Piezoelectric Power Requirements for Active Vibration Control

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ABSTRACT

This paper presents a method for predicting the power consumption of piezoelectric actuators utilized for active vibration control. Analytical developments and experimental tests show that the maximum power required to control a structure using surface-bonded piezoelectric actuators is independent of the dynamics between the piezoelectric actuator and the host structure. The results demonstrate that for a perfectly-controlled system, the power consumption is a function of the quantity and type of piezoelectric actuators and the voltage and frequency of the control law output signal. Furthermore, as control effectiveness decreases, the power consumption of the piezoelectric actuators decreases. In addition, experimental results revealed a non-linear behavior in the material properties of piezoelectric actuators. The material non-linearity displayed a significant increase in capacitance with an increase in excitation voltage. Tests show that if the non-linearity of the capacitance was accounted for, a conservative estimate of the power can easily be determined.

Keywords: piezoelectric, power, vibration, control, non-linear piezoelectric capacitance

1.0 VARIABLES AND NOTATION

C = Capacitance
c = Damping of host structure
D_3 = Electric displacement
d_{31} = Piezoelectric constant
E = Electric field
F_e = External force
F_a = Piezoelectric actuator force
h = Actuator thickness
I = Current
k = Stiffness of host structure
k_a = Stiffness of piezoelectric actuator
L = Actuator length
m = Mass of the host structure
P = Power
Q = Charge
S_1 = Strain of piezoelectric actuator

s = Complex variable
T_1 = Stress acting on piezoelectric actuator
t = Time
V = Voltage
w = Actuator width
x = x-direction, displacement in x-direction
Y = Electrical admittance
Y_{11} = Piezoelectric elastic modulus
y = y-direction
z = z-direction

ε_{33} = Dielectric constant
φ = Phase angle
ω_n = Natural frequency
ω = Radial frequency
ζ = Damping coefficient

2.0 INTRODUCTION

Piezoelectric materials have been investigated extensively as actuators because they can couple electrical energy with mechanical energy. The benefits found in using piezoelectric actuators for active vibration

control have been displayed in numerous ground tests and wind-tunnel demonstrations (1-12). The current research was motivated by the results from the Piezoelectric Aeroelastic Response Tailoring Investigation(PARTI)(13,14). During this study, a four foot long semi-span wing model with 72 distributed piezoelectric actuators was used to demonstrate that strain actuation can effectively control aeroelastic response.

One critical issue raised by this study is whether full-scale airplanes and other large structures will require excessive power to use piezoelectric actuators for active vibration control. The current study seeks to find a method to predict the power requirements of surface-bonded piezoelectric actuators used for active vibration control.

Previous research on piezoelectric power consumption essentially arise from the approaches of two different groups of researchers: Liang et. al.(15-18) and Hagood, Chung and von Flotow(19). Both approaches examined the case of piezoelectric power consumption where the actuators are used to excite the host structure and did not consider the case of active control.

Liang developed an equation of motion of the piezoelectric actuator. The response of the actuator was determined by prescribing boundary conditions of the actuator that coupled it with the host structure. The final solution includes a coupling of the electro-mechanical impedance of the host structure and the piezoelectric actuator.

An alternative method was developed by Hagood, Chung and von Flotow. Their approach determined the equation of motion of piezoelectric actuator and host structure. The response of the structure determines the stress acting on the piezoelectric actuator. The deviation between the two methods is that Liang solves for the explicit equation of motion of the actuator where Hagood, Chung and von Flotow determine the equation of motion of the actuator and host structure. Both models are proven to be highly accurate. The current research applies the model presented by Hagood, Chung and von Flotow because it is conceptually easier to apply to a variety of structures.

Subsequent research concerning the power consumption of piezoelectric actuators is primarily founded from the two previous methods(10,11,12,20). A comprehensive study was presented by Warkentin(20) of the Massachusetts Institute of Technology. Warkentin presented a development of piezoelectric power consumption for active vibration control using the analytical models developed by Hagood, Chung and von Flotow. The conclusion of this and all previous literature for both the excitation of a structure and the closed-loop control of a structure is that the electrical power of piezoelectric actuators is dependent on the mechanical motion of the structure and the electrical characteristics of the piezoelectric material.

The research presented in this paper takes Warkentin's results a step further by developing an analytical model that shows that power consumption of piezoelectric materials used for active vibration control is independent of the mechanical motion of the host structure when the structure is completely controlled. When completely controlled, the structure is motionless, thus the power requirements of the piezoelectric actuator are no longer a function of the mechanical motion of the structure. In this ideal scenario, the power is only dependent on geometry and material properties of piezoelectric actuators and the voltage and frequency of the control law signal. Furthermore, this research finds that as control effectiveness decreases, the power requirements of the actuator decrease. Thus, the results from this ideal scenario provide an upper bound for the power required.

In the present study, an analytical model is developed and used to identify the parameters that characterize the power consumption of piezoelectric materials. This model is then applied to a closed-loop system where the piezoelectric actuators are utilized for active vibration control. Next, the analytical model is verified using a single-degree of freedom (SDOF) experimental model. During these experiments, a non-linearity in the material constants was observed which had a significant influence on the power consumption of the actuator.

3.0 ANALYTICAL DEVELOPMENT OF POWER

3.1 Electrical Power Consumption:

Electrical power, P, is developed from the product of voltage and current: $P=VI$. For sinusoidal voltage, $V=\sin(\omega t)$, and current, $I=\sin(\omega t+\varphi)$, the apparent power using the magnitude of the electrical admittance, Y, is defined as(21):

$$P = V^2 |Y| \sin(\omega t) \sin(\omega t + \varphi) \tag{1}$$

$$Y = \frac{I}{V}$$

where $|Y| = (\text{Re}(Y)^2 + \text{Im}(Y)^2)^{1/2}$ and phase angle, $\varphi = \tan^{-1} \dfrac{\text{Im}(Y)}{\text{Re}(Y)}$ (2)

Equation 1 indicates that given the voltage, the power requirements of a system can be characterized by determining the electrical admittance of the system. When piezoelectric actuators are used for active vibration control, the voltage is defined by the control law output signal. Therefore, the analytical development of power presented below focuses on the development of the electrical admittance of piezoelectric actuators.

The electrical admittance is developed directly from the piezoelectric constitutive equations. The motion of the host structure is coupled with the piezoelectric actuator through the strain of the actuator. The contribution of the strain on the admittance is determined for a structure completely controlled where all vibration has been suppressed and for a less perfectly controlled structure.

3.2 Constitutive Equations:

The constitutive equations for a piezoelectric actuator (reference 15) excited in the z-direction and displaced in the x-direction are (orientation of actuator is illustrated in Figure 1):

$$S_1 = \frac{1}{Y_{11}} T_1 + d_{31} E$$

or (3)

$$T_1 = Y_{11}(S_1 - d_{31}E)$$

Figure 1: Configuration of Piezoelectric Actuator.

$$D_3 = \varepsilon_{33} E + d_{31} T_1 \tag{4}$$

Where the electric field, E, is defined as voltage divided by the piezoelectric material thickness: E=V/h. From equations 3 and 4, the charge and the current can be developed:

$$Q = \iint_{xy} D_3 \, dx \, dy = wL[\varepsilon_{33} E + d_{31} T_1] \tag{5}$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}[\varepsilon_{33}E + d_{31}T_1]wL = \frac{d}{dt}[(\varepsilon_{33} - d_{31}^2 Y_{11})E + d_{31}Y_{11}S_1]wL \tag{6}$$

Again, the admittance is defined as the ratio of current to voltage: Y=I/V. By taking the Laplace Transform of equation 6 and dividing by voltage, the admittance is given by:

$$\frac{\tilde{I}(s)}{\tilde{V}(s)} = \tilde{Y}(s) = s\left[(\varepsilon_{33} - d_{31}^2 Y_{11})\frac{wL}{h} + wLd_{31}Y_{11}\frac{\tilde{S}_1(s)}{\tilde{V}(s)}\right] \quad \text{or}$$

(7)

$$\frac{\tilde{I}(s)}{\tilde{V}(s)} = \tilde{Y}(s) = s\left[C + wLd_{31}Y_{11}\frac{\tilde{S}_1(s)}{\tilde{V}(s)}\right]$$

Equation 7 shows the admittance is comprised of two parts: a constant defined by the geometry and material constants of the piezoelectric actuator, designated by the constant C, and a term that is a function of the ratio of actuator strain to applied voltage:
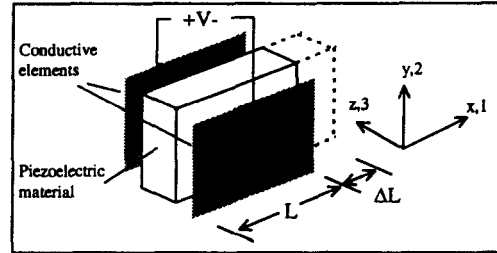
$$wLd_{31}Y_{11}\frac{\tilde{S}_1(s)}{\tilde{V}(s)} \tag{8}$$

To determine the total actuator admittance, an understanding of the actuator strain, S_1, must be developed.

3.3 Actuator Strain

The actuator strain is determined by the interaction of the actuator and host structure. For a single degree of freedom system excited by an external force and a piezoelectric actuator, the equation of motion is:

$$m\ddot{x} + c\dot{x} + [k + k_a]x = F_a + F_e \tag{9}$$

Figure 2: Mass, spring damper system excited by external force and piezoelectric actuator.

The additional stiffness of the actuator on the host structure is defined by k_a. The mass and damping contributions of the piezoelectric actuator are assumed to be negligible. The normalized equation of motion is:

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = (F_a + F_e)/m \tag{10}$$

It is assumed that the strain of the actuator is the same as the strain of the host structure. For this development, the strain of the actuator is given by:

$$S_1 = \frac{\Delta L}{L} = \frac{x}{L} \tag{11}$$

Applying the definition of the strain from equation 11, the actuator strain can be determined by taking the Laplace Transform of equation 10:

$$\tilde{S}_1(s) = \frac{1}{mL(s^2 + 2\xi\omega_n s + \omega_n^2)}(\tilde{F}_a(s) + \tilde{F}_e(s)) \tag{12}$$

The blocking force of the piezoelectric actuator is defined from the stress when the piezoelectric actuator strain is zero:

$$F_a = -T_1(S_1 = 0) \times Area = whd_{31}Y_{11}E = wd_{31}Y_{11}V \tag{13}$$

Combing equations 12 and 13, the solution for the ratio of actuator strain to applied voltage is given by:

$$\frac{\tilde{S}_1(s)}{\tilde{V}(s)} = \frac{1}{mL(s^2 + 2\xi\omega_n s + \omega_n^2)}(wd_{31}Y_{11} + \frac{\tilde{F}_e(s)}{\tilde{V}(s)}) \tag{14}$$

3.4 Structural Control

For a perfectly controlled structure, the displacement of the host structure is equal to zero. As indicated by equation 11, the strain of the actuator will equal zero for this case. Actuator strain is equal to zero when the force generated by the piezoelectric actuator is equal and opposite to the external force:

$$wd_{31}Y_{11}\tilde{V}(s) = -\tilde{F}_e(s) \tag{15}$$

The maximum commanded force of the piezoelectric actuator is defined by the maximum blocking force:

$$\text{maximum } \tilde{F}_a(s) = wd_{31}Y_{11}\tilde{V}_{max}(s) \tag{16}$$

The maximum voltage is generally defined by the maximum voltage the piezoelectric actuator can withstand before it breaks down. For perfect control, the external force must be less than or equal to the actuator blocking force:
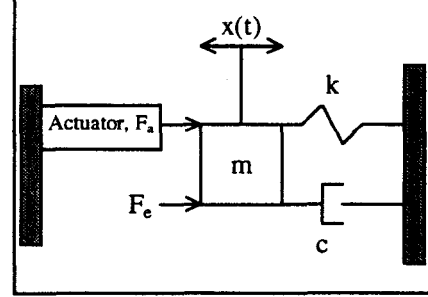
$$|F_a| \geq |F_e| \qquad (17)$$

$$\text{or}$$

$$wd_{31}Y_{11}V_{max} \geq \tilde{F}_e(s)$$

In which case the strain in equation 11 is equal to zero. Consequently, the admittance of the piezoelectric actuator reduces to:

$$\frac{\tilde{I}(s)}{\tilde{V}(s)} = \tilde{Y}(s) = s(\mathcal{E}_{33} - d_{31}^2 Y_{11})\frac{wL}{h} = sC \qquad (18)$$

$$\text{Effective Capacitance} = C = (\mathcal{E}_{33} - d_{31}^2 Y_{11})\frac{wL}{h} \qquad (19)$$

The constant C in equation 19 is the effective capacitance of a piezoelectric actuator. The effective capacitance is made up of dielectric and piezoelectric properties of the piezoelectric actuator. As a note: the capacitance quoted by many manufacturers is defined only by the dielectric properties of the piezoelectric actuator. In this paper, capacitance will refer to the effective capacitance, C, defined in equation 19.

If the external force exceeds the maximum blocking force, control is less than perfect:

$$wd_{31}Y_{11}\tilde{V}_{max}(s) < -\tilde{F}_e(s) \quad \Rightarrow \quad \text{strain, } S_1 > 0$$

In this case, the external force can be redefined as a combination of the actuator blocking force and an additional force, f_e, as illustrated in Figure 3:
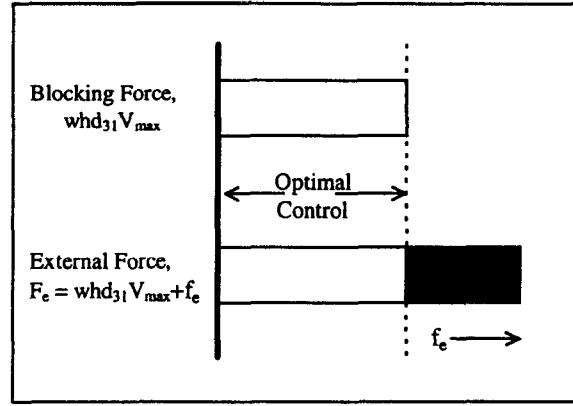

Figure 3: Force Diagram of Actuator and External Forces

$$\tilde{F}_e(s) = -(wd_{31}Y_{11}\tilde{V}_{max}(s) + \tilde{f}(s)_e) \qquad (20)$$

The transform of the strain to voltage can be simplified as:

$$\frac{\tilde{S}(s)}{\tilde{V}(s)} = \frac{-\tilde{f}_e(s)}{\tilde{V}_{max}(s)} \frac{1}{mL(s^2 + 2\xi\omega_n s + \omega_n^2)} \qquad (21)$$

Hence, the admittance for less than perfect control is:

$$\tilde{Y}(s) = s\left[C - \frac{\tilde{f}_e(s)}{\tilde{V}_{max}(s)} \frac{wd_{31}Y_{11}}{m(s^2 + 2\xi\omega_n s + \omega_n^2)}\right] \qquad (22)$$

The magnitude of the added force, f_e, actually reduces the total admittance of the actuator. Accordingly, as control effectiveness decreases, admittance(and thus power) decreases.

The strain in a structure is limited by the failure strength of the structure, after which the structure breaks. Therefore, the strain induced by a force on a structure is limited by the failure strength of the structure. Considering the failure strength of piezoelectric actuators, the magnitude of equation 21 is limited by the piezoelectric actuator failure strength. For this case, the second term in equation 22 is generally limited to a value that is an order of magnitude less than the capacitance, C. Consequently, even at the largest value of f_e, the admittance is only reduced approximately 10%. Most applications of piezoelectric actuators will involve situations far below the ultimate strength of the actuator. Thus, for most control situations, a conservative estimate of the admittance can be made by neglecting the influence of the additional external force, f_e.

3.5 Piezoelectric Power Required for Active Control

Conservative calculations of the power consumption of piezoelectric actuators can be determined without direct consideration of the dynamics of the actuator and host structure. The results of section 3.4 indicate that less than 10% error is introduced by neglecting these dynamics. Using equations 1-19 enables the power requirements to control both simple and complex structures to be determined in an uncomplicated manner. Transforming equation 18 from the complex domain to the frequency domain, the admittance becomes:

$$Y(\omega) = \omega(\varepsilon_{33} - d_{31}^2 Y_{11}) \frac{wL}{h} = \omega C \tag{23}$$

$$\text{phase angle, } \varphi = \frac{\pi}{2} \tag{24}$$

Returning to equation 1, the estimate for the magnitude of piezoelectric power is:

$$P = \frac{\omega C V^2}{2} \tag{25}$$

The voltage is assumed to be sinusoidal, V=sin(ωt). Equation 25 is a conservative estimate of the piezoelectric power consumption for active control given the capacitance of the actuator and the voltage and frequency of the control law output signal. The capacitance of the actuator is determined from equation 19. For multiple piezoelectric actuators, the total capacitance is the sum of the capacitance of each piezoelectric actuator:

$$C_{total} = \sum_{i=1}^{n} C_i \tag{26}$$

C_i = Effective Capacitance of Actuator i

n = Number of Actuators Being Used

The ability to predict the maximum power consumption of piezoelectric actuators can be extremely useful in the design of structures that will use piezoelectric actuators for active control. For example: once a estimate of the number and type of piezoelectric actuators are established, the maximum power required to control vibration on a large airplane wing can be calculated as follows:

$$P_{max} = \frac{1}{2} \omega_{max} V_{max}^2 \sum_{i=1}^{n} C_i \tag{27}$$

Where ω_{max} is the frequency of the control law output signal. Generally, this frequency is defined by the frequency of the highest mode of interest for control purposes.

Note: the above developments assume the capacitance of the piezoelectric actuator is constant with respect to voltage. Although this assumption is commonly made, the experimental tests conducted as a part of the current research show that capacitance actually increases with voltage. Thus, to accurately predict the maximum power consumption using equation 25, the measured maximum capacitance of the piezoelectric actuators must be used. Further description of this phenomena is discussed in the Experimental Results section that follows.

4.0 EXPERIMENTAL RESULTS

4.1 Objective and Apparatus

Experimental tests were conducted to verify that structural dynamics have a minor affect on the closed-loop admittance of piezoelectric actuators. This was accomplished by comparing closed-loop measurements of admittance with admittance calculated (via equation 18) using experimental values for the capacitance. As noted earlier, equation 23 assumes that the structural dynamics of the host structure and actuator are

negligible. This section describes the model, the non-linear capacitancecharacteristics, and the comparison of experimental and actual data.

For the experimental tests, a single-degree-of-freedom model, displayed in Figure 4, was used. The model is a 12 inch long cantilevered beam with an aluminum mass fixed to the tip to ensure single-degree-of-freedom dynamics. One piezoelectric actuator was adhered to each side of the beam near the cantilever. An electro-magnetic force shaker was positioned to apply a force at the center of gravity of the mass. Force output of the shaker was measured through a load cell at the tip of the stinger connecting the shaker to the structure. A strain-feedback control law was used for all tests.

4.2 Non-Linear Capacitance

As noted previously, experimental tests revealed a non-linear characteristic in the material properties of the piezoelectric actuator. Although brief mention of this phenomenon was made by Warkentin in reference 20, no explicit description has been found in the available literature. Tests revealed that the admittance of the piezoelectric actuators was increasing with voltage as indicated from Figure 5. The admittance for this case was measured by fixing the structure to ensure there was no motion. This allows the admittance to be a function of the capacitance only. From the admittance in Figure 5, a relationship between capacitance and voltage was formed, as seen in Figure 6. The experimentally determined values of capacitance were found to increase significantly with voltage. To account for this non-linear behavior, the capacitance of the piezoelectric actuators was re-calibrated as a function of voltage. Using the experimental results in Figure 6, a first order model of capacitance as a function of voltage was developed: (shown as the empirical model in the figure)
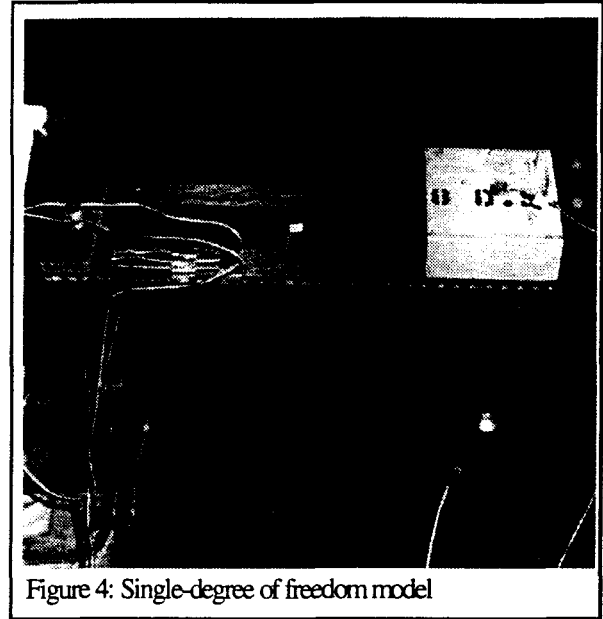

Figure 4: Single-degree of freedom model

$$C_{actual} = C_0 + \frac{\partial C}{\partial V}V(\omega)$$

(28)

The empirical solution for the slope was estimated as:

$$\frac{\partial C}{\partial V} = (0.0044)C_0$$

This translates to a 0.44% increase in capacitance per volt. Piezoelectric actuators are commonly operated in the 100 to 200 volt range. In this voltage range, approximately a 40-90% increase in capacitance can be expected over previously assumed values of capacitance.

4.3 Estimate of Admittance

Since experimental tests showed that capacitance increases with voltage, all values for capacitance were calculated using the empirical model shown above. Two estimates for piezoelectric admittance during active vibration control are presented. Both estimates of admittance are plotted against the actual admittance that was experimentally measured. The first estimate of admittance, as seen in Figure 6, assumes the control law output voltage is unknown; therefore, the maximum voltage of the actuator is used to determine the updated capacitance. From figure 6, the maximum admittance is clearly greater than the actual admittance. The second estimate of admittance, as seen in figure 7, uses the power spectral density of the control law output voltage to update the capacitance. This yields an estimate of the actual admittance for this specific control output voltage. From figure 7, the estimated admittance is greater than or equal to the actual admittance.

This is consistent with what was stated in section 3.4. The control law used for this example displayed a 20% reduction in structural vibration. For this case, the difference between the actual and the estimated admittance was never greater than 3%. Thus, the structural motion has a negligible effect on the total admittance of the piezoelectric actuator for active vibration control. With an estimate of admittance, the power required by the piezoelectric actuator for active vibration control can easily be calculated.
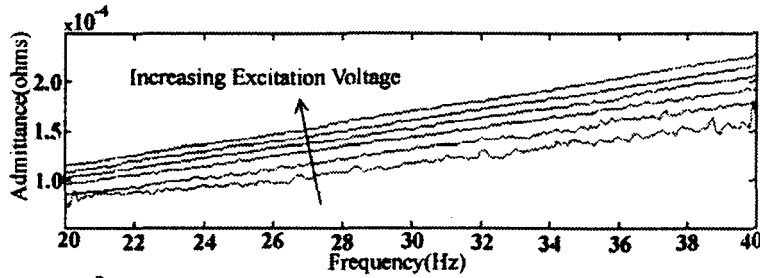

Figure 5: Admittance vs. frequency with increasing excitation voltage.
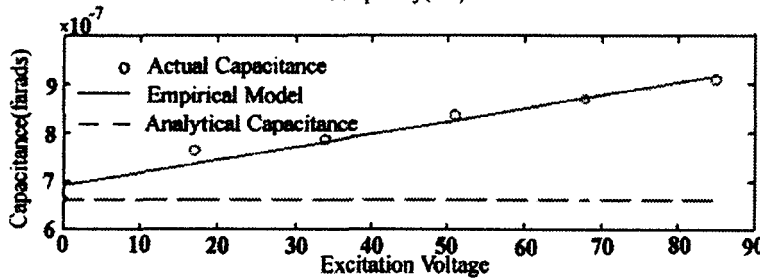
Figure 6: Capacitance vs. excitation voltage for the actual, analytical and the empirical model of capacitance.
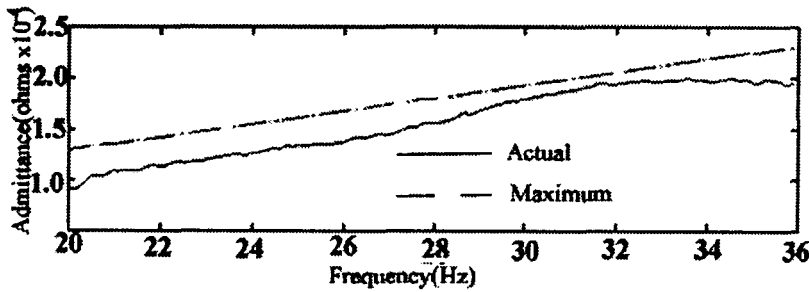
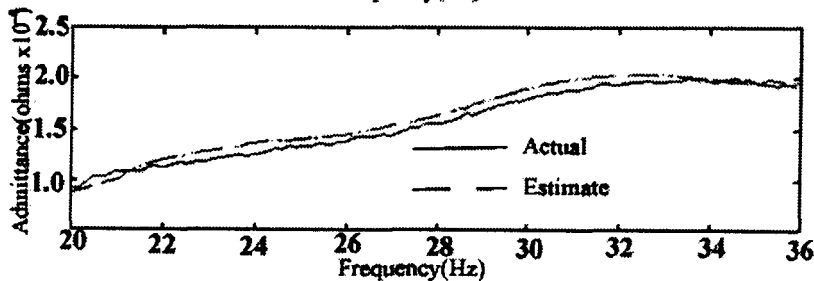Figure 7: Calculated maximum admittance vs. frequency and actual admittance vs. frequency

Figure 8: Estimated admittance vs. frequency and actual admittance vs. frequency. Error is less than or equal to 3%

CONCLUSION

Analytical models of piezoelectric power consumption during active vibration control were developed. These developments showed that piezoelectric power consumption is essentially a function of the material and geometric properties of the piezoelectric actuators. The structural dynamics of the host structure and actuator have a minimal effect on the power consumed by the actuators and in fact, tend to reduce the power consumed by the piezoelectric actuators. Thus, maximum power is consumed when the vibration of the host structure is completely controlled. A single-degree-of-freedom experimental model was used to verify

that a conservative estimate of power consumption can be made by ignoring structural dynamic effects. Experimental results showed that power estimates were greater than actual values by only 3%. In addition to examining piezoelectric power consumption, this research revealed a non-linear behavior in the material properties of piezoelectric materials that showed that piezoelectric material properties increase with voltage. Future work includes a characterization of piezoelectric material properties with a concentration on defining the material non-linearity, validating the power consumption results using multiple-degree-of-freedom experimental models, and an investigation of piezoelectric actuator self-sensing capabilities for feedback control laws.

ACKNOWLEDGEMENTS

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