Multiobjective Aerodynamic Shape Optimization Using Pareto Differential Evolution and Generalized Response Surface Metamodels

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Abstract - Differential Evolution (DE) is a simple, fast, and robust evolutionary algorithm that has proven effective in determining the global optimum for several difficult single-objective optimization problems. The DE algorithm has been recently extended to multiobjective optimization problems by using a Pareto-based approach. In this paper, a Pareto DE algorithm is applied to multiobjective aerodynamic shape optimization problems that are characterized by computationally expensive objective function evaluations. To improve computational efficiency the algorithm is coupled with generalized response surface metamodels based on artificial neural networks. Results are presented for some test optimization problems from the literature to demonstrate the capabilities of the method.

I. INTRODUCTION

Aerodynamic shape optimization refers to the process of determining the shapes of airfoils, wings, or other aerodynamic surfaces that are optimal with regard to certain (one or many) desired characteristics. Major advances in the field of aerodynamic shape optimization have been achieved in recent years by combining improved methods for the simulation of complicated flow fields with efficient numerical optimization techniques and by exploiting the powerful capabilities of modern computers. Both Euler and high-fidelity Navier-Stokes solvers have been combined with various optimization techniques (gradient-based methods, adjoint methods, response surfaces, genetic algorithms, neural networks, etc.) to obtain optimal aerodynamic shapes and designs.

Multiobjective aerodynamic shape optimization is part of a class of optimization problems characterized by the presence of multiple conflicting objectives that must be optimized simultaneously and allow multiple optimal solutions. These multiple solutions are referred to collectively as the non-inferior or non-dominated Pareto-optimal set. They are all optimal in the sense that there are no other solutions in the entire solution domain or search space that are superior to them when all objectives are considered simultaneously. Multiobjective evolutionary algorithms (MOEAs) are population-based methods developed in recent years for solving such problems. These algorithms guide the search process toward the global Pareto-optimal region while maintaining adequate population diversity to capture as many solutions in the Pareto set as possible.

MOEAs can be classified broadly as either non-Pareto or Pareto-based approaches. Non-Pareto methods are based on an aggregating approach where the multiple objective functions are combined into a single function allowing single-objective evolutionary algorithms to be then applied. Methods based on aggregating approaches, such as weighted sum, goal attainment, etc., require multiple single-objective optimization runs with different weights for the various objectives in order to find multiple Pareto-optimal solutions. On the other hand, Pareto-based approaches offer the advantage of generating multiple Pareto solutions simultaneously. These methods use nondominated ranking and selection to evolve a population of solutions toward the Pareto set.

This paper considers the evolutionary strategy (ES) known as Differential Evolution (DE) [1] that has been applied successfully to a wide range of single-optimization problems. Recently, efforts to extend the DE algorithm to multiobjective problems have been reported. In [2] a Pareto DE method is described that modifies the selection procedure in the basic DE algorithm by incorporating a nondominated sorting and ranking selection scheme Alternative approaches have also been suggested by others.

In aerodynamic shape optimization the objective function evaluations are performed typically using compute-intensive Euler and Navier-Stokes analysis codes. In such applications, the routine use of population-based MOEA approaches is impeded by the fact that they often require large numbers of expensive objective function evaluations. However, metamodeling techniques based on the use of approximate models as surrogates for the actual objective functions can be incorporated to reduce the number of calls to the expensive analysis codes. One metamodeling approach that has received much attention is the response surface method (RSM). While traditional RSM uses low-order polynomials for function approximation, generalized response surface methods (GRSM) allow for the inclusion of a wide range of approximations, including polynomials, neural networks, kriging, multivariate adaptive regression splines, radial basis functions, and multiquadrics. Both global and local GRSM approaches have been established for single-objective optimization. In the global approach a GRSM metamodel for the entire design space is used and gradually refined as the optimization progresses. Since developing good global metamodels with validity over the entire design space can be difficult, a local approach based on local approximations and a sequential strategy for iteratively zooming into the region of design space around the

optimum is often preferred. Typically, the optimization process is decomposed into a sequence of cycles and an optimization subproblem is defined within a trust region, i.e., a smaller part of the design space, where local metamodels are used as surrogates for the exact objective functions. The exact objective functions are evaluated only at a limited number of points in each trust region, thus reducing computational cost. The trust regions are resized and/or moved as the optimization progresses. Various single-objective optimization frameworks based on such trust-region and move-limit methods have been developed to strike an appropriate balance between the use of exact and approximate function evaluations. In the case of multiobjective optimization, the linking of evolutionary optimizers and GRSM is not straightforward since we are not dealing with one optimum but a set of Pareto-optimal solutions. Thus, the multiobjective equivalent of the region around the optimum is a complex and often nonconnected area of the design space; extensions of the sequential search space zooming strategy of single-objective optimization are non-trivial.

In this paper the Pareto DE method is applied for the first time to multiobjective aerodynamic shape optimization. Efforts to link a neural network-based GRSM to the multiobjective DE algorithm are also described. Radial basis function neural networks are used here primarily because of the ease with which they can be trained. Results are presented for sample optimization problems from the literature. The problems considered are the multiobjective optimization of a two-dimensional compressor blade, and time permitting, optimization of a two-dimensional turbomachinery cascade.

II. DIFFERENTIAL EVOLUTION

DE uses a population of real-valued parameter vectors of design variables that is usually initialized in a random fashion. The population size is maintained constant throughout the optimization. The key ingredient of DE is mutation. New parameter vectors for the subsequent generation are formed using weighted differences between two (or more) parameter vectors selected randomly from the current population to provide appropriately scaled perturbations that modify another parameter vector (or, comparison vector) selected from the same population. This can be implemented in various forms A discrete recombination strategy is also used in addition to the mutation operator. The selection scheme is deterministic and based on local competition only, with the child trial vector competing against one population member and the survivor entering the new population.

II. PARETO-BASED DIFFERENTIAL EVOLUTION

The Pareto-based Differential Evolution algorithm used in this paper differs from the basic algorithm primarily in the selection procedure used to pick subsequent generations of the population. The nondominated sorting and ranking selection procedure [3] that has been shown to be very effective in guiding the search toward the global Pareto front for several difficult optimization problems is adopted. The method in effect combines the robust and effective DE mutation and crossover operators with the fast nondominated ranking scheme and diversity preservation strategy of the highly successful NSGA-II algorithm [3]. The result is a simple and powerful evolutionary strategy that is self-adaptive, elitist, and can maintain diversity in the Pareto set.

III. COUPLING DE WITH GRSM METAMODELS

In the current study radial basis function neural networks are used as metamodels to approximate the objective functions. Both global and local approaches were implemented. In the global approach metamodels are not used until the population has evolved for several generations. The network is then trained on the exact analyses data and the metamodel is continuously refined as the population further evolves. The implementation of the local approach is substantially more involved. The updating of the search space is done using the sum of n-dimensional hypercubes (n is the number of design variables) around the centers of weight of the current Pareto set. An auxiliary optimization problem is solved to add new points in unexplored areas of the design space.

IV. RESULTS

Figure 1 shows results for the well-known two-objective function problem of Poloni with two nonconvex Pareto fronts that are disconnected in both the objective and decision variable spaces. The present method is able to predict the two disconnected Pareto fronts that lie on the boundaries of the search space.
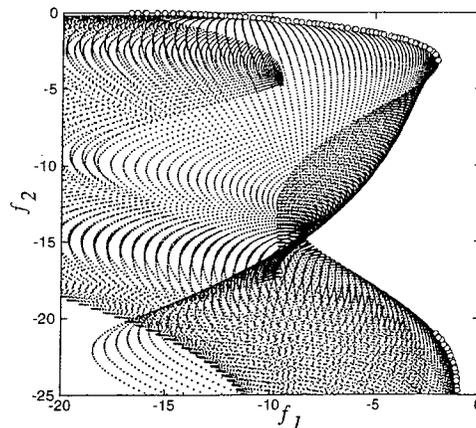
Figure 1. Pareto-optimal solutions in objective spance obtained by the present method for the Poloni test problem. The dots represent solutions obtained on a 200 x 200 grid of uniformly distributed points in parameter space.

The method has also been applied to the multiobjective aerodynamic shape optimization of a compressor blade. Results are currently being obtained interfacing the method with the GRSM and will also be presented in the final paper.

REFERENCES

[1] R. Storn, and K. Price, "Differential Evolution - A simple evolution strategy for fast optimization," Dr. Dobb's Journal Vol. 22:4, pp. 18-24, April 1997.

[2] N. K. Madavan, "Multiobjective Optimization Using a Pareto Differential Evolution Approach," IEEE Congress on Evolutionary Computation, Honolulu, HI, May 12-17, 2002.

[3] K. Deb, S. Agrawal, A. Pratap and T. Meyarivan, "A Fast Elitist Non-dominated sorting genetic algorithm for multiobjective optimization: NSGA-II," Proceedings of the Parallel Problem Solving from Nature VI Conference, pp. 849-858, Paris, France, 16-20 September, 2000.