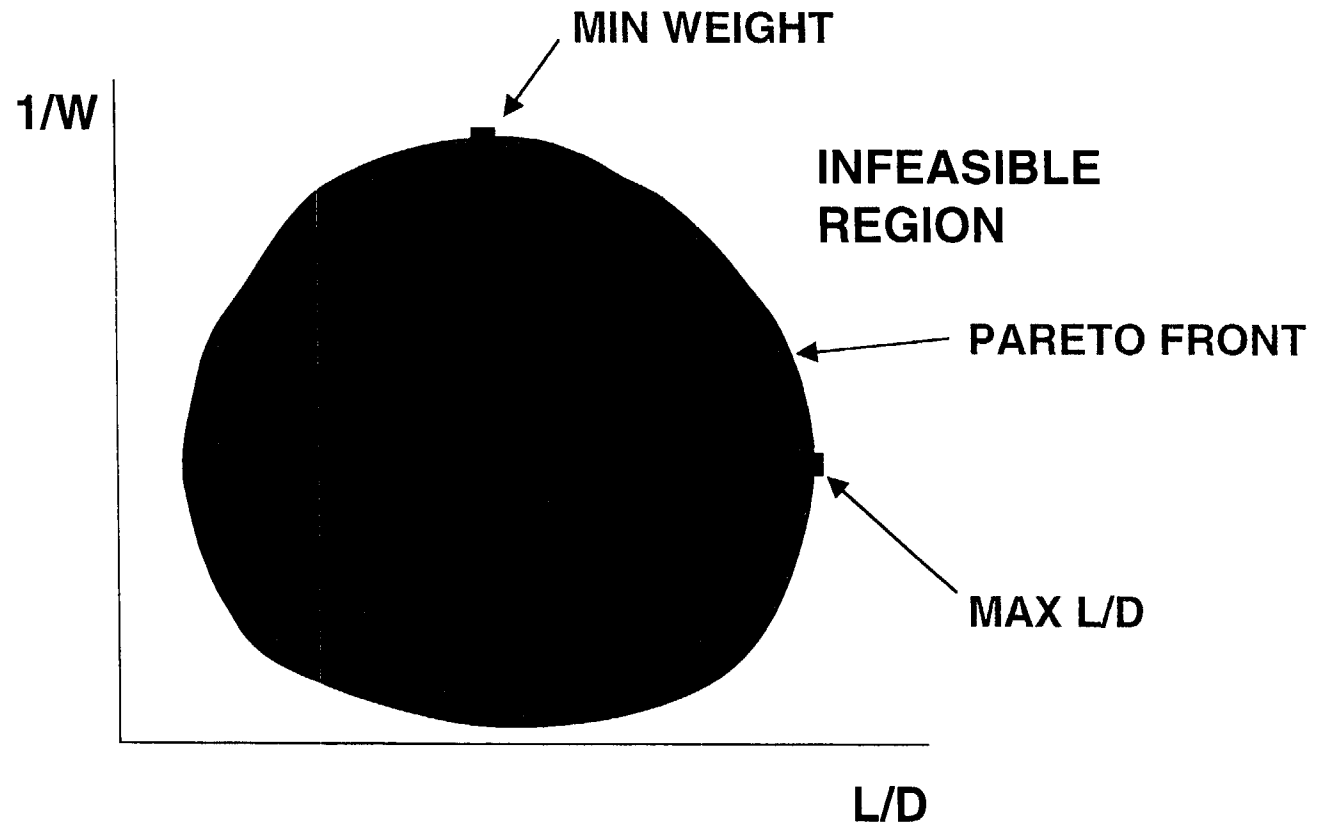




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# **MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM CHARACTERISTICS**

- ▶ EAs ARE USEFUL FOR MULTI-OBJECTIVE OPTIMIZATION, E.G., MAX L/D AND MIN WEIGHT





- ▶ The  $i^{\text{th}}$  gene in the  $j^{\text{th}}$  chromosome of the  $n^{\text{th}}$  EA generation is indicated by

$$X_{i,j}^n$$

- ▶ The  $j^{\text{th}}$  chromosome within the  $n^{\text{th}}$  generation composed of NG genes

$$\mathbf{X}_j^n = (X_{1,j}^n, X_{2,j}^n, \dots, X_{i,j}^n, \dots, X_{NG,j}^n)$$

- ▶ The fitness vector associated with the  $j^{\text{th}}$  chromosome and the  $n^{\text{th}}$  generation

$$\mathbf{F}_j^n = [f_1^n(\mathbf{X}_j^n), f_2^n(\mathbf{X}_j^n), \dots, f_{NOB}^n(\mathbf{X}_j^n)]$$

where NOB is the number of objective functions.



# MULTIPLE OBJECTIVE OPTIMIZATION PARETO FRONT DEFINITIONS

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- ▶ **PARETO OPTIMAL SET or PARETO FRONT :**
  - ▶ The optimal result of a multi-objective optimization
- ▶ Membership in the Pareto Optimal Set determined using the concept of **DOMINANCE:**

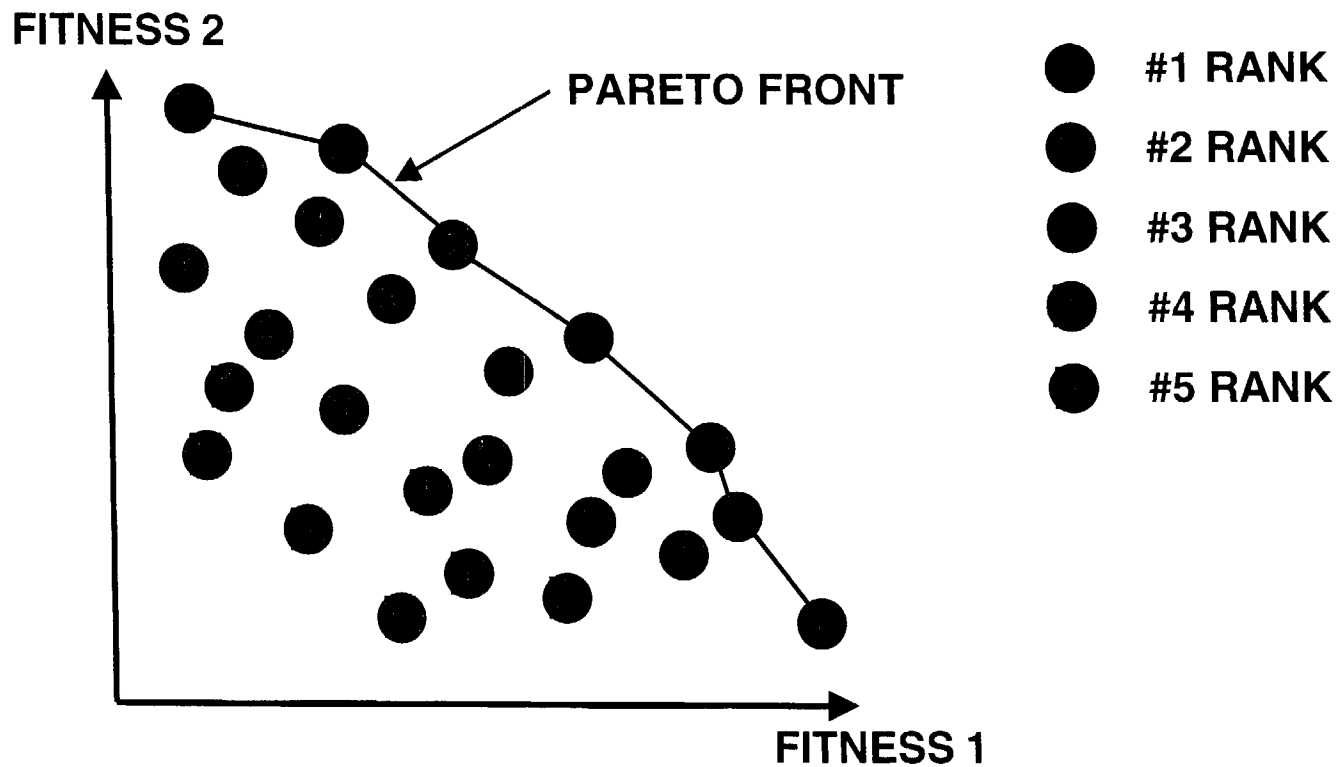
***Chromosome  $X_a$  dominates chromosome  $X_b$   
iff  $f_{a,k} \geq f_{b,k}$  for all  $k$  with  $f_{a,k} > f_{b,k}$  for at least one  $k$***

- ▶ Chromosome rank tied to dominance.
  - ▶ Several ranking algorithms available:
    - ▶ Goldberg ranking
    - ▶ Fonseca and Fleming ranking
    - ▶ Others

# MULTI-OBJECTIVE OPTIMIZATION RANKING



► Goldberg ranking using maximization for two objectives





## MULTIPLE OBJECTIVE OPTIMIZATION ACTIVE AND ACCUMULATION FILES

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### ▶ ACTIVE FILE:

- ▶ Current collection of chromosomes (nth population)

### ▶ ACCUMULATION FILE:

- ▶ Collection of all #1 ranked chromosomes discovered during EA iteration

### ▶ ACCUMULATION FILE development and use:

- ▶ Add all newly discovered #1 ranked chromosomes
- ▶ Cull old individuals that lose dominance
- ▶ Increases in size with EA iteration
- ▶ Used in active file ranking
- ▶ Not used in the EA selection/crossover/mutation process (Some variations do use accumulation file in selection)



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# COMPARISON OF ADJOINT GRADIENT AND EVOLUTIONARY ALGORITHM APPROACHES



# COMPARISON OF EVOLUTIONARY AND ADJOINT GRADIENT METHODS

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## ▶ ADJOINT GRADIENT (AG) METHOD

- ▶ ADJOINT METHOD USED TO DETERMINE DESIGN SPACE GRADIENTS
- ▶ BFGS QUASI-NEWTON APPROACH USED FOR GRADIENT OPTIMIZATION
- ▶ WEIGHTED OBJECTIVE FUNCTION (WOF) USED FOR “MULTI-OBJECTIVE” OPTIMIZATIONS, i.e.,  $OBJ^{NEW} = W*OBJ_1+(1-W)*OBJ_2$

## ▶ EVOLUTIONARY ALGORITHM (EA)

- ▶ WOF AND DOMINANCE PARETO FRONT (DPF) APPROACHES BOTH USED

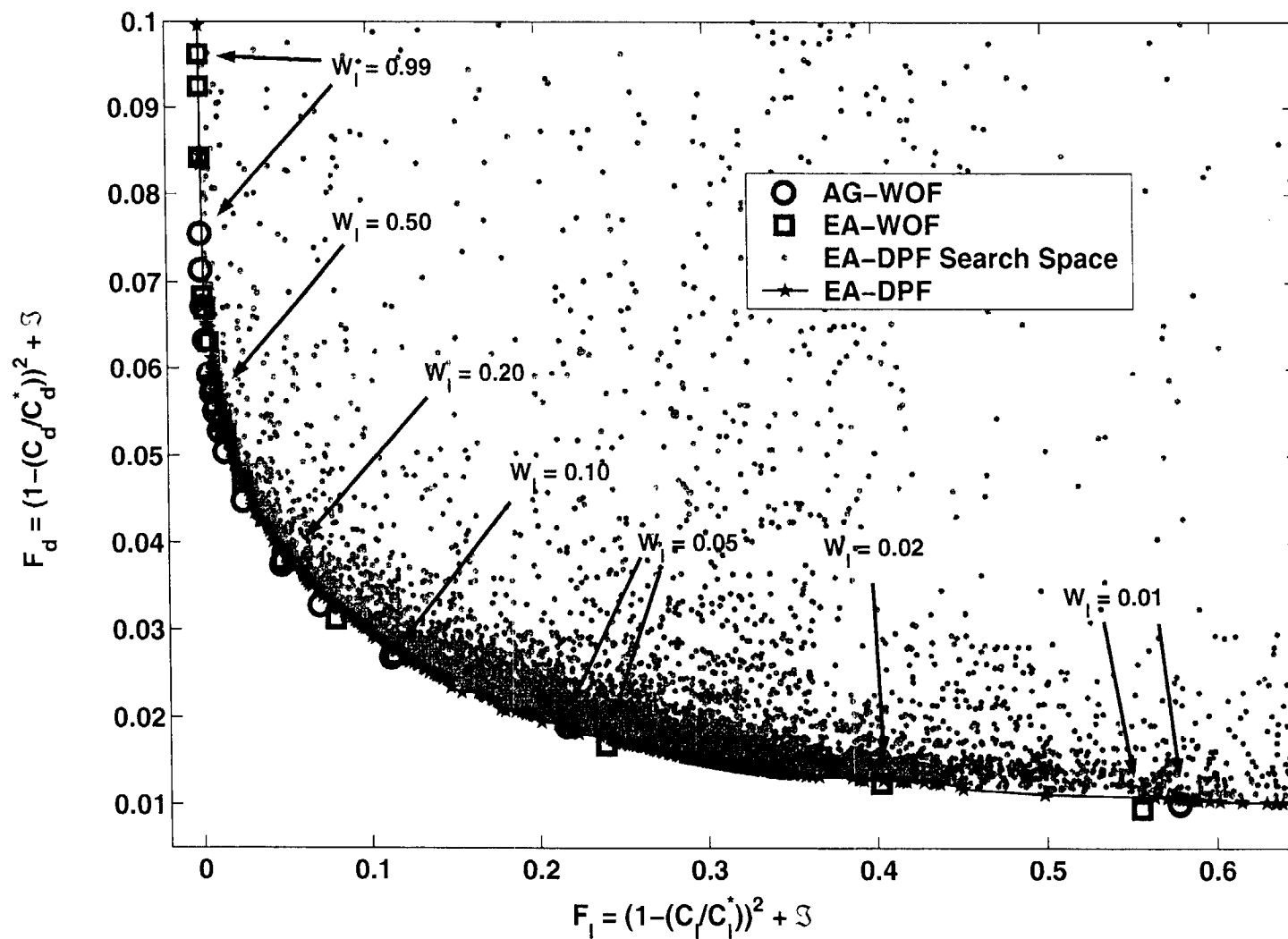
## ▶ MULTI-OBJECTIVE VISCOUS AIRFOIL OPTIMIZATION:

- ▶ ALL FUNCTION EVALUATIONS PERFORMED USING ARC2D
  - ▶ STEADY STATE SOLUTIONS TO NAVIER-STOKES EQUATIONS
  - ▶ SPALART-ALMARAS TURBULENCE MODEL
- ▶ B-SPLINE REPRESENTATION OF AIRFOIL USED
  - ▶ FIVE SPLINE KNOTS ON EACH SURFACE PLUS  $\alpha$  -- TOTAL OF 11 GENES (DECISION VARIABLES)

- ▶ Details found in Pulliam, Nemec, Holst, Zingg, AIAA Paper 2003-0298.

# PARETO FRONT COMPARISONS

$M_\infty = 0.7$ ,  $Re = 9 \times 10^6$ ,  $C_l^* = 0.55$ ,  $C_d^* = 0.0095$

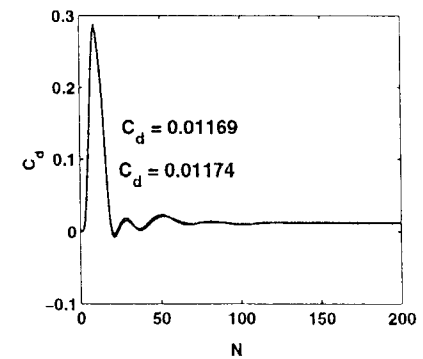
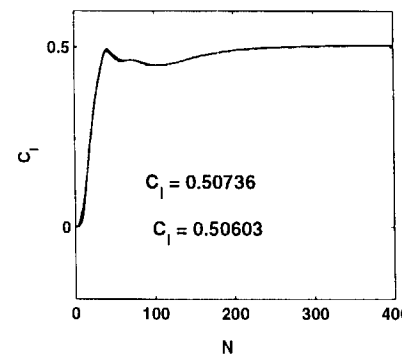
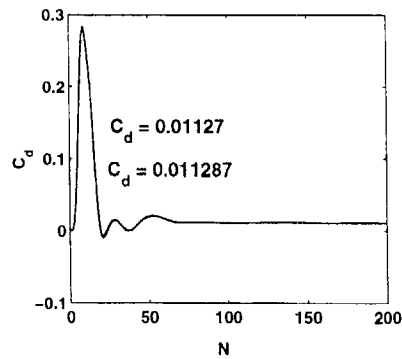
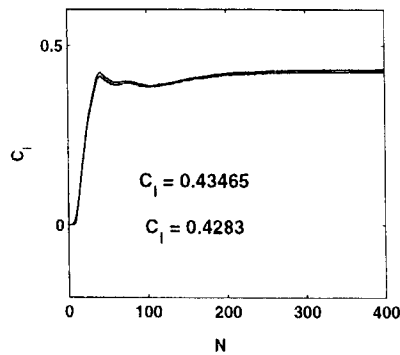
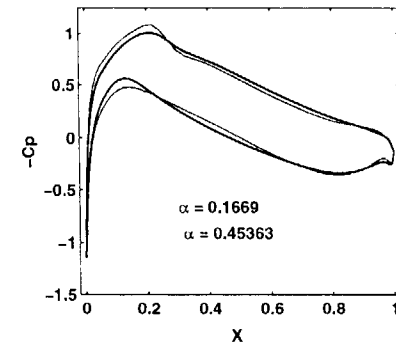
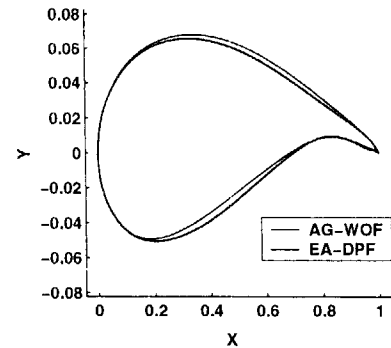
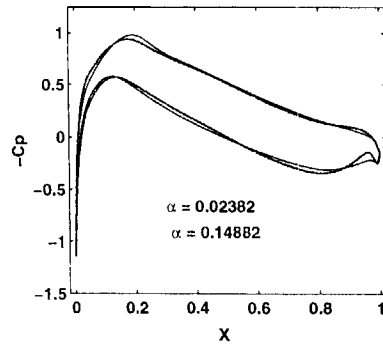
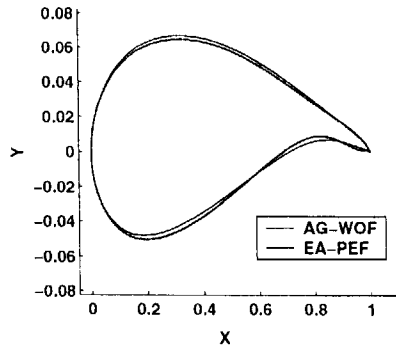


# COMPARISON OF AG-WOF AND EA-DPF RESULTS

$M_\infty = 0.7$ ,  $Re = 9 \times 10^6$ ,  $C_l^* = 0.55$ ,  $C_d^* = 0.0095$

$W = 0.2$

$W = 0.5$







## AG AND EA COMPARISON CONCLUSIONS

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- ▶ ALL METHODS PRODUCED CONSISTENT PARETO FRONTS
- ▶ AG-WOF RESULTS ARE MORE TIGHTLY CONVERGED THAN EA-BASED RESULTS
- ▶ AG-WOF APPROACH INVOLVES A SIGNIFICANT AMOUNT OF CODING FOR EACH IMPLEMENTATION WHEREAS THE TWO EA APPROACHES DO NOT
- ▶ SPEED COMPARISONS:
  - ▶ AG-WOF ~ 30 TIMES FASTER THAN EA-WOF FOR SINGLE-OBJECTIVE OPTIMIZATION
  - ▶ AG-WOF ~ 4 TIMES FASTER THAN EA-DPF FOR TWO-OBJECTIVE OPTIMIZATION
    - ▶ AG-WOF 15 POINTS ON PARETO FRONT POINTS
    - ▶ EA-DPF 500 POINTS ON THE PARETO FRONT



# EA RESULTS IN THREE DIMENSIONS

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## ▶ CASES PRESENTED

▶ SINGLE-OBJECTIVE DRAG MINIMIZATION

▶ TWO-OBJECTIVE SINGLE-DISCIPLINE MINIMIZATION

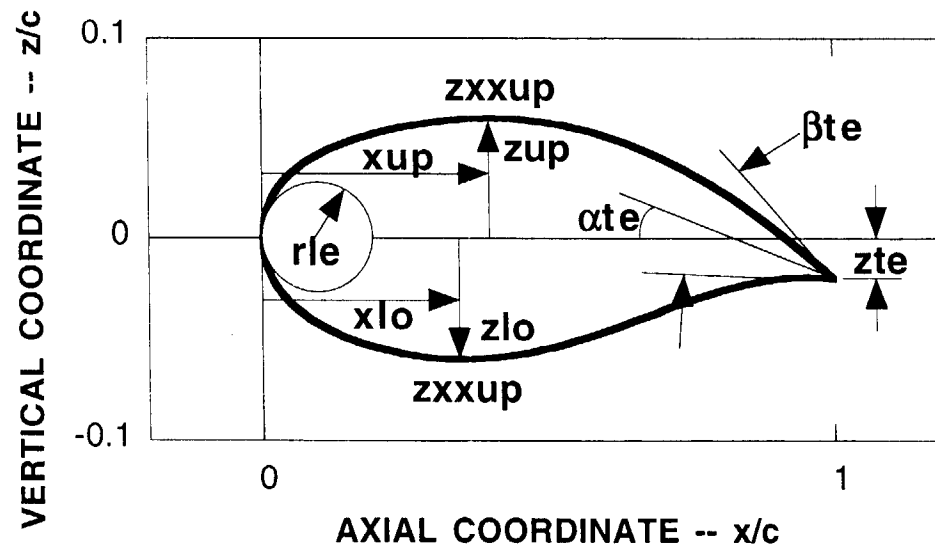
▶ TWO-OBJECTIVE MULTI-DISCIPLINE MINIMIZATION

# WING PARAMETERIZATION



- ▶ Wing defined using N airfoil defining stations
- ▶ Each airfoil defined using Sobieczky parameterization (see definition below)
- ▶ Twist angle added to each defining station >> total number of parameters = 11N
- ▶ Linear lofting used between each defining station

$$z = \sum_{n=1}^6 a_n \cdot x^{n-1/2}$$





# FUNCTION EVALUATIONS

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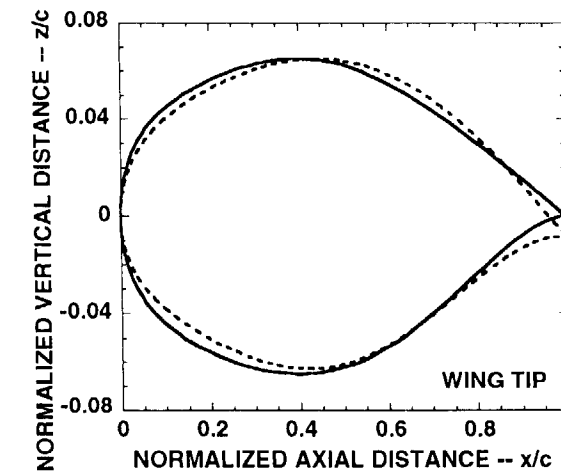
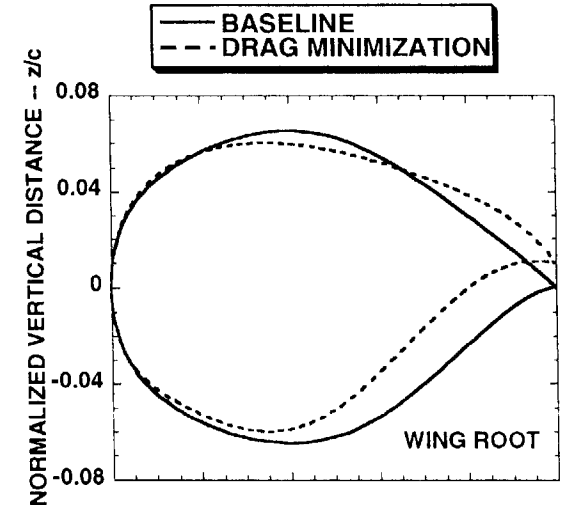
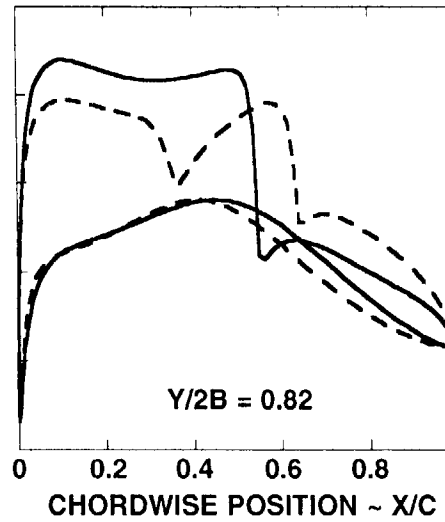
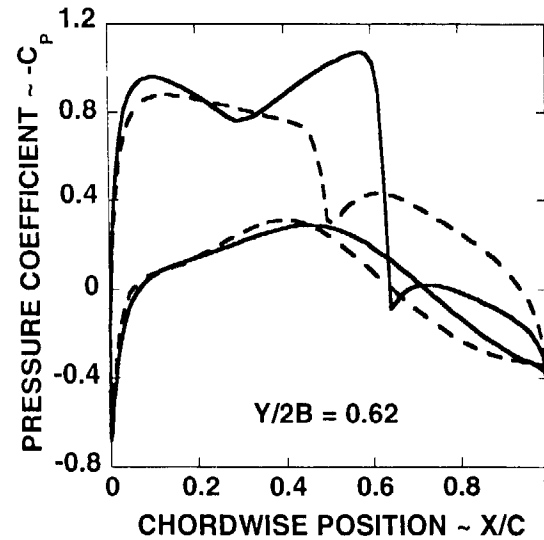
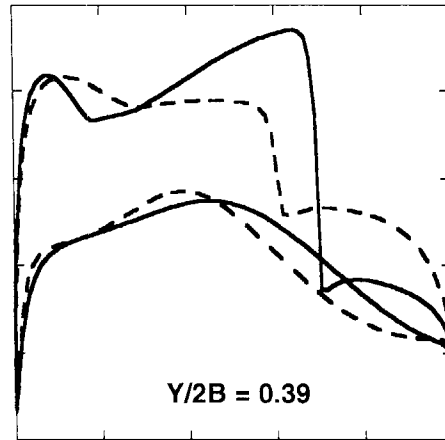
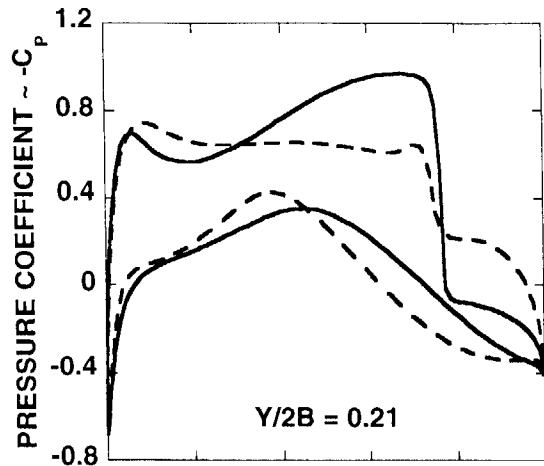
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- ▶ **AERODYNAMIC FUNCTION EVALUATIONS**
  - ▶ **TOPS (TRANSONIC OVERSET POTENTIAL SOLVER)**
  
- ▶ **TWO STATIONS (ROOT AND TIP) USED, I.E., NUMBER OF GENES (NG) IS 22**
  
- ▶ **WEIGHT FUNCTION EVALUATIONS**
  - ▶ **SIMPLE BOX BEAM MODEL**
  - ▶ **USES AERODYNAMIC LOADS TO ESTIMATE WEIGHT SO THAT MAX STRESS\*FOS NOT EXCEEDED**
  - ▶ **SHEAR AND BENDING INCLUDED BUT NOT TORSION**

# SINGLE-OBJECTIVE WING OPTIMIZATION

$M_\infty = 0.84$ ,  $C_L = 0.45$ ,  $R_{MAX} < 10^{-6}$ ,  $NG = 22$ ,  $NC = 20$

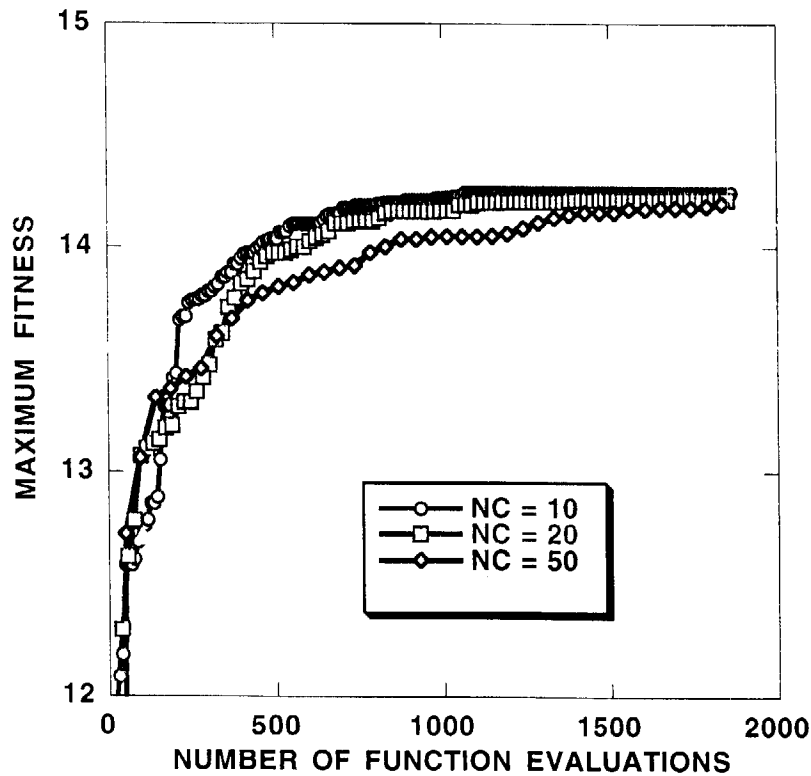


# GA CONVERGENCE CHARACTERISTICS DRAG MINIMIZATION

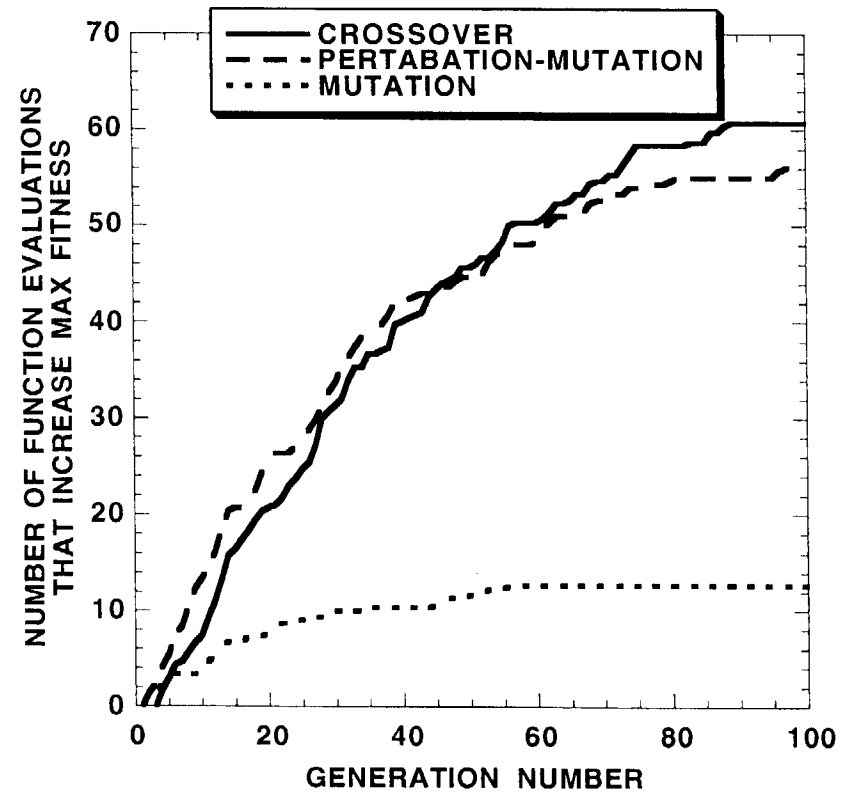


$M_\infty = 0.84$ ,  $C_L = 0.45$ ,  $RMAX < 10^{-6}$ ,  $NG = 22$

### EFFECT OF POPULATION SIZE ON GA CONVERGENCE



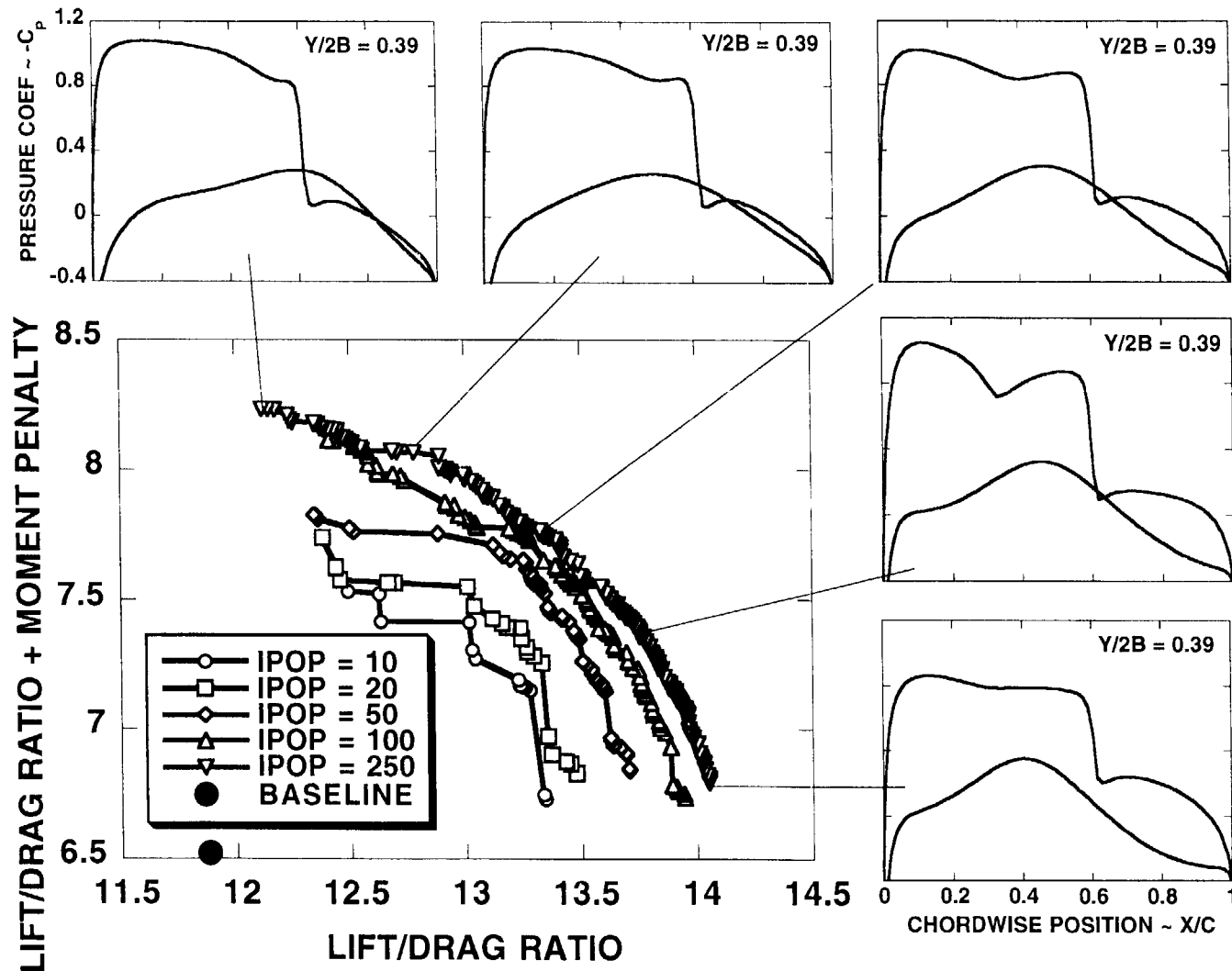
### GA OPERATOR EFFECTIVENESS NC = 20



# GA CONVERGENCE CHARACTERISTICS

## TWO-OBJECTIVE, SINGLE DISCIPLINE OPTIMIZATION

$M_\infty = 0.84, C_L = 0.45, RMAX < 10^{-6}, NG = 22$

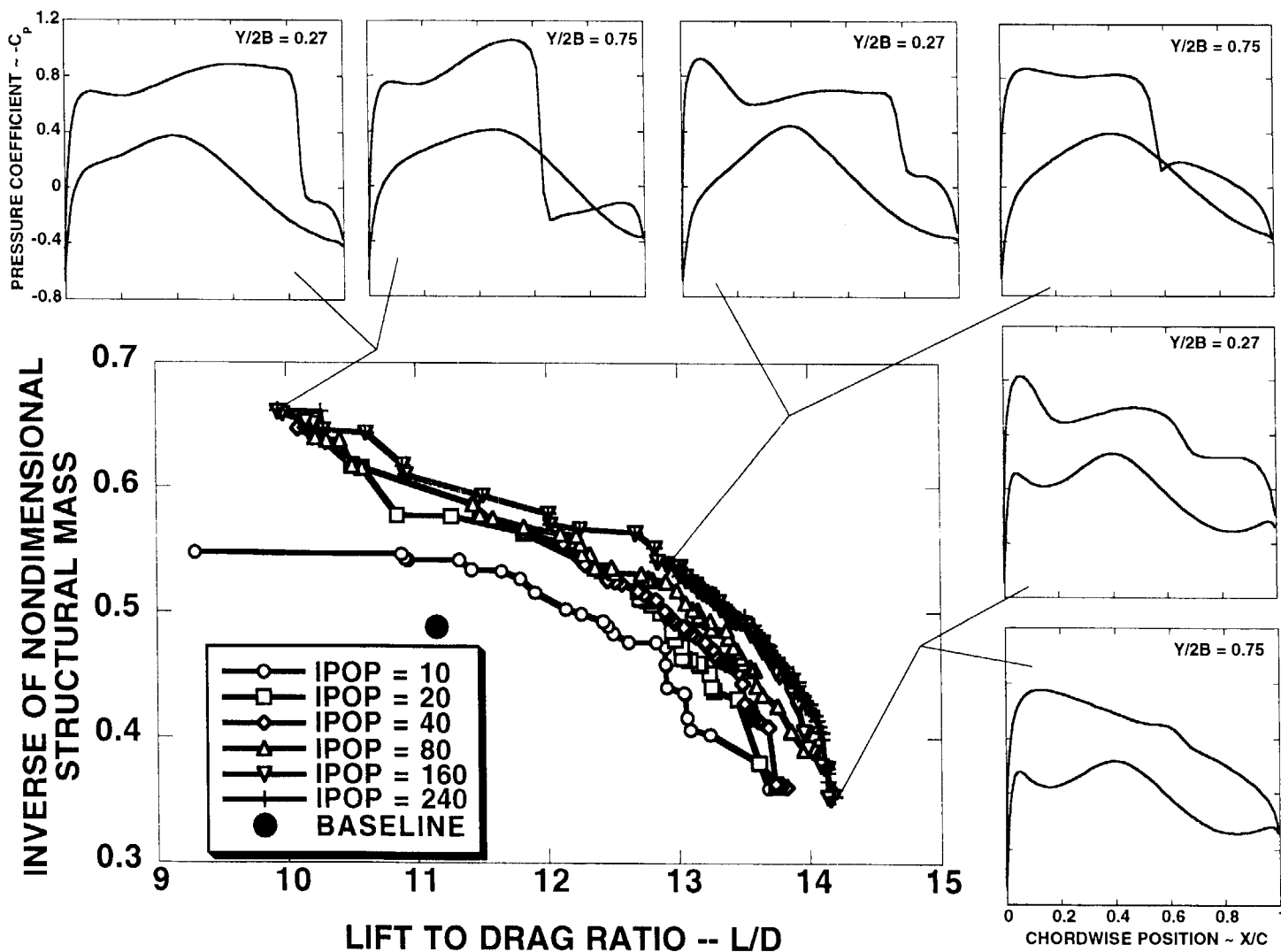


# GA CONVERGENCE CHARACTERISTICS

## TWO-OBJECTIVE, TWO-DISCIPLINE OPTIMIZATION



$M_\infty = 0.84, C_L = 0.45, RMAX < 10^{-6}, NG = 22$







# CONCLUDING REMARKS

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## EVOLUTIONARY ALGORITHMS REPRESENT AN ATTRACTIVE ALTERNATIVE FOR FINDING OPTIMAL SOLUTIONS IN ENGINEERING DESIGN

### Strengths include:

- ▶ Robustness
- ▶ Flexibility
- ▶ Ease of implementation
- ▶ Embarrassingly parallel (ideal for heterogeneous distributed computing)
- ▶ Amenable to multi-modal design spaces
- ▶ Ability to work for multi-objective cases (pareto fronts)

### Weaknesses include:

- ▶ Potentially expensive
- ▶ Difficult to know when convergence is reached

### Future focus on:

- ▶ Efficiency improvements especially for multi-objective cases
- ▶ Parallel implementation (load balancing)
- ▶ Application to other problems