CURVE FITTING SOLAR CELL DEGRADATION DUE TO HARD PARTICLE RADIATION

Edward M. Gaddy,[1] Rebecca Cikoski,[1] Danchai Mekadenaumporn[2]
1. Goddard Space Flight Center, Greenbelt, MD 20771
2. QSS Group Inc., Seabrook, MD 20706

ABSTRACT

This paper investigates the suitability of the equation below for accurately defining solar cell parameter degradation as a function of hard particle radiation.

$$Y = Y_o + C_1 * Log(1+\varphi/\varphi_x) + C_2 * Log(1+\varphi/\varphi_y)$$

The paper also provides methods for determining the constants in the equation and compares results from this equation to those obtained by the more traditionally used:

$$Y = \sum_{i=0}^{m} a_i * [\ln(\varphi)]^i$$

1. INTRODUCTION

The Third Edition of *The Solar Cell Radiation Handbook*, published in 1982, states that commonly used silicon solar cell output parameters, indicated by Y below, degrade as a function of radiation fluence φ as:

$$Y = Y_o + C * Log(1+\varphi/\varphi_x). \quad (1)$$

In this equation, Y may represent the following parameters: open circuit voltage, V_{oc}; short circuit current, I_{sc}; peak power, P_{max}; voltage at peak power, V_{mp}; or current at peak power, I_{mp}. This equation may be slightly modified by dividing by Y_o to obtained normalized functions of the parameter as a function of fluence. In this case the equation takes the form:

$$Y = 1 + C * Log(1+\varphi/\varphi_x). \quad (2)$$

Naturally, the value for C is different for equations (1) and (2) even if the equations describe the same parameter.

In both equations, the quantity φ_x represents the radiation fluence at which Y "starts to change to a linear function of the logarithm of the fluence."[1] The equations are simple, elegant descriptions of the fall off of any of the solar cell parameters with radiation. Unfortunately, the accuracy of the equations is sometimes not sufficient for prediction of the degradation of modern cells even while they continue to predict the general shape of the degradation. By the time the *GaAs Solar Cell Radiation Handbook* was published in 1996[2], the degradation curves were fit for some purposes with four or five order polynomials of the form:

$$Y = \sum_{i=0}^{m} a_i * [\ln(\varphi)]^i \quad (3)$$

Similarly, the degradation for some solar cells in *Assessment of Multijunction Solar Cell Performance in Radiation Environments*[3] was not precise enough to use equation (1) for performing estimations of degradation between data points.

This paper describes an equation, analogous to equation (1), that accurately defines solar cell degradation as a function of radiation when (1) results in too much inaccuracy. The equation is:

$$Y = Y_o + C_1 * Log(1+\varphi/\varphi_x) + C_2 * Log(1+\varphi/\varphi_y) \quad (4)$$

This equation suggested itself to the one of the authors as being correct for V_{oc} degradation of dual junction solar cells. It turned out that the equation provided an excellent fit to not only the open circuit voltage of dual junction solar cells but to all the solar cell parameters of the modern multi-junction solar cells for which the equation was tried. The equation provides well behaved predictions for any reasonable value for φ, which is not true of polynomials which generally give completely erroneous results for any fluence outside of a restricted range.

Again this equation can represent the normalized degradation of parameters by dividing by Y_o. In this case:

$$Y = 1 + C_1 * Log(1+\varphi/\varphi_x) + C_2 * Log(1+\varphi/\varphi_y) \quad (5)$$

The paper describes methods, which are fortunately readily available, for estimating the constants in equation (5). This paper also compares the fit of equations of the form (5) to polynomial equations for describing the degradation of solar cell parameters under hard particle irradiation.

2. COMPARISON

Figure 1 shows the normalized peak power decrease as a function of radiation for a Spectrolab Triple Junction Cell. The Figure uses data taken from *Assessment of Multijunction Solar Cell Performance in Radiation Environments*[3]. The Figure also shows a third order polynomial fit to the data and a fit to the

data using an equation of the form of (5). The sum of the differences squared, between the data and the predictions of the polynomial fit and of the logarithmic fit, are respectively 3.0E-05 and 1.5E-06. The fit of equation (5) is therefore better, which is the true for all of the data in [3] for power, open circuit voltage and short circuit current [4]. The fit of equation (5) was not attempted for voltage at peak power or current at peak power.

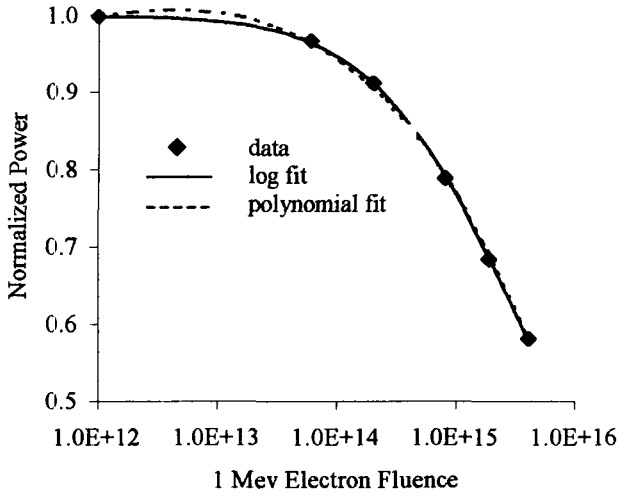

Fig. 1 Normalized Pmax versus 1 MeV Electron Fluence for a Spectrolab Triple Junction Cell. Data fitted with an Equation of Having the Form of Equation (5) and with a Third Order Polynomial.

Outside of the range of the data, the polynomial fit frequently gives answers that exceed a reasonable estimation. However, the logarithmic fit continues to act as expected. For example at a fluence of 1E4 1MeV electrons, the polynomial used for the Fig. 1 fit gives a value of 6.52 for the normalized Pmax, whereas the logarithmic fit gives 1.000. Of course, the polynomial fit is not intended for this fluence; however, for some uses it is convenient to have an equation that will give accurate results for any reasonable fluence. The logarithmic fit achieves results with four constants, the number of constants required by the lowest order polynomials, namely third order, historically used to fit the data.

It might be supposed that a higher order polynomial will provide a better fit. In terms of the sum of the square of the differences between the data and the polynomial prediction, this is undoubtedly true. However, as the order of the polynomial is increased, it may predict unphysical behavior between the points. The data plotted in Fig. 2, the same data plotted in Fig. 1, is fitted to a fifth order polynomial. Clearly, this result is not acceptable for predicting solar cell degradation. This difficulty is ultimately caused because the polynomial does not reflect the physical processes that cause the degradation. It may be possible to rectify this problem by placing conditions on the polynomial that prevent the slope of the curve from increasing with increasing fluence. However, this would make fitting with the polynomial more difficult than using standard methods and computer programs, see Section 3 on Determining Constants. (It is also possible that the unacceptable behavior is due to fitting all six data points. The third order polynomial was fitted without using the highest fluence. [4].)

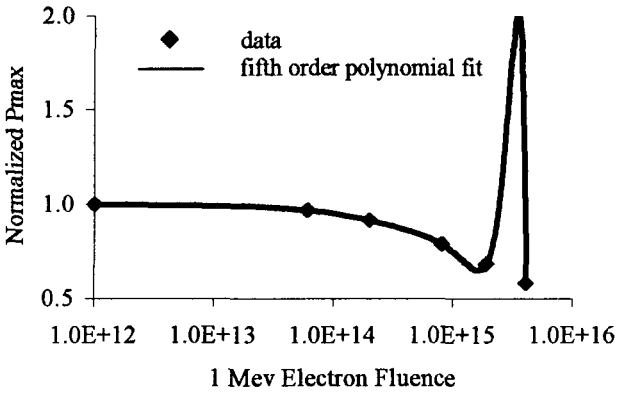

Fig. 2 Normalized Pmax versus 1 MeV Electron Fluence for a Spectrolab Triple Junction Cell with Data Fitted with a Fifth Order Polynomial.

SUMMARY OF RESULTS

Tables I and II summarize the results of fits using equations of the types (2), (3), and (5) for the cell types used in reference [4]. In all cases, equations of type (5) provide the best fit.

Table I shows the largest error that the fitted curve has with respect to any data point in the set, or the maximum residual. With the exception of the Spectrolab three junction cell optimized for end of life power (Spectrolab 3J EOL), the data provided by Dean Marvin[4] was for six fluences: 1E12, 6E13, 2E14, 8E14, 1.9E15 and 4.1E15). All of these data were taken under the direction of the Systems Planning and Engineering Group of The Aerospace Corporation at the request of the Air Force Research Laboratory. The cells are roughly of 1997 vintage. The triple junction solar cells have an efficiency of approximately 25%. The polynomial was not fitted using the highest fluence. However, the log equations were generally successful in fitting well to all six points. As a result the results for equations (2) and (5) were for all the data generated. Even so, these equations always show less error than do the polynomial fits. (The error of the polynomial to the highest fluence was not considered in Table I.)

The data for the Spectrolab 3J EOL cell was taken by Spectrolab. Its data is nonetheless reported in [3].The data for these cells was taken at: 1E13, 1E12, 1E14, 5E14, and 2E15. For the fits shown in the table, an additional point was added at 1E12 that showed no degradation.

Table II provides a similar summary to that of Table I, except the parameter is the sum of the squares between the fitted curve

and each of the data points. Again equations (2) and (5) are fitted and compared to more data points and so if all else were equal would show a greater sum of squares. However, their fit is sufficiently superior that the opposite happens.

Table I. Largest Error Of Fits (Maximum Residual)

Cell Type	Polynomial Fit Against 5 Points (%)	Two Parameter Log Fit Against 6 Points (%)	Four Parameter Log Fit Against 6 Points (%)
Spectrolab 2J	.53	1.0	.15
Spectrolab 2J EOL	1.1	2.1	.32
Tecstar 3J	.33	2.0	.23
Spectrolab 3J	.56	3.1	.08
Spectrolab 3J EOL	.85	.60	.66

Table II. Sum of Squares Multiplied by 1E5

Cell Type	Polynomial Fit Against 5 Points	Two Parameter Log Fit Against 6 Points	Four Parameter Log Fit Against 6 Points
Spectrolab 2J	3.8	14	.35
Spectrolab 2J EOL	16	62	2.8
Tecstar 3J	1.7	83	1.2
Spectrolab 3J	3.0	90	.15
Spectrolab 3J EOL	17	11	.6

3. DETERMINING CONSTANTS

One advantage of the polynomial is that there is a well established closed form solution for determining the constants that will minimize the sum of the differences squared between the data and the predictions of the polynomial fit. These methods are readily available in a number of computer programs and spread sheets.

In general, determining the constants in non-linear equations such as equation (5) is more difficult even though there are several widely used methods to solve this problem. These are the method of steepest descent, the Gaussian-Newton method, and the more comprehensive method of Levenberg-Marquardt [5], [6]. Fortunately, the method of Levenberg-Marquardt is available in several programs to determine parameters such as C_1 and C_2 in user defined equations [7]. Even so, the procedure is not quite as straightforward as that of determining the predictions of a polynomial fit.

To successfully use the programs that use Marquardt's method, it is sometimes necessary to divide the 1MeV electron fluence by 1E15 for the program to work. It is also necessary to use equations of the form of equation (6), because if the programs use $1/\phi_x$ they will sometimes halt due to division by zero.

$$Y = 1 + C_1 * Log(1 + C_3 * \varphi) + C_2 * Log(1 + C_4 * \varphi) \qquad (6)$$

In addition, initial guesses must be provided for the constants. If the guesses are not sufficiently accurate, the program will not converge to a solution. After using equation (6), the constants φ_x and φ_y are manually recomputed to obtain the desired values.

As stated above, the initial form of equation C_3 and C_4 was derived from physical considerations regarding the degradation of the cell's open circuit voltage. However, the best fits sometimes do not match a physical solution. Specifically, C_3 or C_4 will sometimes be negative for the best fits. Nonetheless, the fits are generally excellent.

In addition to the commercial programs, one of the authors wrote a computer program which estimates the values of C_1 and C_2. Because of knowledge of the specifics of the problem, the user does not need to enter initial guesses regarding the constants and convergence is assured. The program has not yet been fully optimized; its final results are not quite as accurate as those provided by the commercial program.

CONCLUSION

Equations of the form:

$$Y = 1 + C_1 * Log(1 + \varphi/\varphi_x) + C_2 * Log(1 + \varphi/\varphi_y)$$

may be used to accurately fit data representing the degradation of multi-junction solar cells after exposure to hard particle radiation. This equation form is more accurate than a polynomial within the range of the data and can be used to extrapolate outside the range of the data. It is slightly more difficult to determine the constants in this equation than the constants in a polynomial.

REFERENCES

[1] H. Y. Tada, J. R. Carter, Jr., B. E. Anspaugh, *R. G. Downing, Solar Cell Radiation Handbook, Third Edition,* JPL Publication 82-69, 1982, p. 3-18.

[2] B. E. Anspaugh, *GaAs Solar Cell Radiation Handbook,* JPL Publication 96-9, 1996.

[3] D. C. Marvin, *Assessment of Multijunction Solar Cell Performance in Radiation Environments,* Aerospace Report No. TOR-2000(1210)-1, 2000 and Errata Sheet dated 29 February, 2000.

[4] Dean Marvin kindly provided the raw data from reference [3] to the authors.

[5] Donald W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters", *SIAM J. Appl. Math.* vol 11, 431-441, (1963).

[6] Levenberg. K., "A Method for the Solution of Certain Problems in Least Squares," *Quart. Appl. Math.* Vol. 2, pp. 164-168, 1944.

[7] This paper used GraphPad Prism and CurveExpert.